Physics 105, Spring 2021, Reinsch

Homework Assignment 6

Due Thursday, March 11, 11:59 pm

Problem 1

The definitions of an origin and x and y axes are shown in Figure 9.3. The z axis is orthogonal to the page. Equation (9.13) defines U_{tid} as a function of x, y and z, with d equaling $\sqrt{(d_o + x)^2 + y^2 + z^2}$.

(a) Expand U_{tid} to second order in x, y and z.

(b) Knowledge of spherical harmonics is not necessary to solve this problem. However, for those students that have seen spherical harmonics it may be interesting to see how they relate to our tidal calculations. For $\ell = 2$ the spherical harmonics $Y_{\ell m}$ are

You do not have to do part (b). Just appreciate that it can be done.

$$\begin{pmatrix} \frac{1}{4}\sqrt{\frac{15}{2\pi}}e^{-2i\phi}\sin^2(\theta) \\ \frac{1}{2}\sqrt{\frac{15}{2\pi}}e^{-i\phi}\sin(\theta)\cos(\theta) \\ \frac{1}{4}\sqrt{\frac{5}{\pi}}\left(3\cos^2(\theta) - 1\right) \\ -\frac{1}{2}\sqrt{\frac{15}{2\pi}}e^{i\phi}\sin(\theta)\cos(\theta) \\ \frac{1}{4}\sqrt{\frac{15}{2\pi}}e^{2i\phi}\sin^2(\theta) \end{pmatrix} \tag{1}$$

where m ranges from -2 to 2.

Expressed in terms of the Cartesian coordinates, the $r^2 Y_{\ell m}$ are

$$\begin{pmatrix} \frac{1}{4}\sqrt{\frac{15}{2\pi}}(x - iy)^2 \\ \frac{1}{2}\sqrt{\frac{15}{2\pi}}z(x - iy) \\ -\frac{1}{4}\sqrt{\frac{5}{\pi}}\left(x^2 + y^2 - 2z^2\right) \\ -\frac{1}{2}\sqrt{\frac{15}{2\pi}}z(x + iy) \\ \frac{1}{4}\sqrt{\frac{15}{2\pi}}(x + iy)^2 \end{pmatrix} \tag{2}$$

Because the Moon's mass density is zero outside of the Moon, the Laplacian of U_{tid} is zero outside of the Moon, and the Laplacian of the answer to part (a) is zero. Express your answer to part (a) as a linear combination of the above 5 functions. [Hint: you will not need the $m = \pm 1$ spherical harmonics]

Problem 2

Taylor, Problem 9.22 on page 363.

Problem 3

Taylor, Problem 9.23 on page 363.

Problem 4

In this problem we will ignore the rotation of the Earth. A Merry-Go-Round is set up on a horizontal playground, and it can rotate about a vertical axis. As in Fig. 9.8, we define an S_o frame and an S frame. At time $t = 0$ the two sets of coordinate axes coincide. The origin is at the center of the Merry-Go-Round. The z and z_o axes point upwards, coinciding with the axis of rotation. The angular velocity of the Merry-Go-Round is $\mathbf{\Omega}$.

An extremely small toy cannon is attached to the Merry-Go-Round a distance R from the center. The configuration is such that if $\mathbf{\Omega}$ is constant and zero, the projectile leaves the surface of the Merry-Go-Round (with velocity vector at angle α relative to the horizontal) and lands at the midpoint between the cannon and the center of the Merry-Go-Round. Based on this statement you can calculate the magnitude of the muzzle velocity, v_o, in terms of α and other parameters.

Now we consider a small constant nonzero value for $\mathbf{\Omega}$. The projectile is launched at the point $(R, 0, 0)$ in both coordinate systems at $t = 0$.

(a) In the S_o frame, calculate the initial velocity vector and the exact trajectory of the projectile. Calculate the coordinates of the landing point, and then from these values calculate the coordinates of the landing point in the S frame.

(b) Next we will do an approximate calculation of the trajectory in the S frame. Use the methods discussed on page 353 to calculate the approximate landing point in the S frame and compare this with your result from part (a).