Q. M. H. W. # 5

Griffiths : # 7.20, 7.21, 7.24, 7.28

Liboff :

13.14 The eigenstates of a rotating dumbbell, with moment of inertia I,

$$E_l = \frac{\hbar^2 l(l+1)}{2I}$$

are $(2l + 1)$-fold degenerate. In the event that the dumbbell is equally and oppositely charged at its ends, it becomes a dipole. The interaction energy between such a dipole and a constant, uniform electric field $\mathcal{E}$ is

$$\hat{H}' = -\mathbf{d}\cdot\mathcal{E} \qquad (\hat{H} = \hat{H}_0 - \mathbf{d}\cdot\mathcal{E})$$

The dipole moment of the dumbbell is $\mathbf{d}$. Show that to terms of first order, this perturbing potential *does not separate* the degenerate E_l eigenstates.

13.15 Consider again the dipole moment described in Problem 13.14. If both ends are equally charged, the rotating dipole constitutes a magnetic dipole. If the dipole has angular momentum $\mathbf{L}$, the corresponding magnetic dipole moment is

$$\boldsymbol{\mu} = \frac{e}{2mc}\mathbf{L}$$

where e is the net charge of the dipole. The interaction energy between this magnetic dipole and a constant, uniform magnetic field $\mathcal{B}$ is

$$\hat{H}' = -\hat{\boldsymbol{\mu}}\cdot\mathcal{B} = -\frac{e}{2mc}\hat{\mathbf{L}}\cdot\mathcal{B} \qquad (\hat{H} = \hat{H}_0 - \hat{\boldsymbol{\mu}}\cdot\mathcal{B})$$

z

(a) If $\mathcal{B}$ points in the z direction, show that $\hat{H}'$ separates the $(2l + 1)$-fold degenerate E_l energies of the rotating dipole.

(b) Apply these results to one-electron atoms to find the splitting of the P states. (Neglect spin-orbit coupling.) (*Note*: This phenomenon is an example of the *Zeeman effect* discussed previously in Problems 12.15 et seq.)