Problem Set 7

Physics 110A, UC Berkeley, Spring 2021

Due Monday, 3/15, at 11:59PM

Problem 1

Show that the quadrupole term in the multipole expansion can be written as

$$V_{\text{quad}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} \sum_{i,j=1}^3 \hat{r}_i \hat{r}_j Q_{ij},$$

where

$$Q_{ij} = \int \left[\frac{3}{2} r_i' r_j' - \frac{1}{2} \left(r' \right)^2 \delta_{ij} \right] \rho \left(\mathbf{r}' \right) d\tau'.$$

Note that Q_{ij} is a rank-two tensor, so it is possible to express it as a matrix. Also show that Q_{ij} is traceless.

Problem 2

Show that the quadupole and moment Q_{ij} is independent of origin if the monopole and dipole moments both vanish.

Problem 3

A circular disk has a radius R and a uniform surface charge density σ . The disk is lying on the x-y plane, with its center fixed at the origin. Find the potential $V(\mathbf{r})$ of the disk for large r, up to the $1/r^3$ term.

Below are selected optional problems from Griffiths. We do not collect your work, but you are encouraged to do as many practice problems as you can.

- Problem 3.47
- Problem 3.48
- Problem 3.49
- Problem 4.2
- Problem 4.4
- Problem 4.8
- Problem 4.10
- \bullet Problem 4.12