# 7.1, 7.2, 7.3, 7.5



A particle of mass m is confined to a one-dimensional infinite square potential well that extends from x = 0 to x = L. The energy eigenvalues for the (nonrelativistic) Hamiltonian are (see Section 3.2)

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

If the mass of the particle is small or if the length L is small, the energy eigenvalues will be large, and the particle may become relativistic (this happens if the energy is comparable with or larger than,  $mc^2$ ). The relativistic Hamiltonian is

$$H = \sqrt{p^2 c^2 + m^2 c^4} - mc^2$$

- (a) Use first-order perturbation theory to find the new energy eigenvalues that correspond to this relativistic Hamiltonian.
- (b) The energy eigenvalues obtained by first-order perturbation theory are actually the exact eigenvalues for the relativistic Hamiltonian. Explain carefully why this is so.
- 12. Suppose that the electron in a hydrogen atom is perturbed by a repulsive potential concentrated at the origin. Assume that the potential has the form of a delta function, so the perturbed Hamiltonian is

$$H = \frac{\mathbf{p}^2}{2m} - \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r} + A \,\delta(\mathbf{r})$$

where A is a constant.

- (a) To first order in A, find the change in the energy of the state with quantum numbers  $n \ge 1$ , l = 0. [Hint:  $\psi_{n00}(0) = 2/\sqrt{4\pi} (na_0)^{3/2}$ .]
- (b) Find the change in the wavefunction.