ML Exercise Sheet 4

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Exercise 1.1

- To be proven :
$$\sum_{n=1}^{N}(x_n - \bar{x})(y_n - \bar{y}) = \sum_{n=1}^{N}(y_n - \bar{y})x_n$$

- Start from the left side
$$\begin{aligned}\sum_{n=1}^{N}(x_n - \bar{x})(y_n - \bar{y}) &= \sum_{n=1}^{N} x_n y_n - x_n\bar{y} - \bar{x}y_n + \bar{x}\bar{y} \\ &= \sum_{n=1}^{N} \left(x_n y_n - x_n\bar{y}\right) - \sum_{n=1}^{N}\bar{x}y_n + \sum_{n=1}^{N}\bar{x}\bar{y}\end{aligned}$$

- We show that
$$-\sum_{n=1}^{N}\bar{x}y_n + \sum_{n=1}^{N}\bar{x}\bar{y} = 0$$
then the claim would be verified
$$\begin{aligned}-\sum_{n=1}^{N}\bar{x}y_n + \sum_{n=1}^{N}\bar{x}\bar{y} &= -\sum_{n=1}^{N}\bar{x}y_n + N\cdot\frac{1}{N}\left(\sum_{i=1}^{N}x_i\right)\cdot\frac{1}{N}\left(\sum_{i=1}^{N}y_i\right) \\ &= -\sum_{n=1}^{N}\frac{1}{N}\left(\sum_{i=1}^{N}x_i\right)y_n + \frac{1}{N}\left(\sum_{i=1}^{N}x_i\right)\cdot\left(\sum_{i=1}^{N}y_i\right) \\ &= -\frac{1}{N}\left(\sum_{n=1}^{N}\sum_{i=1}^{N}x_i y_n - \left(\sum_{i=1}^{N}x_i\right)\left(\sum_{i=1}^{N}y_i\right)\right)\end{aligned}$$

- Because of the distributive property
$$\left(\sum_{i=1}^{N}x_i\right)\left(\sum_{i=1}^{N}y_i\right) = \left(\sum_{i=1}^{N}x_i\right)(y_1 + y_2 + \ldots + y_N) = \sum_{n=1}^{N}\left(\sum_{i=1}^{N}x_i\right)\cdot y_n$$

- That's why

$$
\begin{aligned}
-\frac{1}{N}\left(\sum_{n=1}^{N}\sum_{i=1}^{N} x_i y_n - \left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right)\right) &= -\frac{1}{N}\left(\sum_{n=1}^{N}\sum_{i=1}^{N} x_i y_n - \sum_{n=1}^{N}\sum_{i=1}^{N} x_i y_n\right) \\
&= -\frac{1}{N}\cdot 0 \\
&= 0 \qquad \text{(Which is what we wanted to show)}
\end{aligned}
$$