

McMASTER UNIVERSITY

ELEC ENG 4CL4

CONTROL SYSTEMS DESIGN - FALL 2018

Phase 4 - State Feedback Control Synthesis of Inverted Pendulum

Decemeber 3rd, 2018

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Abstract

For phase 4, the observer was implemented for the inverted pendulum model. In phase 4, concepts of state feedback control and control using a state estimator were introduced. First, the open loop response of the linearized system was analyzed for stability. This showed that the system was not stable due to poles in the right half plane. In order to stabilize the system response, state feedback control was introduced. This involved adding a gain controller to the closed loop response, so that the A matrix was replaced with $A-BK$. By calculating values of the K matrix, poles were moved to desired locations in the left half plane, stabilizing the system. To design the observer, a similar method was implemented. However, to make the system stable the observer poles were placed to converge ten times faster than the feedback controller. This pole placement minimizes the error from the observer. Two kinds of observers were simulated (state-estimator and numerical differentiator). The results indicated that a state estimator is a better design for an observer.

1 Equilibrium Positions

Case One

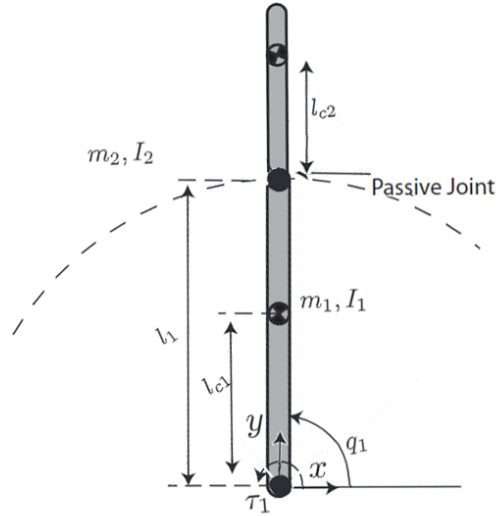


Figure 1: Equilibrium Case One Position of Dual Link Inverted Pendulum

Case Two

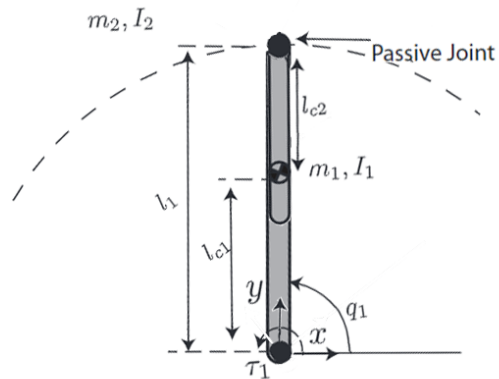


Figure 2: Equilibrium Case Two Position of Dual Link Inverted Pendulum

2 State Estimator (Observer)

2.1 Observability of both equilibrium positions.

using the Matlab function `obsv(A,C)` we were able to analyze the observability of both equilibrium positions within the linearized dynamics. For position one we have the following matrices A_{pos1} ,

$$A_{pos1} = \begin{bmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 1.8600 & -4.8679 & -3.3267 & 0.0033 \\ 115.1393 & 141.3019 & 12.9365 & -0.0739 \end{bmatrix} \quad (1)$$

$$A_{pos2} = \begin{bmatrix} 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 1.8600 & -4.8679 & -3.3267 & -0.0016 \\ -118.8593 & -131.5660 & -6.2831 & -0.0642 \end{bmatrix} \quad (2)$$

with the consistent C matrix defined by,

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (3)$$

the observability matrix for position one was,

$$P_{observ1} = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 1.8600 & -4.8679 & -3.3267 & 0.0033 \\ 115.1393 & 141.3019 & 12.9365 & -0.0739 \\ -5.8106 & 16.6571 & 12.9695 & -4.8791 \\ 15.5524 & -73.4171 & 71.1469 & 141.3497 \end{bmatrix} \quad rank(P_{observ1}) = 4 \quad (4)$$

while the observability for position two was,

$$P_{observ2} = \begin{bmatrix} 1.0000 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 1.8600 & -4.8679 & -3.3267 & -0.0016 \\ -118.8593 & -131.5660 & -6.2831 & -0.0642 \\ -5.9986 & 16.4036 & 12.9371 & -4.8625 \\ -4.0589 & 39.0285 & -97.5540 & -131.5519 \end{bmatrix} \quad rank(P_{observ2}) = 4 \quad (5)$$

Therefore, from the results of the observability test we can conclude that the linearization of both equilibrium positions are observable.

2.2 Designing The Observer

To design the observer, we place the eigen values of the new system A-LC such the the observer error reaches zero much faster than the state feedback controller. To do this, we use the function $place(A^T, C^T, p)$ where p are the pole positions 10 times larger than the state feedback controller such to reduce error. The values for the unknown angular velocities will be calculated via numerical integration. The gain of the observer for both positions are as follows.

$$P_{o1} = \begin{bmatrix} 219.2 & 24.08 \\ 38.64 & 227.4 \\ 11304.06 & 2512.8 \\ 5727.29 & 13004.71 \end{bmatrix} \quad P_{o2} = \begin{bmatrix} 220.53 & -24.02 \\ -32.25 & 226.08 \\ 11443.12 & -2518.21 \\ -4307.87 & 12430.02 \end{bmatrix} \quad (6)$$

Based on these results as well as the results from phase 3 the observer was designed as such.

