Physics 105, Spring 2021, Reinsch

Homework Assignment 7

Due Thursday, March 18, 11:59 pm

Problem 1

Repeat Example 10.4 on page 390, but now with a non-uniform mass density $\rho(x, y, z) = k\, x^4\, y^4\, z^4$, where k is a constant. Of course, outside of the cube the density is zero.

Problem 2

Taylor, Problem 10.2, with the following change. Rather than a uniform wheel, we will use a uniform sphere (mass M, radius R). You must show how to calculate both of the moments of inertia, I and I'. If you use a theorem you must state the theorem.

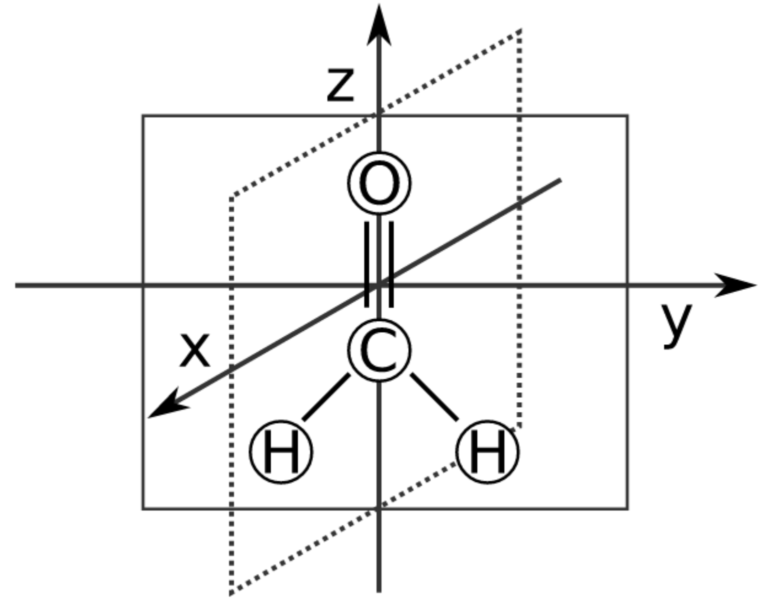
Problem 3

Taylor, Problem 10.5, with the following change. The density is $k_1 + k_2 z$, where k_1 and k_2 are positive constants. You should check that if you set k_2 to zero you get the result on page 764.

Problem 4

Taylor, Problem 10.23

Problem 5



https://en.wikipedia.org/wiki/File:Formaldehyde_symmetry_elements.svg

As you know, formaldehyde is a trigonal planar molecule. In this problem we study a single molecule of formaldehyde using a classical description as a collection of point masses, one for each atom. The center of mass is at the origin. At $t = 0$ the molecule is in the yz plane, as shown in the diagram

We will use m for the mass of a hydrogen atom, and for simplicity we will use $12m$ and $16m$ for the other two atoms. We assume that the HCH bond angle is $2\pi/3$. We use d for the distance from a hydrogen atom to the carbon atom, and for simplicity we use the same value for the distance from the carbon atom to the oxygen atom.

(a) Calculate the coordinates of all of the atoms in the diagram above.

(b) Calculate the moment of inertia tensor.

(c) If at $t = 0$ the $\boldsymbol{\omega}$ vector points in the z direction, what is the angular momentum? Now assume we have steady rotation with this $\boldsymbol{\omega}$ (that is, $\boldsymbol{\omega}$ is constant). Calculate the moment of inertia tensor as a function of time. Note that the moment of inertia tensor is a function of time because we are currently using the "space frame" tensor rather than the "body frame" tensor.

(d) For the time-dependent moment of inertia tensor you found in part (c), calculate normalized eigenvectors as a function of time.

Problem 6

We’ll do one more amazing problem from Chapter 9 of our textbook.

Taylor, Problem 9.31, Compton Generator