MIDTERM EXAM 1, Physics 105, Spring 2021, Reinsch

- The exam is open-book, open-notes, and you may look at all files on our bCourses site. You are not allowed to use the internet or consult with anyone except the instructors.
- No Mathematica, Wolfram Alpha, or any other computer help. You may use a calculator (a small device that computes numerical results like 1 / 7 = 0.1428571428571429).
- We will not be doing Zoom monitoring or any sort of video proctoring.
- Explain how you got your answers. Put a box around your answers.
- At the end of the exam you must create a pdf file that contains all of your work. After you upload it you must download it to verify that it is legible and contains all of your work.

Problem 1

A simple pendulum is mounted on a cart. The cart is forced to accelerate to the right with constant acceleration a by an external agent; the horizontal coordinate of the pendulum's pivot is X(t) = (½) $a\,t^2$, as shown in the diagram. The familiar gravitational field $\boldsymbol{g}$ points straight down and has magnitude g. The rod is massless and has length L. The pendulum bob has mass m. As our generalized coordinate we will use the usual angle ϕ, measured relative to the vertical as shown in the diagram on page 155.

(a) Calculate the Lagrangian.

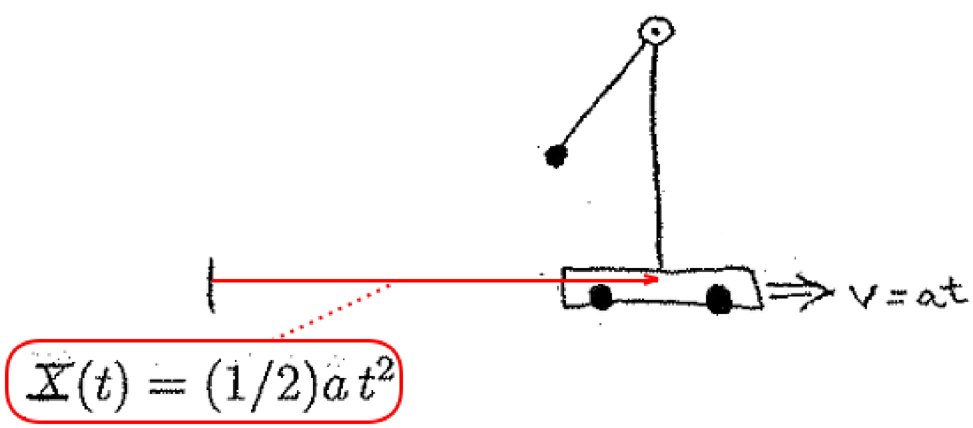
(b) Show how to get the equation of motion (a differential equation) from the Lagrangian.

(c) Find a stable equilibrium. [Hint: The value of ϕ is between 0 and $-\pi/2$]

(d) Calculate the period of small oscillations about the equilibrium you found in part (c).

(e) The total mechanical energy (kinetic plus potential) is not conserved for this system. Prove this by computing the time derivative of the total mechanical energy and showing that you get something nonzero.

(f) We define C = kinetic energy + A cos(ϕ) + B sin(ϕ), where A and B are constants. Find values for A and B in terms of some or all of the constant parameters mentioned above so that C is conserved (C is a "constant of the motion"). Clarification added: In this sub-problem, the kinetic energy term in C is the kinetic energy in the frame of the cart.



Problem 2

One of the roofs of a laboratory is planar and described by the formula $z = c_x x + c_y y$, where c_x and c_y are constants. A particle of mass m slides without friction on this roof. The familiar gravitational field $\boldsymbol{g}$ points straight down (that's the negative z direction) and has magnitude g.

(a) Using x and y as generalized coordinates, calculate the Lagrangian.

(b) Write out the Euler-Lagrange equations (the equations of motion) gotten from the Lagrangian.

(c) Solve the differential equations you found in part (b), that is, find $x(t)$ and $y(t)$. [Hint: There should be a total of four free parameters.]

(d) Is the total mechanical energy conserved? Explain.

(e) We define $C = A\, dx/dt + B\, dy/dt$, where A and B are constants. Find values for A and B in terms of some or all of the constant parameters mentioned above so that C is conserved (C is a "constant of the motion").

(f) Now we consider different generalized coordinates. We define $q_1 = x^3$ and $q_2 = y^5$. What is the Lagrangian in these new coordinates? You do not have to repeat parts (b), (c) or any of the other parts above.