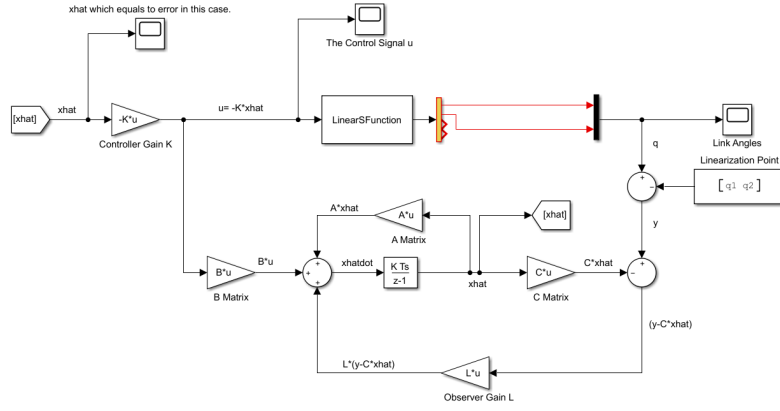
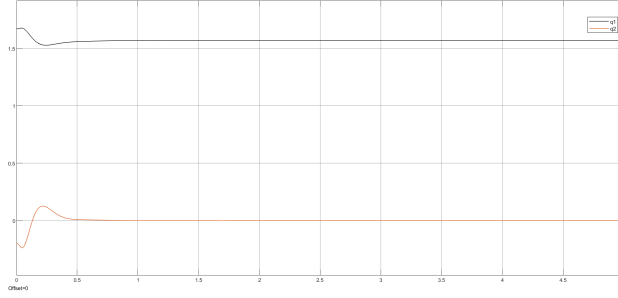


Figure 3: Closed loop observer and state feedback controller Simulink design

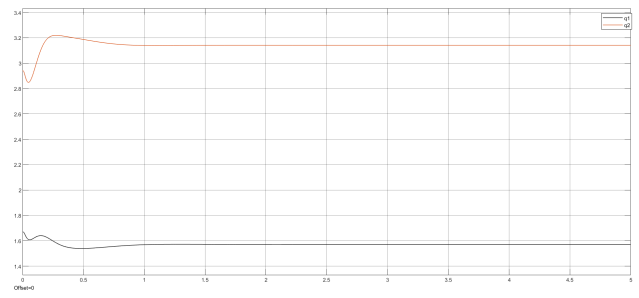
3 State-feedback Control using State Estimator

3.1 Simulating Linearized Equilibrium Positions

When simulating the linearized model with the observer/state-estimator design, We observed the same response as the non-linear model with the state-estimator/observer. Please note that for these simulations, q1 is in black and q2 is in red.



(a) Linearized model at equilibrium point 1 simulation.



(b) Linearized model at equilibrium point 2 simulation.

as we can see from the figures above, both links reach equilibrium quite fast.

3.2 Simulating Full Non-Linear Model

To simulate the full non-linear model, we had to swap the LinearSFunction from figure 3 with a non-linear sfunction. The simulation setup is displayed below.

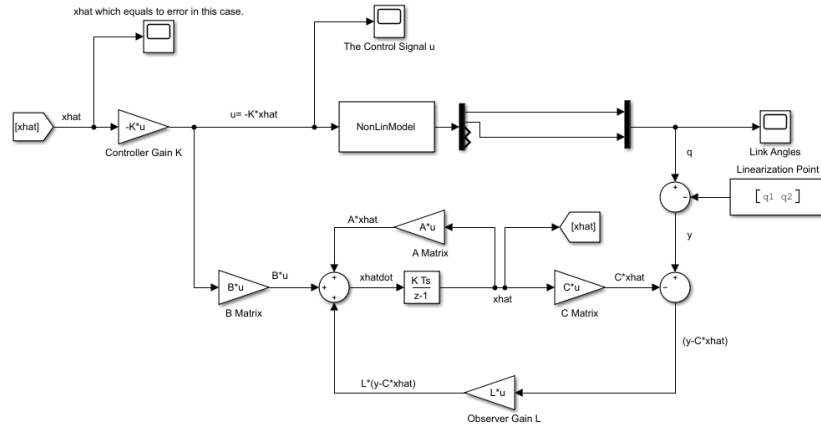
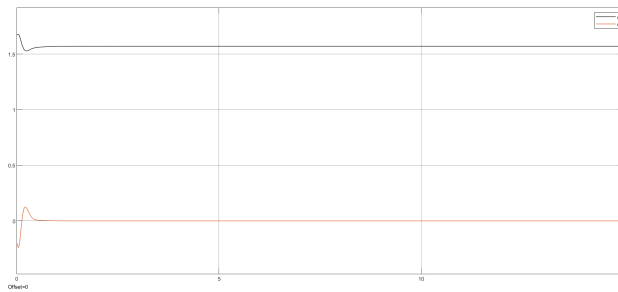
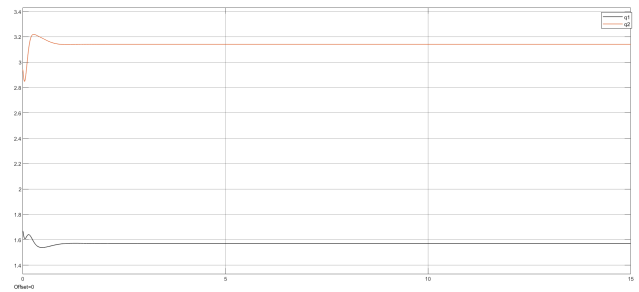


Figure 5: Closed loop observer and state feedback controller Simulink design for non-linear model



(a) Non-Linear model at equilibrium point 1 simulation.



