Physics 105, Spring 2021, Reinsch

Homework Assignment 11

Due Tuesday, May 4, 11:59 pm

Homework Assignment 11 is based on Section 12.9 and involves computer programming. You upload a single pdf file containing all of your work, including programs you have written.

Because of the Covid pandemic, we have placed the following file in the Homework 11 folder.

```
logistic_map-Spring2021.ipynb
```

You should study this file and experiment with changing parameters. You are welcome to use this code in your solutions. It should be quite helpful for the problems below.

Problem 1

Write a program that produces Fig. 12.40 on page 510. You should plot both "theoretical" curves and "experimental" dots (these dots are not the same as the dashed curves in Fig. 12.40. The dashing indicates lack of stability). The theoretical curves are given by formulas [for example, Eq. (12.47)], and the experimental dots are obtained by iterations (for example, see page 507). It will be necessary to introduce a parameter M which is a large integer. This large integer is the number of iterations that are computed for each r value. You should plot experimental points for r values going from 0 to 4 in steps of 0.1. Explain how you chose your M value. Typically you would adjust such a parameter so that you are convinced the calculations have converged. At that point, making M larger should not change the plot. These are some of the realities we face in numerical work. We cannot set M to infinity.

The first big paragraph on page 510 explains how to produce the experimental dots. What we call M in the problem statement is called t_{max} in that paragraph.

Further Remarks on Problem 1

Using the ideas explained in the text, you can write down fairly reasonable formulas for the curves in Fig. 12.40 (the curves for the fixed-points and the two-cycles as a function of r). It would be nice if you could indicate where the curves go unstable using dashed lines, but if this is too inconvenient in your plotting environment you don't have to.

The right-most region of Fig. 12.40 shows the curves all going unstable. In that region the experimental dots will get messy. However we are going in coarse steps of 0.1 so we are not plotting the full blown Fig. 12.41. Producing something like Fig. 12.41 requires more effort because care must be taken to launch enough runs for each r value so that all the cycles are found. If this is not done thoroughly there will be gaps in the diagram. Also, some effort is required to convince oneself that the number of iterations is sufficient for each portion of the diagram. For these reasons we are looking for something closer to Fig. 12.40 with a "messy" region of experimental dots

Problem 2

Extend your work to produce Fig. 12.41 on page 511. Describe the difficulties you faced and how you overcame them.

Problem 3

Extend your work to produce Fig. 12.44 on page 513. Describe the new challenges you encountered in doing this.

Problem 4

In this problem we'll work on zooming in even further. In the Homework 11 Folder there is a diagram called "Figs1241and1244.png" containing copies of two figures in the text. Red color has been used to add a new rectangle to Fig. 12.44 that was not present in the original figure. The red rectangle is defined by two values on the r axis (3.8537 and 3.8542) and two values on the x axis (0.489 and 0.508). Use your program to zoom in on this rectangle and produce the next plot in the sequence of plots begun in "Figs1241and1244.png." Note the words "upside-down copy" in the Fig. 12.44 caption; do you see another flipping over? It is likely that you will encounter new numerical challenges for this plot. You do not have to resolve all the problems with numerical artifacts, but you should comment on what you think the causes are.

Some of the keywords from Chapter 12 and our homework are

- Iterations
- Fractals
- Self-similarity

You can experiment further in many ways. For example, in Mathematica

`http://demonstrations.wolfram.com/FiveFamousFractals/`

Maxima

`http://maxima.sourceforge.net/docs/manual/de/maxima_48.html`

Python

`http://csc.ucdavis.edu/~chaos/courses/nlp/Projects2009/Projects2009.html`

Matlab

`http://ocw.mit.edu/courses/mathematics/18-s997-introduction-to-matlab-programming-fall-2011/fractals-and-chaos/`