色彩科學導論 作業六

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Please proof that

$$\frac{1}{2^{2k}}\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i+j)^2 = \frac{2^{2k}-1}{6}$$

Proof

$$\frac{1}{2^{2k}}\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i-j)^2$$

$$=\frac{1}{2^{2k}}\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}(i^2-2ij+j^2)$$

$$=\frac{1}{2^{2k}}(\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}i^2 - 2\sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}ij + \sum_{i=0}^{2^k-1}\sum_{j=0}^{2^k-1}j^2)$$

$$=\frac{1}{2^{2k}}(\sum_{i=0}^{2^k-1}2^k i^2 - 2\sum_{i=0}^{2^k-1}i\frac{(2^k-1)2^k}{2} + \sum_{i=0}^{2^k-1}\frac{(2^k-1)(2^k)(2^{k+1}-1)}{6})$$

$$=\frac{1}{2^{2k}}(2^k\frac{(2^k-1)(2^k)(2^{k+1}-1)}{6} - 2\frac{(2^k-1)2^k}{2}\frac{(2^k-1)2^k}{2}$$

$$+2^k\frac{(2^k-1)(2^k)(2^{k+1}-1)}{6})$$

$$=\frac{(2^k-1)(2^{k+1}-1)}{6} - (2^k-1)\frac{(2^k-1)}{2} + \frac{(2^k-1)(2^{k+1}-1)}{6}$$

$$=(2^k-1)(\frac{2(2^{k+1}-1)}{6} - \frac{(2^k-1)}{2})$$

$$=(2^k-1)(\frac{2(2^{k+1}-1)}{6} - \frac{3(2^k-1)}{6})$$

$$=(2^k-1)(\frac{4*2^k-2-3*2^k+3}{6})$$

$$=\frac{1}{6}(2^k-1)(2^k+1)$$

$$=\frac{2^{2k}-1}{6}$$

End of proof