(b) Non-Linear model at equilibrium point 2 simulation.

3.3 Simulating Non-Linear Model with Numerical Differentiator

The second task of this section was to simulate the nonlinear model with a numerical differentiator. The Simulink setup is shown below.

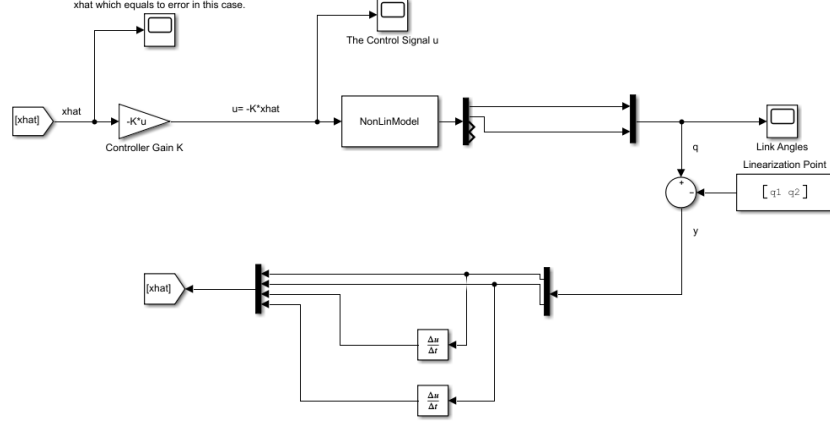
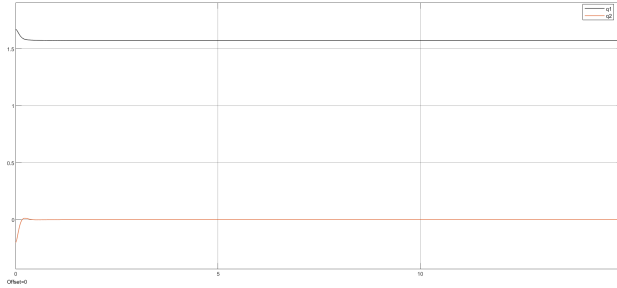
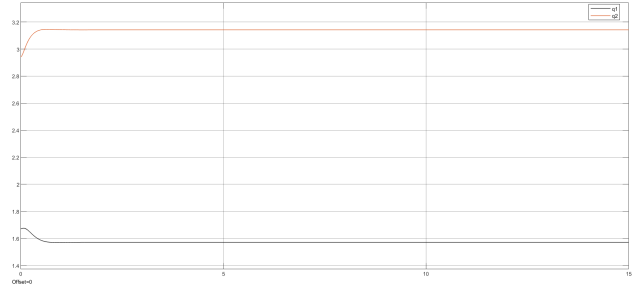


Figure 7: Closed loop observer and state feedback controller Simulink design for non-linear model with a numerical differentiator



(a) Non-Linear model at equilibrium point 1 simulation with numerical differentiator.



(b) Non-Linear model at equilibrium point 2 simulation with numerical differentiator.

Again, the results for the numerical differentiator agree with all previous simulations. However, one thing to note with a numerical differentiator is that the system is very sensitive to noise. We go on to simulate the system with added gaussian white noise and for both inputs the system is highly unstable.

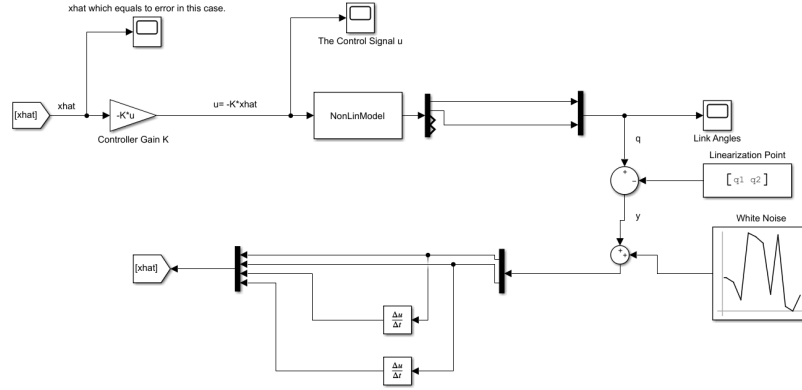
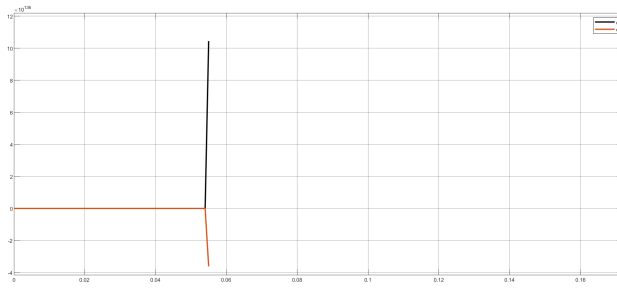
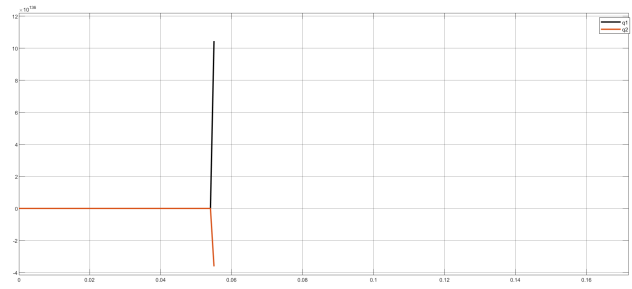


