Solutions

Homework #1

137 B

2) Griffiths 5.4

a) $\psi_\pm = A\left(\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) \pm \psi_b(\vec{r}_1)\psi_a(\vec{r}_2)\right)$

$$1 = \int d^3r_1\, d^3r_2\, |\psi_\pm|^2 = |A|^2 \int d^3r_1\, d^3r_2 \left(\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) \pm \psi_b(\vec{r}_1)\psi_a(\vec{r}_2)\right)\left(\psi_a^*(\vec{r}_1)\psi_b^*(\vec{r}_2) \pm \psi_b^*(\vec{r}_1)\psi_a^*(\vec{r}_2)\right)$$

$$= |A|^2 \int d^3r_1\, d^3r_2 \Big(|\psi_a(\vec{r}_1)|^2 |\psi_b(\vec{r}_2)|^2 \pm \left(\psi_a^*(\vec{r}_1)\psi_b(\vec{r}_1)\right)\left(\psi_a(\vec{r}_2)\psi_b^*(\vec{r}_2)\right)$$
$$\pm \left(\psi_a(\vec{r}_1)\psi_b^*(\vec{r}_1)\right)\left(\psi_a^*(\vec{r}_2)\psi_b(\vec{r}_2)\right) + |\psi_b(\vec{r}_1)|^2 |\psi_a(\vec{r}_2)|^2\Big)$$

$$= |A|^2 \Big[\underbrace{\int d^3r_1 |\psi_a(\vec{r}_1)|^2}_{=1} \underbrace{\int d^3r_2 |\psi_b(\vec{r}_2)|^2}_{=1} \pm \underbrace{\int d^3r_1\, \psi_a^*(\vec{r}_1)\psi_b(\vec{r}_1)}_{=0} \underbrace{\int d^3r_2\, \psi_a(\vec{r}_2)\psi_b^*(\vec{r}_2)}_{=0}$$
$$\pm \underbrace{\int d^3r_1\, \psi_a(\vec{r}_1)\psi_b^*(\vec{r}_1)}_{=0} \underbrace{\int d^3r_2\, \psi_a^*(\vec{r}_2)\psi_b(\vec{r}_2)}_{=0} + \underbrace{\int d^3r_1 |\psi_b(\vec{r}_1)|^2}_{=1} \underbrace{\int d^3r_2 |\psi_a(\vec{r}_2)|^2}_{=1}\Big]$$

$$= 2|A|^2, \quad \text{so} \quad \boxed{A = \frac{1}{\sqrt{2}}}$$

b) If $\psi_a = \psi_b$, then $\psi = 2\psi_a(\vec{r}_1)\psi_a(\vec{r}_2)$

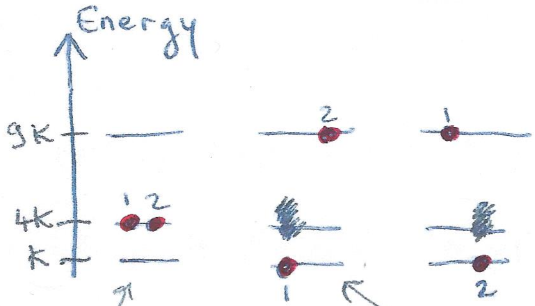
$$1 = 4|A|^2 \underbrace{\int d^3r_1 |\psi_a(\vec{r}_1)|^2}_{=1} \underbrace{\int d^3r_2 |\psi_a(\vec{r}_2)|^2}_{=1} \Longrightarrow \boxed{A = \frac{1}{2}}$$

3) Griffiths 5.5b

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$$K = \frac{\hbar^2 \pi^2}{2ma^2}$$

Distinguishable



$\psi(x_1, x_2) = \psi_3(x_1)\psi_1(x_2)$

have the same energy

= doubly degenerate

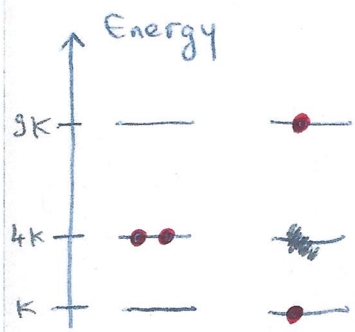
$E = K + 9K = \underline{10K}$

$\psi(x_1, x_2) = \psi_1(x_1)\psi_3(x_2)$

$\psi(x_1, x_2) = \psi_2(x_1)\psi_2(x_2)$

$E = 4K + 4K = \underline{8K}$

Bosons



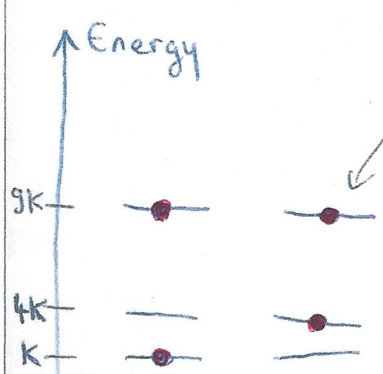
$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\psi_1(x_1)\psi_3(x_2) + \psi_3(x_1)\psi_1(x_2)\right)$

$E = K + 9K = \underline{10K}$

$\psi(x_1, x_2) = \psi_2(x_1)\psi_2(x_2)$

$E = 4K + 4K = \underline{8K}$

Fermions



$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\psi_2(x_1)\psi_3(x_2) - \psi_2(x_2)\psi_3(x_1)\right)$

$\to E = 4K + 9K = \underline{13K}$

$\psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\psi_1(x_1)\psi_3(x_2) - \psi_1(x_2)\psi_3(x_1)\right) \to E = K + 9K = \underline{10K}$

4) Griffiths 5.7

a) $\psi(x_1, x_2, x_3) = \psi_a(x_1)\,\psi_b(x_2)\,\psi_c(x_3)$ (distinguishable)

b) $\psi(x_1, x_2, x_3) = \frac{1}{\sqrt{6}}\Big[\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_1)\psi_b(x_3)\psi_c(x_2) + \psi_a(x_2)\psi_b(x_1)\psi_c(x_3)$
$+ \psi_a(x_2)\psi_b(x_3)\psi_c(x_1) + \psi_a(x_3)\psi_b(x_1)\psi_c(x_2) + \psi_a(x_3)\psi_b(x_2)\psi_c(x_1)\Big]$

(bosons)

c) $$\psi(x_1, x_2, x_3) = \frac{1}{\sqrt{6}} \operatorname{Det}\begin{pmatrix} \psi_a(x_1) & \psi_a(x_2) & \psi_a(x_3) \\ \psi_b(x_1) & \psi_b(x_2) & \psi_b(x_3) \\ \psi_c(x_1) & \psi_c(x_2) & \psi_c(x_3) \end{pmatrix}$$ Slater determinant

$$= \frac{1}{\sqrt{6}}\Big(\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_2)\psi_b(x_3)\psi_c(x_1)$$
$$+ \psi_a(x_3)\psi_b(x_1)\psi_c(x_2) - \psi_a(x_1)\psi_b(x_3)\psi_c(x_2)$$
$$- \psi_a(x_3)\psi_b(x_2)\psi_c(x_1) - \psi_a(x_2)\psi_b(x_1)\psi_c(x_3)\Big)$$

(fermions)