Physics 105, Spring 2021, Reinsch

Homework Assignment 8

Due Thursday, April 1, 11:59 pm

<u>Problem 1(a)</u>: Taylor, Problem 10.42. Although not stated clearly, it is intended that you solve the problem in the body frame. It is much more difficult to solve in the space frame. To get a feeling for the level of difficulty, compare the frequencies Omega_b and Omega_s for the much simpler problem on page 400. This relates to the famous “Feyman dinner plate,” discussed on the next page of this homework assignment.

<u>Problem 1(b)</u>: Taylor, Problem 10.44.

In the literature, you may see references to Richard Feynman (Nobel Laureate) and the dinner plate. I would recommend reading the following quote from Feynman:

Then I had another thought: Physics disgusts me a little bit now, but I used to enjoy doing physics. Why did I enjoy it? I used to play with it. I used to do whatever I felt like doing - it didn't have to do with whether it was important for the development of nuclear physics, but whether it was interesting and amusing for me to play with. When I was in high school, I'd see water running out of a faucet growing narrower, and wonder if I could figure out what determines that curve. I found it was rather easy to do. I didn't have to do it; it wasn't important for the future of science; somebody else had already done it. That didn't make any difference. I'd invent things and play with things for my own entertainment.

So I got this new attitude. Now that I am burned out and I'll never accomplish anything, I've got this nice position at the university teaching classes which I rather enjoy, and just like I read the Arabian Nights for pleasure, I'm going to play with physics, whenever I want to, without worrying about any importance whatsoever.

Within a week I was in the cafeteria and some guy, fooling around, throws a plate in the air. As the plate went up in the air I saw it wobble, and I noticed the red medallion of Cornell on the plate going around. It was pretty obvious to me that the medallion went around faster than the wobbling.

I had nothing to do, so I start to figure out the motion of the rotating plate. I discover that when the angle is very slight, the medallion rotates twice as fast as the wobble rate - two to one [Note: Feynman mis-remembers here---the factor of 2 is the other way]. It came out of a complicated equation! Then I thought, ``Is there some way I can see in a more fundamental way, by looking at the forces or the dynamics, why it's two to one?''

I don't remember how I did it, but I ultimately worked out what the motion of the mass particles is, and how all the accelerations balance to make it come out two to one.

I still remember going to Hans Bethe and saying, ``Hey, Hans! I noticed something interesting. Here the plate goes around so, and the reason it's two to one is ...'' and I showed him the accelerations.

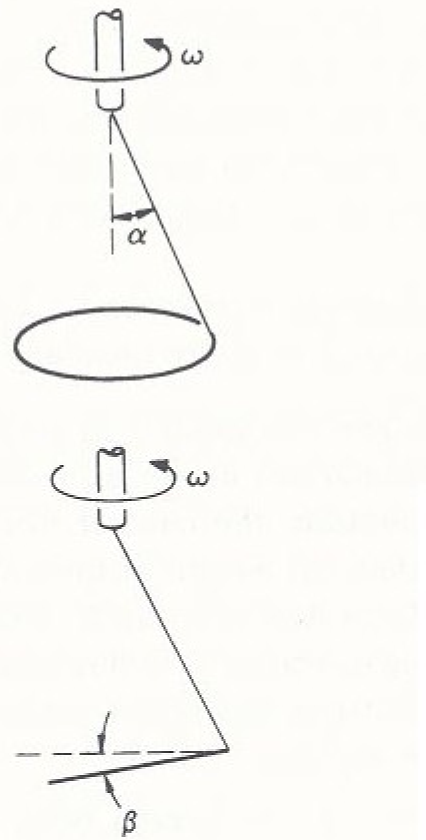
He says, ``Feynman, that's pretty interesting, but what's the importance of it? Why are you doing it?''

``Hah!'' I say. ``There's no importance whatsoever. I'm just doing it for the fun of it.'' His reaction didn't discourage me; I had made up my mind I was going to enjoy physics and do whatever I liked.

I went on to work out equations of wobbles. Then I thought about how electron orbits start to move in relativity. Then there's the Dirac Equation in electrodynamics. And then quantum electrodynamics. And before I knew it (it was a very short time) I was ``playing'' - working, really - with the same old problem that I loved so much, that I had stopped working on when I went to Los Alamos: my thesis-type problems; all those old-fashioned, wonderful things.

It was effortless. It was easy to play with these things. It was like uncorking a bottle: Everything flowed out effortlessly. I almost tried to resist it! There was no importance to what I was doing, but ultimately there was. The diagrams and the whole business that I got the Nobel Prize for came from that piddling around with the wobbling plate.

Problem 2: Read this problem and then fill in the red boxes on the next page



A thin hoop of mass M and radius R is suspended from a string through a point on the rim of the hoop. If the support is turned with high angular velocity ω, the hoop will spin as shown, with its plane nearly horizontal and its center nearly on the axis of the support. The string makes angle α with the vertical.

a. Find, approximately, the small angle β between the plane of the hoop and the horizontal.

b. Find, approximately, the radius of the small circle traced out by the center of mass about the vertical axis. (With skill you can demonstrate this motion with a rope. It is a favorite cowboy lariat trick.)

Solution using Euler's equations (10.88)

Define $\mathcal{O}$ as shown in diagram.