Figure 9: Closed loop observer and state feedback controller Simulink design for non-linear model with a numerical differentiator and input noise



(a) Non-Linear model at equilibrium point 1 simulation with numerical differentiator and input noise.

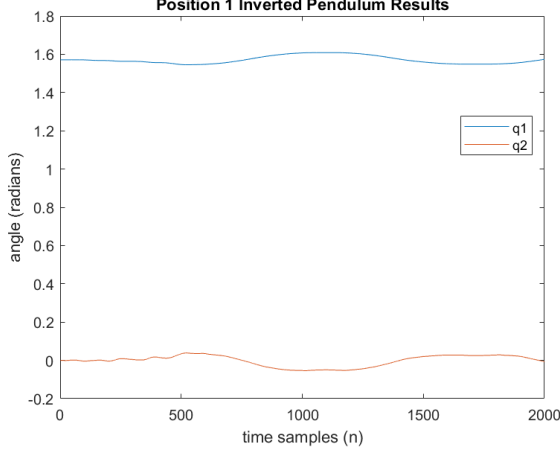


(b) Non-Linear model at equilibrium point 2 simulation with numerical differentiator and input noise.

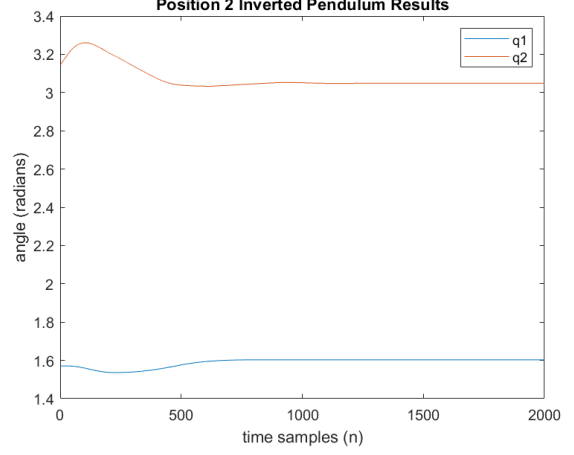
As you can tell from the simulation results, the presence of noise in the observer produces high instability because the differentiators essentially amplify the noise.

4 Experimental Evaluation of the Controllers

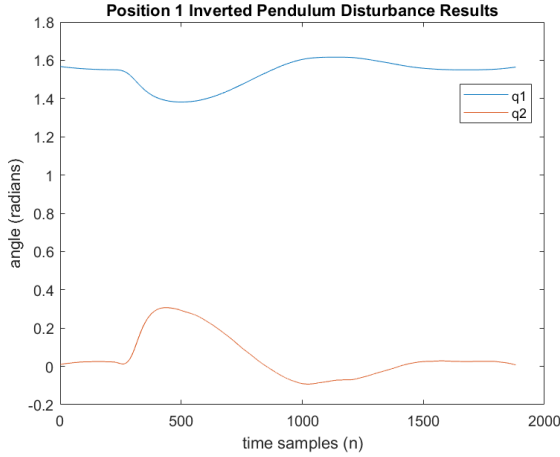
4.1 Results from system actualization



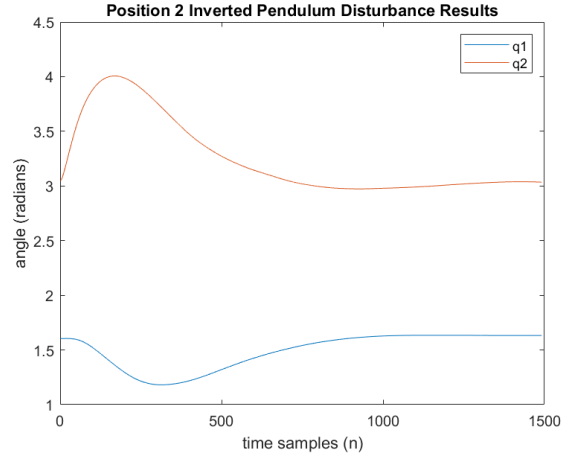
(a) System actualization output at equilibrium position 1.



(b) System actualization output at equilibrium position 2.



(c) System actualization output at equilibrium position 1 with disturbance input.



(d) System actualization output at equilibrium position 2 with disturbance input.

From the results, it is evident that position 2 behaves almost identical to the simulations. However position one has a slight oscillation in its steady state response. This is probably because position one is more unstable than position two. Therefore, when position one reaches equilibrium, it has a greater tendency to slightly deviate and thus have to be rectified by the controller.

