1. Let S, *T* be non-empty sets. Show that a function f: $S_\omega \to T_\omega$ is monotonic iff it is a strict or a constant function. Does this also hold for functions f: $S_\omega{}^n \to T_\omega$ with $n > 1$?

2. Suppose $\mathscr{I}$ is the 'usual' interpretation. In particular the symbols $\wedge$, $\vee$, and $\neg$ represent the strict ω-extensions of their usual meaning. Which of the following pairs are simplification schemes for $\mathscr{I}$? Which are schemes of the standard simplification rule for $\mathscr{I}$?
   (a) $(\neg\neg x, x)$ (b) (x $\vee$ *true, true)*
   (c) (x $\wedge\neg$x, *false)* (d) $(x \wedge x, x)$
   What happens when the symbols $\wedge$ and $\vee$ represent non-strict ω-extensions of their usual meaning ?

3. Let $\mathscr{I}$ be the 'usual' interpretation and let $\gamma \in \Gamma$ be an assignment with $\gamma(x) = 3$ and $\gamma(y) = 4$.
   For each of the following λ-terms *t* calculate $\mathscr{I}(t)(\gamma)$:
   (i) $[\lambda x,y.(x + y)]$ (ii) $[\lambda x.[\lambda y.\ (x + y)]]$
   (iii) $[\lambda x.[\lambda x.(x + y)]]$ (iv) $[\lambda x.[\lambda y.\ (x + y)]](y)$
   (v) $[\lambda x.[\lambda y.(x + y)](y)]$

4. Let B = (F, P) be a basis for predicate logic with F = {0,1,...,+} and P = {$\leq$}. Let $\mathscr{I}$ be an usual interpretation for B. Given following while program:

   ```
   y1:=0; y2:=1; y3:=1;
   while y3<=x do
       y1:= y1+1;
       y2:=y2+2;
       y3:=y3+y2
   od
   ```

   Try to give an example of a computation of this program according to its abstract machine semantics.

5. Using usual interpretation over natural numbers, what is the semantic functional Φ of the following while program?

   ```
   while x!=0 do x:=x-1 od
   ```

   And what is the semantics of this program?

6. Prove that
   if $\sigma(x) \leq 6$ then
   $\mathscr{I}$(while $x \leq 5$ do x := x + 1)(σ) = σ[x /6].
   [hint: by induction, by operational semantics]

7. What is the weakest precondition of the program x:=x+1 and postcondition x>=60? What is the weakest liberal precondition?

8. Prove that *{true} while x!=10 do x++ od {x=10}* is a valid Hoare formula in usual interpretation.

9. Prove that
   $$\frac{p \to r, \{r \wedge e\} S \{r\}, (r \wedge \neg e) \to q}{\{p\} while\ e\ do\ S\ od\ \{q\}} \quad \text{for all } p,r,q \in WFF_B,\ e \in QFF_B$$
   is a valid derivation rule for Hoare calculus.

10. Let S be a program which is totally correct with respect to the formulas x=y and x = y! in the usual interpretation (as Natural numbers). Under which conditions does S calculate the factorial function? (Note that the variable y may occur in S.)

11. What does it mean for a program to be partially or totally correct with respect to the following pairs of formulas:
(i) *true* and true; (ii) *true* and *false;*
(iii) *false* and *true;* (iv) *false* and *false.*
What does it mean for a program to terminate with respect to the formula:
(v) *true;* (vi) *false.*

12. Let S be a program, which is totally correct with respect to the formulas $x = y$ and $x = y!$ (in the usual interpretation). Under which conditions does S calculate the factorial function? (Note that the variable *y* may occur in S.)

13. Let S be the recursive program
F(x)$\Leftarrow$if x = 0 then 1 else x$*$F(x–1) fi.
Write out the steps of simplifications and substitutions for F(3) call.

14. According to the denotation semantics, gire the definition of the semantic functional for the following recursive program.
F1(x)$\Leftarrow$if x = 0 then 0 else F2(x–1) fi.
F2(x)$\Leftarrow$if x = 0 then 1 else F1(x–1) fi.
with F1 as the main function variable.

15. *The inductive definition for while-programs in our class is not free. Prove that nevertheless the ω-extended meaning of a while-program—which is based on this inductive definition—is well-defined.