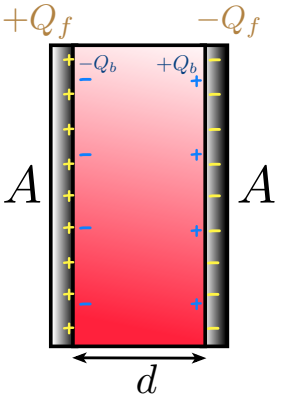
Problem Set 8

Physics 110A, UC Berkeley, Spring 2021

Due Monday, 4/5, at 11:59PM

Problem 1



Consider a parallel plate capacitor where the plates have area A and are separated by a distance d. The gap is filled with a linear dielectric material of electric susceptibility χ_e. Suppose the capacitor is charged so that the plates carry free charges $\pm Q_f$.

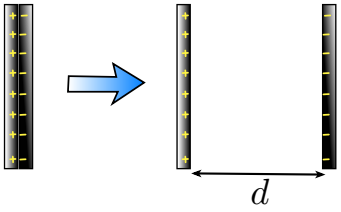
(a) Find the bound charge Q_b originated from the polarization in terms of Q_f and χ_e.

(b) Find the *net* electric field (caused by both the free and bound charges) between the plates. Express your answer in terms of Q_f, A and ϵ, where $\epsilon = (1 + \chi_e)\epsilon_0$.

(c) Compute the work needed to establish the system from zero free charge to a final free charge Q_f by gradually moving infinitesimal free charges dq_f from the right plate to the left plate, overcoming the electric potential of the net field. Express your answer in terms of Q_f, ϵ, A and d.

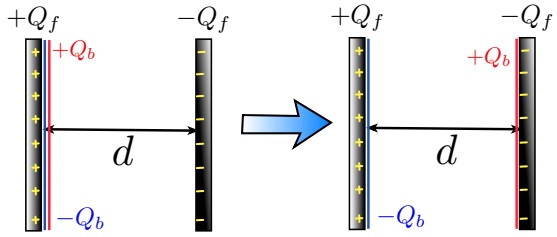
(d) Calculate $W = \frac{1}{2}\int \boldsymbol{D}\cdot\boldsymbol{E}\, d\tau$ for this system. It should agree with your result in part (c).

The difference between $W = \frac{1}{2}\int \boldsymbol{D}\cdot\boldsymbol{E}\, d\tau$ and $W = \frac{\epsilon_0}{2}\int \boldsymbol{E}\cdot\boldsymbol{E}\, d\tau$ is that the latter only take into account the energy needed to put the charges in place, while ignoring the potential energy associated with *microscopic* interactions between the molecules, or within the molecules, which is what we need to overcome when when polarize the material. Specifically, consider the following two-step process. First we separate the free charges $\pm Q_f$ from each other as shown in the right figure.



(e) What is the energy needed to separate the $\pm Q_f$ charges a distance d apart as shown above?

We now separate the bound charges by pulling the $\pm Q_b$ apart from each other within the two charged plates. Along the way, we need to overcome the force between the bound charges, and the force exerted on $+Q_b$ due to $\pm Q_f$.



(f) What is the energy needed to separate the $\pm Q_b$ charges a distance d apart as shown above?

(g) Compute the $W = \frac{\epsilon_0}{2}\int \boldsymbol{E}\cdot\boldsymbol{E}\, d\tau$ for the final configuration. It should be equal to the sum of part (e) and (f).

Problem 2

A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a permanent polarization

$$\boldsymbol{P}(\boldsymbol{r}) = \frac{k}{r}\,\hat{\boldsymbol{r}}$$

where k is a constant and r is the distance from the center. (There is no free charge in the problem.) Find the electric field in all three regions, $r < a$, $a < r < b$, and $r > b$.

Problem 3

An uncharged conducting sphere of radius a is coated with a thick insulating shell out to radius b. The insulating shell has a permanent polarization $\boldsymbol{P} = P_0\,\hat{z}$. Find the electric potential in the three regions, $r < a$, $a < r < b$, and $r > b$.

Below are selected optional problems from Griffiths. We do not collect your work, but you are encouraged to do as many practice problems as you can.

- Problem 4.16
- Problem 4.19
- Problem 4.24
- Problem 4.26
- Problem 4.28
- Problem 4.35
- Problem 4.39
- Problem 4.41