# Supplementary materials

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What follows includes comprehensive tables from the paper, featuring additional samples experiments. The best results for each metric are highlighted in bold. In addition, the deep learning plots for each dataset have been represented as bar plots instead of line charts to improve the readability.

### 1. Deep learning plots

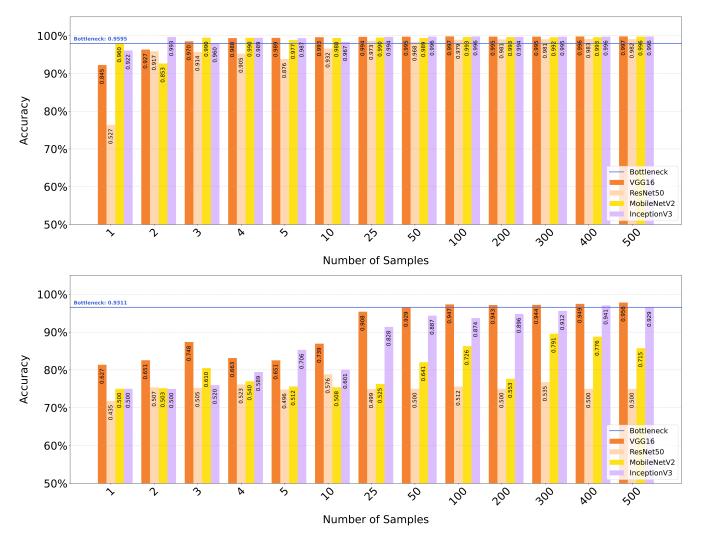


Figure 1: Bar chart illustrating the accuracy comparison between the Bottleneck method and deep learning models across varying numbers of training samples. Above is the Cracks dataset, and below is the Malaria dataset.

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## ${\bf 2.}\,$ Deep learning performances for both Cracks and Malaria dataset

Comples	Method	Cracks dataset				Malaria dataset			
Samples	Method	Acc.	Prec.	Rec.	$\mathbf{F1}$	Acc.	Prec.	Rec.	$\mathbf{F1}$
1	VGG16	0.8446	0.9975	0.6910	0.8164	0.6272	0.7203	0.4158	0.5273
	ResNet50	0.5274	0.9294	0.0592	0.1114	0.4354	0.3976	0.2507	0.3075
	${\bf Mobile Net V2}$	0.9596	0.9952	0.9238	0.9581	0.5000	0.0000	0.0000	0.0000
	${\bf Inception V3}$	0.9221	0.9994	0.8448	0.9156	0.5000	0.5000	0.0004	0.0007
	VGG16	0.9268	0.9991	0.8542	0.9210	0.6511	0.7198	0.4949	0.5865
2	ResNet50	0.9175	0.9997	0.8352	0.9101	0.5074	0.5040	0.9380	0.6557
2	${\bf Mobile Net V2}$	0.8531	0.9982	0.7075	0.8281	0.5031	0.5044	0.3520	0.4146
	${\bf Inception V3}$	0.9934	0.9972	0.9895	0.9933	0.4996	0.4706	0.0058	0.0115
	VGG16	0.9704	0.9813	0.9590	0.9700	0.7476	0.8794	0.5740	0.6946
3	ResNet50	0.9144	1.0000	0.8288	0.9064	0.5049	0.5616	0.0446	0.0827
э	${\bf Mobile Net V2}$	0.9899	0.9905	0.9892	0.9899	0.6098	0.6120	0.5998	0.6058
	InceptionV3	0.9595	0.9984	0.9205	0.9579	0.5198	0.5126	0.8030	0.6258
	VGG16	0.9875	0.9947	0.9802	0.9874	0.6629	0.6061	0.9303	0.7340
4	ResNet50	0.9048	1.0000	0.8095	0.8947	0.5232	0.6151	0.1241	0.2065
4	${\bf Mobile Net V2}$	0.9896	0.9868	0.9925	0.9897	0.5399	0.5915	0.2580	0.3593
	${\bf Inception V3}$	0.9891	0.9895	0.9888	0.9891	0.5889	0.6259	0.4419	0.5181
	VGG16	0.9889	0.9952	0.9825	0.9888	0.6506	0.6123	0.8211	0.7015
5	ResNet50	0.8762	1.0000	0.7525	0.8588	0.4962	0.4981	0.9844	0.6615
5	${\bf Mobile Net V2}$	0.9771	0.9704	0.9842	0.9773	0.5125	0.7226	0.0406	0.0769
	InceptionV3	0.9872	0.9974	0.9770	0.9871	0.7061	0.7363	0.6422	0.6860
	VGG16	0.9925	0.9982	0.9868	0.9925	0.7388	0.8142	0.6187	0.7031
10	ResNet50	0.9321	0.9997	0.8645	0.9272	0.5764	0.5619	0.6930	0.6206
10	${\bf Mobile Net V2}$	0.9884	0.9853	0.9915	0.9884	0.5082	0.7922	0.0221	0.0431
	${\bf Inception V3}$	0.9671	0.9387	0.9995	0.9682	0.6014	0.6206	0.5218	0.5669
	VGG16	0.9942	0.9992	0.9892	0.9942	0.9078	0.9599	0.8512	0.9023
25	ResNet50	0.9732	0.9971	0.9492	0.9726	0.4987	0.4720	0.0214	0.0410
20	${\bf Mobile Net V2}$	0.9904	0.9871	0.9938	0.9904	0.5254	0.7134	0.0849	0.1518
	InceptionV3	0.9942	0.9952	0.9932	0.9942	0.8275	0.8302	0.8233	0.8267