4.2 Actualization with a numerical differentiator

Because of the high instability of a numerical differentiator, we did not actually apply it to the physical model. Refer to section 3.3. to see our results in simulation.

5 Results/Conclusion

The results from our simulations and physical actualization concluded that a state-estimator is the best design for an observer for the dual-link inverted pendulum system. It is essential when designing an observer to verify that the system is observable. This is easily accomplished by checking the rank of the observability matrix defined in section 2.1. The observer pole must be place so that the observer system converges much faster than the feedback controller. This minimizes the error in the system and allows the system to perform better under input disturbance.

The physical actualization for position 2 behaved identical to the simulations. However position 1 showed slightly different dynamics. We propose this to be because of the greater instability in position 1 than position 2. Therefore, smaller disturbances in position 1 will cause the system to react. This produces a slight oscillation in the dynamics for equilibrium position one which we do not see in position 2. Both physical actualization's responded as predicted to slight disturbances in equilibrium.

6 Contributions

MICHAEL BAGNOWSKI — 400026446 : SIMULINK MODELS, ABSTRACT , RESULTS/CONCLUSION

ANDY F.GONZALEZ — 400044420 : OBSERVER DESIGN, OBSERVABILITY CALCULATIONS, POLE POSITIONING

Appendix

Observer Design

```
1 %% Linearization Point
2 q1 = pi/2;
3 q2 = 0;
4 g = 9.81;
5 X0=[q1+.1 q2-.2 0 0];
6 T=.001;
7 NewPanto=1;
8 %% System Parameters
9 Parameters = [11.9253 0.4203 0.1455 7.2462 1.8150 35.1492 0.0089];
10 %% Linear System Definition
11 M = [Parameters(1)+2*Parameters(2)*cos(q2), Parameters(3)+Parameters(2)*cos(q2);
12      Parameters(3)+Parameters(2)*cos(q2), Parameters(3)];
13
14 F = [Parameters(6), 0;
15      0, Parameters(7)];
16
17 K = [-Parameters(4)*g*sin(q1)-sin(q1+q2)*g*Parameters(5), -sin(q1+q2)*g*Parameters(5);
18      -sin(q1+q2)*g*Parameters(5), -sin(q1+q2)*g*Parameters(5)];
19
20 A = [zeros(2,2), eye(2,2);
21      -M^-1*K, -M^-1*F];
22
23 B = [zeros(2,2);
24      M^-1]*[1;0];
25
26 C = [1 0 0 0;
27      0 1 0 0];
28
29 %% Stability Analysis
30 EigenValues=eig(A)
31
32 %% Controllability and Controller Gain Design
33 Pc=ctrb(A,B)
34 rank(Pc)
35 p=[-15 -13 -10 -8];
36 K = place(A,B,p)
37 %% Observability and Estimator Gain Design
38 Po=obsv(A,C)
39 rank(Po)
40 po=10*p;
```

```
41 L = transpose(place(A', C',po))
```

Simulation Results

```
1 % simulation results
2 close all;
3 %% position 1
4 pos1 = load('equilibriumpositon_1_realsim.mat','outputy');
5 pos1 = pos1.outputy.signals.values;
6 figure;
7 range = 1:2000;
8 disturbance = 43170:45050;
9 pos1(:,1) = pos1(:,1) + pi/2;
10 plot(pos1(range,:));
11 title('Position 1 Inverted Pendulum Results');
12 xlabel("time samples (n)");
13 ylabel("angle (radians)");
14 legend("q1","q2");
15 figure;
16 plot(pos1(disturbance,:));
17 title('Position 1 Inverted Pendulum Disturbance Results');
18 xlabel("time samples (n)");
19 ylabel("angle (radians)");
20 legend("q1","q2");
21 %% position 2
22 pos2 = load('equilibriumpositon_2_realsim.mat','outputy');
23 pos2 = pos2.outputy.signals.values;
24 disturbance = 38570:40060;
25 figure;
26 range = 1:2000;
27 pos2(:,1) = pos2(:,1) + pi/2;
28 pos2(:,2) = pos2(:,2) + pi;
29 plot(pos2(range,:));
30 title('Position 2 Inverted Pendulum Results');
31 xlabel("time samples (n)");
32 ylabel("angle (radians)");
33 legend("q1","q2");
34 figure;
35 plot(pos2(disturbance,:));
36 title('Position 2 Inverted Pendulum Disturbance Results');
37 xlabel("time samples (n)");
38 ylabel("angle (radians)");
39 legend("q1","q2");
```

