ML Exercise Sheet 4

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Exercise 1.1

- To be proven :

$$\sum_{n=1}^{N}(x_n - \bar{x})(y_n - \bar{y}) = \sum_{n=1}^{N}(y_n - \bar{y})y_n$$

- Start from the left side

$$\sum_{n=1}^{N}(x_n - \bar{x})(y_n - \bar{y}) = \sum_{n=1}^{N} x_n y_n - x_n \bar{y} - \bar{x} y_n + \bar{x}\bar{y}$$

$$= \sum_{n=1}^{N}\left(x_n y_n - x_n \bar{y}\right) - \sum_{n=1}^{N} \bar{x} y_n + \sum_{n=1}^{N} \bar{x}\bar{y}$$

- We show that

$$-\sum_{n=1}^{N} \bar{x} y_n + \sum_{n=1}^{N} \bar{x}\bar{y} = 0$$

then the claim would be verified

$$-\sum_{n=1}^{N} \bar{x} y_n + \sum_{n=1}^{N} \bar{x}\bar{y} = -\sum_{n=1}^{N} \bar{x} y_n + N \cdot \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right) \cdot \frac{1}{N}\left(\sum_{i=1}^{N} y_i\right)$$

$$= -\sum_{n=1}^{N} \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right) y_n + \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right) \cdot \left(\sum_{i=1}^{N} y_i\right)$$

$$= -\frac{1}{N}\left(\sum_{n=1}^{N}\sum_{i=1}^{N} x_i y_n - \left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right)\right)$$

- Because of the distributive property

$$\left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right) = \left(\sum_{i=1}^{N} x_i\right)(y_1 + y_2 + \ldots + y_N) = \sum_{n=1}^{N}\left(\sum_{i=1}^{N} x_i\right) \cdot y_n$$

- That's why

$$-\frac{1}{N}\left(\sum_{n=1}^{N}\sum_{i=1}^{N}x_iy_n - \left(\sum_{i=1}^{N}x_i\right)\left(\sum_{i=1}^{N}y_i\right)\right) = -\frac{1}{N}\left(\sum_{n=1}^{N}\sum_{i=1}^{N}x_iy_n - \sum_{n=1}^{N}\sum_{i=1}^{N}x_iy_n\right)$$

$$= -\frac{1}{N}\cdot 0$$

$$= 0 \qquad \text{(Which is what we wanted to show)}$$