Table 1: Performance metrics for deep learning models for the Cracks and Malaria datasets across various number of samples per class ranging from 1 to 25.

Samples	25.12.1	Cracks dataset				Malaria dataset				
	Method	Acc.	Prec.	Rec.	$\mathbf{F1}$	Acc.	Prec.	Rec.	$\mathbf{F1}$	
50	VGG16	0.9952	0.9965	0.9940	0.9952	0.9287	0.9084	0.9536	0.9304	
	ResNet50	0.9676	0.9973	0.9378	0.9666	0.5002	0.5001	0.9989	0.6665	
	MobileNetV2	0.9886	0.9897	0.9875	0.9886	0.6410	0.6462	0.6230	0.6344	
	InceptionV3	0.9955	0.9945	0.9965	0.9955	0.8872	0.8949	0.8774	0.8860	
	VGG16	0.9965	0.9985	0.9945	0.9965	0.9465	0.9624	0.9292	0.9455	
100	ResNet50	0.9785	0.9948	0.9620	0.9781	0.5118	0.5337	0.1869	0.2768	
100	MobileNetV2	0.9934	0.9920	0.9948	0.9934	0.7257	0.9430	0.4804	0.6365	
	InceptionV3	0.9959	0.9955	0.9962	0.9959	0.8743	0.8799	0.8668	0.8733	
	VGG16	0.9954	0.9945	0.9962	0.9954	0.9430	0.9685	0.9158	0.9414	
200	ResNet50	0.9806	0.9921	0.9690	0.9804	0.5000	0.5000	1.0000	0.6667	
200	MobileNetV2	0.9930	0.9893	0.9968	0.9930	0.5530	0.9966	0.1063	0.1921	
	InceptionV3	0.9940	0.9933	0.9948	0.9940	0.8960	0.8905	0.9031	0.8968	
	VGG16	0.9951	0.9945	0.9958	0.9951	0.9441	0.9356	0.9539	0.9447	
300	ResNet50	0.9811	0.9931	0.9690	0.9809	0.5352	0.6054	0.2021	0.3030	
300	MobileNetV2	0.9921	0.9886	0.9958	0.9922	0.7915	0.9763	0.5976	0.7414	
	InceptionV3	0.9952	0.9918	0.9988	0.9953	0.9120	0.9192	0.9035	0.9113	
	VGG16	0.9962	0.9940	0.9985	0.9963	0.9492	0.9602	0.9372	0.9486	
400	ResNet50	0.9825	0.9906	0.9742	0.9824	0.5000	0.0000	0.0000	0.0000	
400	MobileNetV2	0.9925	0.9920	0.9930	0.9925	0.7759	0.9929	0.5559	0.7127	
	InceptionV3	0.9956	0.9945	0.9968	0.9956	0.9410	0.9606	0.9198	0.9398	
	VGG16	0.9965	0.9955	0.9975	0.9965	0.9563	0.9708	0.9409	0.9556	
500	ResNet50	0.9820	0.9876	0.9762	0.9819	0.4998	0.4999	0.9996	0.6665	
500	MobileNetV2	0.9955	0.9935	0.9975	0.9955	0.7150	0.9837	0.4372	0.6054	
	InceptionV3	0.9959	0.9958	0.9960	0.9959	0.9291	0.9368	0.9202	0.9284	

