Q.M. HW #10

Griffiths: 10.8

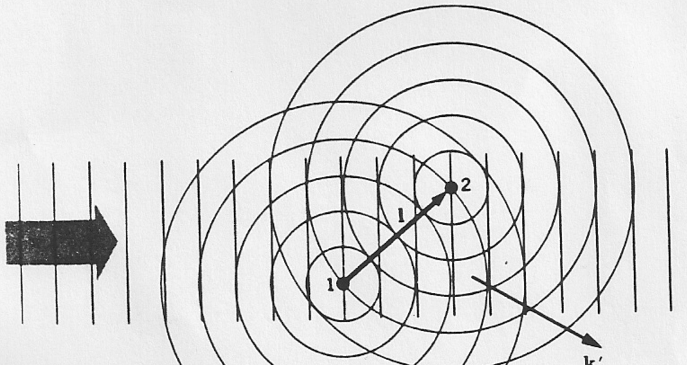
From class: Show that combining $f_{\vec{k}}(\hat{r}) = \frac{-m}{2\pi\hbar^2}\int e^{-ik\hat{r}\cdot\vec{r}'}V(\vec{r}')\psi_{\vec{k}}(\vec{r}')\,d^3\vec{r}'$ with $\psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f_{\vec{k}}(\hat{r})\frac{e^{ikr}}{r}$ leads to the Born approximation in the limit of weak scattering

Park:

Problem 8.18. A molecule of a homonuclear diatomic gas may roughly be regarded as composed of two identical spherically symmetric scattering centers separated by a (vectorial) distance **l** (Fig. 8.15). If the scattering amplitude for a certain kind of particle directed against the atom is known, what scattering cross section is measured for the molecules of the gas? Contrast the atomic and molecular cross sections, and neglect effects of multiple scattering, that is, of particles which bounce back and forth between the two centers.

Hint: The calculation is in two parts: first the calculation of σ for a molecule in a particular spatial orientation and then the average over all orientations. The first thing to note is the difference in phase between the waves arriving at the two scattering centers: Compared to the phase at the center of the molecule, that at **l** is $\frac{1}{2}\mathbf{k}\cdot\mathbf{l}$ early, while that at 2 is equally late. Thus the wave at the counter is

$$\psi_s = \exp\left(\tfrac{1}{2}i\mathbf{k}\cdot\mathbf{l}\right)\left[\frac{e^{ikR_1}}{R_1}f(\vartheta)\right] + \exp\left(-\tfrac{1}{2}i\mathbf{k}\cdot\mathbf{l}\right)\left[\frac{e^{ikR_2}}{R_2}f(\vartheta)\right]$$



The denominators can be set equal to r without error (see Fig. 8.13), while in the exponents, with k' as in (8.51),

$$kR_1 = kr - \tfrac{1}{2}\mathbf{k}'\cdot\mathbf{l} \qquad kR_2 = kr + \tfrac{1}{2}\mathbf{k}'\cdot\mathbf{l}$$

Thus,

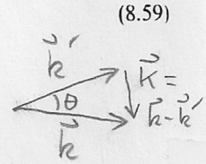
$$\psi_s = \frac{e^{ikr}}{r}\{\exp\left[\tfrac{1}{2}i(\mathbf{k}\cdot\mathbf{l} - \mathbf{k}'\cdot\mathbf{l})\right] + \exp\left[-\tfrac{1}{2}i(\mathbf{k}\cdot\mathbf{l} - \mathbf{k}'\cdot\mathbf{l})\right]f(\vartheta)\}$$

$$= 2\frac{e^{ikr}}{r}\cos\left[\tfrac{1}{2}(\mathbf{k}-\mathbf{k}')\cdot\mathbf{l}\right]f(\vartheta) \qquad (8.59)$$

Eq. 8.33

$\vec{K} = 2k\sin\frac{1}{2}\theta$

so that



$$\frac{d\sigma}{d\Omega} = 4\cos^2\left[\tfrac{1}{2}(\mathbf{k}-\mathbf{k}')\cdot\mathbf{l}\right]|f(\vartheta)|^2$$

The vector $\mathbf{k} - \mathbf{k}' := \mathbf{K}$ is shown in Fig. 8.9, and its length is given by Eq. (8.33). The average over the directions of **l** and the rest of the discussion are left to you.

Extra Info for HW #10

$$f(\vartheta) = -\frac{1}{4\pi}\frac{2\mu}{\hbar^2}\int \exp(-i\mathbf{k}'\cdot\mathbf{r}')V(\mathbf{r}')\psi(\mathbf{r}')\,dv' \qquad (8.51)$$

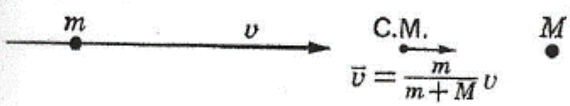


FIGURE 8.3
The center of mass of m and M and its motion.

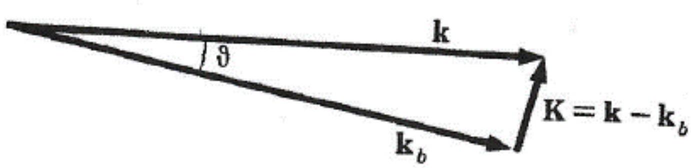


FIGURE 8.9
Relation between $\mathbf{k}$, $\mathbf{k}_b$, and $\mathbf{K}$.