

Q.M. H.W. #12

From Griffiths : # 9.1, 9.3, 9.8

(WKB problems)

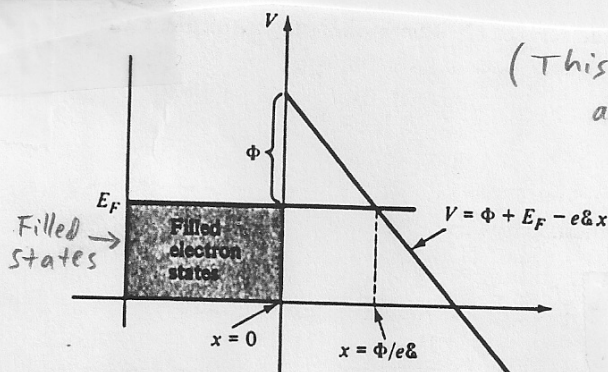
From Liboff :

7.62 In the phenomenon of *cold emission*, electrons are drawn from a metal (at room temperature) by an externally supported electric field. The potential well that the metal presents to the free electrons before the electric field is turned on is depicted in Fig. 2.5. After application of the constant electric field \mathcal{E} , the potential at the surface slopes down as shown in Fig. 7.38, thereby allowing electrons in the Fermi sea to "tunnel" through the potential barrier. If the surface of the metal is taken as the $x = 0$ plane, the new potential outside the surface is

$$V(x) = \Phi + E_F - e\mathcal{E}x$$

where E_F is the Fermi level and Φ is the work function of the metal.

(a) Use the WKB approximation to calculate the transmission coefficient for cold emission.



(This is also known as "Field Emission")

$$T \equiv \left| \frac{J_{\text{Trans.}}}{J_{\text{inc.}}} \right|$$

(b) Estimate the field strength \mathcal{E} , in volt/cm, necessary to draw current density of the order of mA/cm² from a potassium surface. For J_{inc} (see Eq. 7.108) use the expression $J_{\text{inc}} = env$, where n is electron density and v is the speed of electrons at the top of the Fermi sea. The relevant expression for E_F may be found in Problem 2.42. Data for potassium is given in Section 2.3.

$$E_F = \frac{\hbar^2}{2m} \left(\frac{3n}{8\pi} \right)^{2/3} \quad \text{Potassium: } E_F = 2.1 \text{ eV}$$

$$\Phi = 2.1 \text{ eV}$$