Non-Linear S-Function

```

1  function [sys,x0,str,ts,simStateCompliance] = NonLinModel(t,x,u,↵
    flag,x0,Parameters)
2  %SFUNTMPL General MATLAB S-Function Template
3  %   With MATLAB S-functions, you can define you own ordinary ↵
    differential
4  %   equations (ODEs), discrete system equations, and/or just about
5  %   any type of algorithm to be used within a Simulink block ↵
    diagram.
6  %
7  %   The general form of an MATLAB S-function syntax is:
8  %       [SYS,X0,STR,TS,SIMSTATECOMPLIANCE] = SFUNC(T,X,U,FLAG,P1↵
    ,...,Pn)
9  %
10 %   What is returned by SFUNC at a given point in time, T, depends↵
    on the
11 %   value of the FLAG, the current state vector, X, and the ↵
    current
12 %   input vector, U.
13 %
14 %   FLAG      RESULT          DESCRIPTION
15 %   -----
16 %   0          [SIZES,X0,STR,TS] Initialization, return system sizes ↵
    in SYS,
17 %                                     initial state in X0, state ordering ↵
    strings
18 %                                     in STR, and sample times in TS.
19 %   1          DX              Return continuous state derivatives ↵
    in SYS.
20 %   2          DS              Update discrete states SYS = X(n+1)
21 %   3          Y              Return outputs in SYS.
22 %   4          TNEXT          Return next time hit for variable ↵
    step sample
23 %                                     time in SYS.
24 %   5                                     Reserved for future (root finding).
25 %   9          []             Termination, perform any cleanup SYS↵
    =[].
26 %
27 %
28 %   The state vectors, X and X0 consists of continuous states ↵
    followed
29 %   by discrete states.
30 %
31 %   Optional parameters, P1,...,Pn can be provided to the S-↵
    function and

```

```

32 %      used during any FLAG operation.
33 %
34 %      When SFUNC is called with FLAG = 0, the following information
35 %      should be returned:
36 %
37 %          SYS(1) = Number of continuous states.
38 %          SYS(2) = Number of discrete states.
39 %          SYS(3) = Number of outputs.
40 %          SYS(4) = Number of inputs.
41 %          Any of the first four elements in SYS can be ←
specified
42 %          as -1 indicating that they are dynamically sized. ←
The
43 %          actual length for all other flags will be equal to←
the
44 %          length of the input, U.
45 %          SYS(5) = Reserved for root finding. Must be zero.
46 %          SYS(6) = Direct feedthrough flag (1=yes, 0=no). The s-←
function
47 %          has direct feedthrough if U is used during the ←
FLAG=3
48 %          call. Setting this to 0 is akin to making a ←
promise that
49 %          U will not be used during FLAG=3. If you break the←
promise
50 %          then unpredictable results will occur.
51 %          SYS(7) = Number of sample times. This is the number of rows←
in TS.
52 %
53 %
54 %          X0      = Initial state conditions or [] if no states.
55 %
56 %          STR      = State ordering strings which is generally ←
specified as [].
57 %
58 %          TS       = An m-by-2 matrix containing the sample time
59 %          (period, offset) information. Where m = number of ←
sample
60 %          times. The ordering of the sample times must be:
61 %
62 %          TS = [0      0,      : Continuous sample time.
63 %               0      1,      : Continuous, but fixed in ←
minor step
64 %               sample time.
65 %               PERIOD OFFSET, : Discrete sample time where
66 %               PERIOD > 0 & OFFSET < ←
PERIOD.

```

```

67 %             -2      0];           : Variable step discrete ←
sample time
68 %                                     where FLAG=4 is used to get←
time of
69 %                                     next hit.
70 %
71 %             There can be more than one sample time providing
72 %             they are ordered such that they are monotonically
73 %             increasing. Only the needed sample times should be
74 %             specified in TS. When specifying more than one
75 %             sample time, you must check for sample hits ←
explicitly by
76 %             seeing if
77 %             abs(round((T-OFFSET)/PERIOD) - (T-OFFSET)/←
PERIOD)
78 %             is within a specified tolerance, generally 1e-8. ←
This
79 %             tolerance is dependent upon your model's sampling ←
times
80 %             and simulation time.
81 %
82 %             You can also specify that the sample time of the S←
-function
83 %             is inherited from the driving block. For functions←
which
84 %             change during minor steps, this is done by
85 %             specifying SYS(7) = 1 and TS = [-1 0]. For ←
functions which
86 %             are held during minor steps, this is done by ←
specifying
87 %             SYS(7) = 1 and TS = [-1 1].
88 %
89 %             SIMSTATECOMPLIANCE = Specifies how to handle this block ←
when saving and
90 %                                     restoring the complete simulation ←
state of the
91 %                                     model. The allowed values are: '←
DefaultSimState',
92 %                                     'HasNoSimState' or 'DisallowSimState'.←
If this value
93 %                                     is not specified, then the block's ←
compliance with
94 %                                     simState feature is set to '←
UnknownSimState'.
95
96
97 % Copyright 1990-2010 The MathWorks, Inc.

```

```

98 %    $Revision: 1.18.2.5 $
99
100 %
101 % The following outlines the general structure of an S-function.
102 %
103 switch flag,
104
105     %%%%%%%%%%%%%%%
106     % Initialization %
107     %%%%%%%%%%%%%%%
108     case 0,
109         [sys,x0,str,ts,simStateCompliance]=mdlInitializeSizes(X0);
110
111     %%%%%%%%%%%%%%%
112     % Derivatives %
113     %%%%%%%%%%%%%%%
114     case 1,
115         sys=mdlDerivatives(t,x,u,X0,Parameters);
116
117     %%%%%%%%%%%%%%%
118     % Update %
119     %%%%%%%%%%%%%%%
120     case 2,
121         sys=mdlUpdate(t,x,u);
122
123     %%%%%%%%%%%%%%%
124     % Outputs %
125     %%%%%%%%%%%%%%%
126     case 3,
127         sys=mdlOutputs(t,x,u);
128
129     %%%%%%%%%%%%%%%
130     % GetTimeOfNextVarHit %
131     %%%%%%%%%%%%%%%
132     case 4,
133         sys=mdlGetTimeOfNextVarHit(t,x,u);
134
135     %%%%%%%%%%%%%%%
136     % Terminate %
137     %%%%%%%%%%%%%%%
138     case 9,
139         sys=mdlTerminate(t,x,u);
140
141     %%%%%%%%%%%%%%%
142     % Unexpected flags %
143     %%%%%%%%%%%%%%%
144     otherwise