Table 2: Performance metrics for deep learning models for the Cracks and Malaria datasets across various number of samples per class ranging from 50 to 500.

## 3. Machine learning (w/wo Persistent homology) for the Cracks dataset

		Cracks dataset							
Samples	Method	ML				persIm ML			
		Acc.	Prec.	Rec.	$\mathbf{F1}$	Acc.	Prec.	Rec.	$\mathbf{F1}$
	Log Reg	0.6940	0.6999	0.6792	0.6894	0.9140	0.9952	0.8320	0.9063
1	SVM	0.6940	0.6999	0.6792	0.6894	0.9140	0.9952	0.8320	0.9063
	RF	0.5919	0.6284	0.4498	0.5243	0.7220	1.0000	0.4440	0.6150
	Log Reg	0.7522	0.8171	0.6500	0.7240	0.9440	0.9625	0.9240	0.9429
5	SVM	0.7704	0.7299	0.8585	0.7890	0.9600	0.9528	0.9680	0.9603
	RF	0.6738	0.7395	0.5365	0.6218	0.8800	0.9750	0.7800	0.8667
	Log Reg	0.7492	0.7726	0.7065	0.7381	0.9280	0.9908	0.8640	0.9231
10	SVM	0.8080	0.8596	0.7362	0.7932	0.9780	0.9723	0.9840	0.9781
	RF	0.7151	0.7318	0.6792	0.7045	0.4960	0.4980	0.9880	0.6622
	Log Reg	0.7562	0.7889	0.6998	0.7417	0.9540	0.9595	0.9480	0.9537
25	SVM	0.8635	0.8444	0.8912	0.8672	0.9620	0.9358	0.9920	0.9631
	RF	0.7061	0.6917	0.7438	0.7168	0.9660	0.9569	0.9760	0.9663
	Log Reg	0.8028	0.8421	0.7452	0.7907	0.9680	0.9570	0.9800	0.9684
50	SVM	0.8966	0.8959	0.8975	0.8967	0.972	0.9574	0.988	0.9724
	RF	0.7569	0.7510	0.7685	0.7597	0.9680	0.9570	0.9800	0.9684
	Log Reg	0.8336	0.8919	0.7592	0.8203	0.9680	0.9606	0.9760	0.9683
100	SVM	0.9430	0.9847	0.9000	0.9404	0.9700	0.9572	0.9840	0.9704
	RF	0.8934	0.9319	0.8488	0.8884	0.9680	0.9570	0.9800	0.9684
	Log Reg	0.8669	0.9467	0.7775	0.8538	0.9740	0.9798	0.9680	0.9738
200	SVM	0.9559	0.9810	0.9298	0.9547	0.9760	0.9798	0.9720	0.9759
	RF	0.9461	0.9832	0.9078	0.9440	0.9760	0.9648	0.9880	0.9763
	Log Reg	0.8459	0.8918	0.7872	0.8363	0.9576	0.9607	0.9542	0.9575
300	SVM	0.9549	0.9822	0.9265	0.9536	0.9654	0.9600	0.9712	0.9656
	RF	0.9296	0.9280	0.9315	0.9298	0.9648	0.9523	0.9785	0.9652
400	Log Reg	0.8721	0.9288	0.8060	0.8631	0.9616	0.9622	0.9610	0.9616
	SVM	0.9586	0.9784	0.9380	0.9578	0.9674	0.9574	0.9782	0.9677
	RF	0.9596	0.9698	0.9488	0.9592	0.9620	0.9538	0.9710	0.9623
500	Log Reg	0.8734	0.9290	0.8085	0.8646	0.9614	0.9547	0.9688	0.9617
	SVM	0.9582	0.9801	0.9355	0.9573	0.9670	0.9532	0.9822	0.9675
	RF	0.9668	0.9799	0.9530	0.9663	0.9585	0.9526	0.9650	0.9588

Table 3: Performance metrics for machine learning models for the Cracks dataset across various number of samples per class, applied to raw and persistence images.