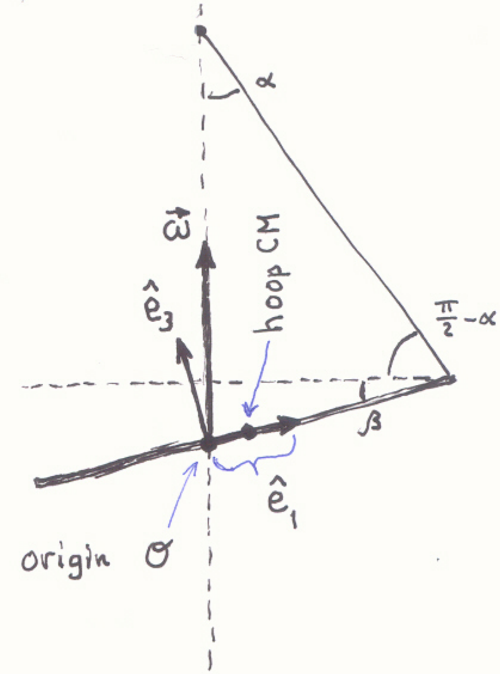
Define body frame as shown.

$\hat{e}_2$ points into page.

$\vec{\omega}$ = const in space frame.

$\vec{\omega}\cdot\hat{e}_i$ = const in space frame.

Thus $\vec{\omega}$ = const in body frame.



<u>In body frame</u>

$$\vec{\omega} = \omega\,(\sin\beta,\ 0,\ \cos\beta)$$

$$\lambda_1 = \boxed{\quad}$$

R_c = radius of circle traced out by CM in space frame = $\boxed{\quad}$

distance from $\mathcal{O}$ to CM = $\boxed{\quad}$

$$\lambda_3 = \boxed{\quad}$$

$$\Gamma_2 = R_c M g - (R + R_c/\cos\beta)\,T\,\overbrace{\sin\left(\tfrac{\pi}{2} - \alpha + \beta\right)}^{\cos(\alpha-\beta)}$$

Now plug into 2^{nd} equation in (10.88)

$$\boxed{\quad} = \Gamma_2$$

Everything has been exact so far. Now begin approximations.

Using $R_c \ll R$ we get

$$-\frac{1}{4} M R^2 \omega^2 \sin(2\beta) = -R\,\frac{Mg}{\cos\alpha}\cos(\alpha-\beta)$$

$$R\,\omega^2 \sin(2\beta) = 4g\,\frac{\cos(\alpha-\beta)}{\cos\alpha}$$

Using $\beta \ll 1$ we get

$$\beta \approx \frac{2g}{\omega^2 R}$$

Problem 3

A molecule consists of six point masses located in a Cartesian coordinate system as follows.

- A mass m is located at $(0, a, 0)$.
- A mass m is located at $(0, -a, 0)$.
- A mass $2m$ is located at $(0, 0, 2a)$.
- A mass $2m$ is located at $(0, 0, -2a)$.
- A mass $3m$ is located at $(a, 0, a)$.
- A mass $3m$ is located at $(-a, 0, -a)$.

(a) Calculate the inertia tensor. It will behoove you to use the symmetry considerations we discussed in lecture.

(b) Calculate the principal moments and principal axes. [You may wish to study Problem 10.35 and its solution on page 765. Please do not hand in a solution to Problem 10.35.]

Problem 4

In this problem we consider an arbitrary rigid body with its center of mass at the origin (see Fig. 10.8 and the caption there). At $t = 0$, we have $\mathbf{e}_1 = \hat{\mathbf{x}}$, $\mathbf{e}_2 = \hat{\mathbf{y}}$ and $\mathbf{e}_3 = \hat{\mathbf{z}}$.

The angular velocity in the space frame, $\boldsymbol{\omega}_{space} = (\omega_x, \omega_y, \omega_z)$, as a function of time is given by

$$\boldsymbol{\omega}_{space}(t) = \begin{cases} \frac{3\omega_o}{2\tau_o^2}\, t(\tau_o - t)\,\hat{\mathbf{z}}, & 0 \le t \le \tau_o \\ \frac{3\omega_o}{2\tau_o^2}\,(t - \tau_o)(2\tau_o - t)\,\hat{\mathbf{x}}, & \tau_o \le t \le 2\tau_o \end{cases}$$

where ω_o is a positive constant, and τ_o is defined to be $\tau_o = 2\pi/\omega_o$.

(a) For $0 \le t \le 2\tau_o$, find the components of the unit vector $\mathbf{e}_1$ with respect to the space frame axes. Your answer will consist of two separate formulas as above.

(b) Repeat part (a) for $\mathbf{e}_2$ and $\mathbf{e}_3$.

(c) The angular velocity in the body frame, $\boldsymbol{\omega}_{body} = (\omega_1, \omega_2, \omega_3)$ can now be found as a function of time. Write out formulas for $\omega_1(t)$, $\omega_2(t)$ and $\omega_3(t)$.

(d) Use Euler's equations, Eq. (10.88), to find the components of the torque in the body frame.

(e) Transform your results from part (d) back to the space frame to get the torque in the space frame as a function of time. Do the results agree with what you expect?

Reminders on notation, Problem 4

There is only one angular velocity vector $\boldsymbol{\omega}$. It is the angular velocity of the body frame relative to the space frame. It can be expressed relative to the space-fixed basis $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ as

$$\boldsymbol{\omega} = \omega_x\,\hat{\mathbf{x}} + \omega_y\,\hat{\mathbf{y}} + \omega_z\,\hat{\mathbf{z}}$$

and it can be expressed relative to the body-fixed basis $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ as

$$\boldsymbol{\omega} = \omega_1\,\mathbf{e}_1 + \omega_2\,\mathbf{e}_2 + \omega_3\,\mathbf{e}_3$$

In the space frame, the unit vectors $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ are constant and the unit vectors $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ are functions of time.

In the body frame, the unit vectors $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ are constant and the unit vectors $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ are functions of time.

Sometimes for clarity we will write $\boldsymbol{\omega}_{space}$ or $\boldsymbol{\omega}_{body}$ to emphasize which frame we are thinking about.