```



```

145     DASTudio.error('Simulink:blocks:unhandledFlag', num2str(flag))↵
146     ;
147 end
148
149 % end sfuntmpl
150
151 %
152 %↵
=====

153 % mdlInitializeSizes
154 % Return the sizes, initial conditions, and sample times for the S↵
    -function.
155 %↵
=====

156 %
157 function [sys,x0,str,ts,simStateCompliance]=mdlInitializeSizes(X0)
158
159 %
160 % call simsizes for a sizes structure, fill it in and convert it ↵
    to a
161 % sizes array.
162 %
163 % Note that in this example, the values are hard coded. This is ↵
    not a
164 % recommended practice as the characteristics of the block are ↵
    typically
165 % defined by the S-function parameters.
166 %
167 sizes = simsizes;
168
169 sizes.NumContStates = 4;
170 sizes.NumDiscStates = 0;
171 sizes.NumOutputs = 4;
172 sizes.NumInputs = 1;
173 sizes.DirFeedthrough = 0;
174 sizes.NumSampleTimes = 1; % at least one sample time is needed
175
176 sys = simsizes(sizes);
177
178 %
179 % initialize the initial conditions
180 %
181 x0 = X0;
182

```

```

183 %
184 % str is always an empty matrix
185 %
186 str = [];
187
188 %
189 % initialize the array of sample times
190 %
191 ts = [0 0];
192
193 % Specify the block simStateCompliance. The allowed values are:
194 %     'UnknownSimState', < The default setting; warn and assume ←
        DefaultSimState
195 %     'DefaultSimState', < Same sim state as a built-in block
196 %     'HasNoSimState',   < No sim state
197 %     'DisallowSimState' < Error out when saving or restoring the ←
        model sim state
198 simStateCompliance = 'UnknownSimState';
199
200 % end mdlInitializeSizes
201
202 %
203 %←
        =====

204 % mdlDerivatives
205 % Return the derivatives for the continuous states.
206 %←
        =====

207 %
208 function sys=mdlDerivatives(t,x,u,X0,Parameters)
209 q1=x(1); q2=x(2); q1dot=x(3); q2dot=x(4);
210 g=9.81;
211 bf1=Parameters(6);
212 bf2=Parameters(7);
213 bf1=0;
214 bf2=0;
215
216 % bf1=20/.32*bf2;
217 % kk=.4*.052*9/.4*1000;
218 kk=1;
219 h = -Parameters(2)*sin(q2);
220 d11 = Parameters(1)+2*Parameters(2)*cos(q2);
221 d12 = Parameters(2)*cos(q2)+Parameters(3);
222 d21 = d12;
223 d22 = Parameters(3);

```

```

224 g1 = Parameters(4)*g*cos(q1)+Parameters(5)*g*cos(q1+q2);
225 g2 = Parameters(5)*g*cos(q1+q2);
226 D   = [d11 d12;d21 d22];
227 C   = [h*q2dot    h*(q1dot+q2dot);
228        -h*q1dot    0];
229 G   = [g1;g2];
230
231 sys(1)= x(3);
232 sys(2)= x(4);
233
234 qddot=inv(D)*([u/kk;0]-C*[q1dot;q2dot]-[bf1*q1dot;bf2*q2dot]-G);
235 sys(3:4)=qddot;
236
237 % end mdlDerivatives
238
239 %
240 %←
=====

241 % mdlUpdate
242 % Handle discrete state updates, sample time hits, and major time ←
    step
243 % requirements.
244 %←
=====

245 %
246 function sys=mdlUpdate(t,x,u)
247
248 sys = [];
249
250 % end mdlUpdate
251
252 %
253 %←
=====

254 % mdlOutputs
255 % Return the block outputs.
256 %←
=====

257 %
258 function sys=mdlOutputs(t,x,u)
259
260 sys = x;
261

```

```

262 % end mdlOutputs
263
264 %
265 %↵
=====

266 % mdlGetTimeOfNextVarHit
267 % Return the time of the next hit for this block. Note that the ↵
    result is
268 % absolute time. Note that this function is only used when you ↵
    specify a
269 % variable discrete-time sample time [-2 0] in the sample time ↵
    array in
270 % mdlInitializeSizes.
271 %↵
=====

272 %
273 function sys=mdlGetTimeOfNextVarHit(t,x,u)
274
275 sampleTime = 1;    % Example, set the next hit to be one second ↵
    later.
276 sys = t + sampleTime;
277
278 % end mdlGetTimeOfNextVarHit
279
280 %
281 %↵
=====

282 % mdlTerminate
283 % Perform any end of simulation tasks.
284 %↵
=====

285 %
286 function sys=mdlTerminate(t,x,u)
287
288 sys = [];
289
290 % end mdlTerminate