## 4. Machine learning performances for the Malaria dataset

		Malaria dataset								
Samples	Method	$\mathbf{ML}$				persIm ML				
		Acc.	Prec.	Rec.	$\mathbf{F1}$	Acc.	Prec.	Rec.	$\mathbf{F1}$	
	Log Reg	0.3881	0.4007	0.4517	0.4247	0.9087	0.9581	0.8549	0.9035	
1	SVM	0.3881	0.4007	0.4517	0.4247	0.9089	0.9574	0.8560	0.9038	
	RF	0.3975	0.4087	0.4590	0.4324	0.6063	0.9508	0.2242	0.3629	
	Log Reg	0.6020	0.6200	0.5269	0.5696	0.8959	0.9576	0.8284	0.8883	
5	SVM	0.6009	0.6161	0.5352	0.5728	0.9272	0.9308	0.9231	0.9269	
	RF	0.6216	0.6144	0.6528	0.6330	0.8641	0.9593	0.7605	0.8484	
	Log Reg	0.5709	0.5821	0.5029	0.5396	0.9082	0.9518	0.8599	0.9035	
10	SVM	0.5776	0.6192	0.4035	0.4886	0.9311	0.9314	0.9307	0.931	
	RF	0.5923	0.5900	0.6052	0.5975	0.9151	0.9525	0.8737	0.9114	
	Log Reg	0.5847	0.5888	0.5617	0.5749	0.8988	0.9541	0.8378	0.8922	
25	SVM	0.6272	0.6699	0.5015	0.5736	0.9311	0.9314	0.9307	0.931	
	RF	0.6477	0.6709	0.5798	0.6220	0.9156	0.9540	0.8734	0.9119	
	Log Reg	0.6054	0.6097	0.5860	0.5976	0.9231	0.9397	0.9042	0.9216	
50	SVM	0.5971	0.6322	0.4641	0.5353	0.9311	0.9314	0.9307	0.9310	
	RF	0.6139	0.6098	0.6328	0.6211	0.9111	0.9572	0.8607	0.9064	
	Log Reg	0.6306	0.6343	0.6168	0.6255	0.9066	0.9491	0.8592	0.9019	
100	SVM	0.6549	0.6545	0.6564	0.6554	0.9311	0.9314	0.9307	0.931	
	RF	0.6798	0.6981	0.6335	0.6643	0.9165	0.9505	0.8788	0.9133	
	Log Reg	0.6179	0.6270	0.5824	0.6038	0.9142	0.9499	0.8745	0.9106	
200	SVM	0.6646	0.6679	0.6546	0.6612	0.9307	0.9332	0.9278	0.9305	
	RF	0.6856	0.7007	0.6480	0.6733	0.9260	0.9437	0.9060	0.9245	
	Log Reg	0.6226	0.6327	0.5849	0.6078	0.9242	0.9452	0.9006	0.9223	
300	SVM	0.6704	0.6667	0.6814	0.6740	0.9309	0.9317	0.9300	0.9308	
	RF	0.6836	0.6869	0.6749	0.6808	0.9251	0.9483	0.8991	0.9231	
400	Log Reg	0.6313	0.6360	0.6143	0.6250	0.9189	0.9415	0.8933	0.9168	
	SVM	0.6782	0.6783	0.6777	0.6780	0.9312	0.9325	0.9282	0.9304	
	RF	0.6877	0.6935	0.6628	0.6778	0.9244	0.9479	0.8985	0.9222	
500	Log Reg	0.6136	0.6220	0.5791	0.5998	0.9224	0.9429	0.8991	0.9205	
	SVM	0.6711	0.6597	0.7068	0.6824	0.9300	0.9322	0.9274	0.9298	
	RF	0.7037	0.7032	0.7050	0.7041	0.9276	0.9416	0.9118	0.9265	

Table 4: Performance metrics for machine learning models for the Malaria dataset across various number of samples per class, applied to raw and persistence images.