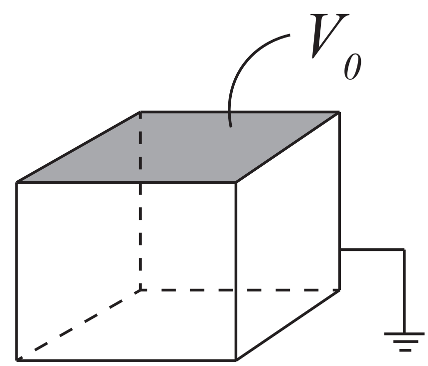
Problem Set 6

Physics 110A, UC Berkeley, Spring 2021

Due Monday, 3/8, at 11:59PM

Problem 1

A hollow cube has six square faces. Five faces are grounded ($V = 0$), while the top face is held at a constant potential $V = V_0$. Find the potential at the center of the cube. *Hint*: duplicate five other such cubes. And then use superposition principle.



Problem 2

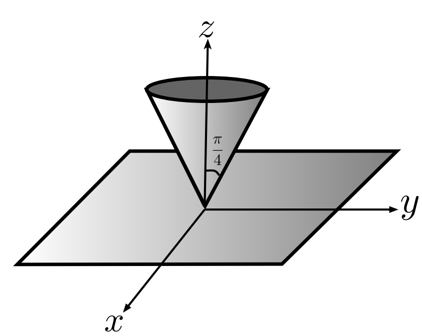
A sphere centered at the origin has a radius R. The electric field within the sphere is given by

$$\boldsymbol{E} = -\frac{V_0 x}{R^2}\,\hat{\boldsymbol{x}} - \frac{V_0 y}{R^2}\,\hat{\boldsymbol{y}} + \frac{2V_0 z}{R^2}\,\hat{\boldsymbol{z}} \qquad \text{for } x^2 + y^2 + z^2 \le R^2. \tag{1}$$

Find the volume charge density $\rho(r, \theta, \phi)$ (confined to $r < R$) and the surface charge density $\sigma(\theta)$ (confined to $r = R$) that produce the electric field given above. Express your answer in terms of spherical coordinates. Is your answer unique? If not, find the general charge distributions that produce such electric field.

Problem 3

Consider a capacitor formed by an infinitely large plate on $z = 0$ with $V = 0$, and an infinite, solid, conducting cone with an interior angle $\pi/4$ held at potential $V = V_0$. Note that the tip of the cone vertex and the infinitely large plate are insulated.



(a) Based on symmetries, explain why $V(r, \theta, \phi) = V(\theta)$ in the space between the cone and the plate.

(b) Integrate Laplace's equation explicitly to find the potential between the cone and the plate. (Note that the general solution Eq.(3.65) in Griffiths does not apply to the case here, since we have charges distributed at $\theta = \pi/4$ and $\pi/2$.)

Below are selected optional problems from Griffiths. We do not collect your work, but you are encouraged to do as many practice problems as you can.

- Problem 3.17
- Problem 3.18
- Problem 3.19
- Problem 3.21
- Problem 3.22
- Problem 3.27
- Problem 3.29
- Problem 3.36
- Problem 3.43