```

Linear S-Function

```

1  function [sys,x0,str,ts,simStateCompliance] = LinearSFunction(t,x,u,flag,X0,A,B,C)
2
3  switch flag,
4
5      case 0,
6          [sys,x0,str,ts,simStateCompliance]=mdlInitializeSizes(X0);
7
8          %%%%%%%%%%%%%%
9          % Derivatives %
10         %%%%%%%%%%%%%%
11     case 1,
12         sys=mdlDerivatives(t,x,u,X0,A,B);
13
14         %%%%%%%%%%%%%%
15         % Update %
16         %%%%%%%%%%%%%%
17     case 2,
18         sys=mdlUpdate(t,x,u);
19
20         %%%%%%%%%%%%%%
21         % Outputs %
22         %%%%%%%%%%%%%%
23     case 3,
24         sys=mdlOutputs(t,u,x,C);
25
26         %%%%%%%%%%%%%%
27         % GetTimeOfNextVarHit %
28         %%%%%%%%%%%%%%
29     case 4,
30         sys=mdlGetTimeOfNextVarHit(t,x,u);
31
32         %%%%%%%%%%%%%%
33         % Terminate %
34         %%%%%%%%%%%%%%
35     case 9,
36         sys=mdlTerminate(t,x,u);
37
38         %%%%%%%%%%%%%%
39         % Unexpected flags %
40         %%%%%%%%%%%%%%
41     otherwise
42         DASTudio.error('Simulink:blocks:unhandledFlag', num2str(flag))↵
43         ;
44 end
45

```

```

46 % end sfuntmpl
47
48 %
49 %↵
=====

50 % mdlInitializeSizes
51 % Return the sizes, initial conditions, and sample times for the S↵
    -function.
52 %↵
=====

53 %
54 function [sys,x0,str,ts,simStateCompliance]=mdlInitializeSizes(X0)
55
56 %
57 % call simsizes for a sizes structure, fill it in and convert it ↵
    to a
58 % sizes array.
59 %
60 % Note that in this example, the values are hard coded. This is ↵
    not a
61 % recommended practice as the characteristics of the block are ↵
    typically
62 % defined by the S-function parameters.
63 %
64 sizes = simsizes;
65
66 sizes.NumContStates = 4;
67 sizes.NumDiscStates = 0;
68 sizes.NumOutputs = 4;
69 sizes.NumInputs = 1;
70 sizes.DirFeedthrough = 0;
71 sizes.NumSampleTimes = 1; % at least one sample time is needed
72
73 sys = simsizes(sizes);
74
75 %
76 % initialize the initial conditions
77 %
78 x0 = X0;
79
80
81
82
83 %
84 % str is always an empty matrix

```

```

85 %
86 str = [];
87
88 %
89 % initialize the array of sample times
90 %
91 ts = [0 0];
92
93 % Specify the block simStateCompliance. The allowed values are:
94 %     'UnknownSimState', < The default setting; warn and assume ↵
      DefaultSimState
95 %     'DefaultSimState', < Same sim state as a built-in block
96 %     'HasNoSimState',   < No sim state
97 %     'DisallowSimState' < Error out when saving or restoring the ↵
      model sim state
98 simStateCompliance = 'UnknownSimState';
99
100 % end mdlInitializeSizes
101
102 %
103 %↵
      =====

104 % mdlDerivatives
105 % Return the derivatives for the continuous states.
106 %↵
      =====

107 %
108 function sys=mdlDerivatives(t,x,u,X0,A,B)
109 sys=A*x+B*u;
110
111 % end mdlDerivatives
112
113 %
114 %↵
      =====

115 % mdlUpdate
116 % Handle discrete state updates, sample time hits, and major time ↵
      step
117 % requirements.
118 %↵
      =====

119 %
120 function sys=mdlUpdate(t,x,u)

```

```

121
122
123
124 sys = [];
125
126 % end mdlUpdate
127
128 %
129 %↵
=====

130 % mdlOutputs
131 % Return the block outputs.
132 %↵
=====

133 %
134 function sys=mdlOutputs(t,u,x,C)
135
136 sys = x;
137
138 % end mdlOutputs
139
140 %
141 %↵
=====

142 % mdlGetTimeOfNextVarHit
143 % Return the time of the next hit for this block. Note that the ↵
    result is
144 % absolute time. Note that this function is only used when you ↵
    specify a
145 % variable discrete-time sample time [-2 0] in the sample time ↵
    array in
146 % mdlInitializeSizes.
147 %↵
=====

148 %
149 function sys=mdlGetTimeOfNextVarHit(t,x,u)
150
151 sampleTime = 1;    % Example, set the next hit to be one second ↵
    later.
152 sys = t + sampleTime;
153
154 % end mdlGetTimeOfNextVarHit
155

```



```
156 %  
157 %↵
```

=====

```
158 % mdlTerminate  
159 % Perform any end of simulation tasks.  
160 %↵
```

=====

```
161 %  
162 function sys=mdlTerminate(t,x,u)  
163  
164 sys = [];  
165  
166 % end mdlTerminate
```
