Physics 110A, Spring 2021
Solution to Homework 10
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1. The system has *plane* symmetry, so we apply Ampére's law for $\mathbf{H}$ with an Amperian loop as in Griffiths Fig. 5.33, and get $\mathbf{H} = \mathbf{0}$ (there is no free current). Using $\mathbf{H} = (1/\mu_0)\,\mathbf{B} - \mathbf{M}$, we get $\boxed{\mathbf{B} = \mathbf{0}}$ outside the slab. You can also locate the bound current and get the same answer.

2. The system has *cylindrical* (type-1) symmetry, so we applay Ampére's law for $\mathbf{H}$ with an Amperian loop as in Griffiths Fig. 5.37, and we get $\boxed{\mathbf{H} = \mathbf{0}}$ everywhere. So

$$\boxed{\mathbf{B} = \mu_0 \mathbf{M} = \begin{cases} \mu_0 k s^2 \hat{\mathbf{z}}, & s < R, \\ \mathbf{0}, & s > R. \end{cases}}$$

   Since $\mathbf{H} = \mathbf{0}$ and $I_f = 0$ everywhere, Griffiths Eq. 6.20 $\boxed{\oint \mathbf{H} \cdot d\mathbf{l} = I_f}$ holds. You can also locate the bound current and get the same answer.

3. The problem assumes $\mathbf{J}$ is the total current, so using Ampére's law, $\mathbf{J} = \dfrac{1}{\mu_0}\boldsymbol{\nabla}\times\mathbf{B} = \dfrac{3ks}{\mu_0}\hat{\mathbf{z}}$, we get $\boxed{J(s) = \dfrac{3ks}{\mu_0}.}$ (If the current $\mathbf{J}$ is the free current, then $\mathbf{J} = \boldsymbol{\nabla}\times\mathbf{H} = \boldsymbol{\nabla}\times\left(\dfrac{1}{\mu}\mathbf{B}\right) = \dfrac{3ks}{\mu}\hat{\mathbf{z}}$, so $J(s) = \dfrac{3ks}{\mu}$.)