Problem Set 2

Physics 110A, UC Berkeley, Spring 2021

Due Monday, 2/8, at 11:59PM

Problem 1

The electric field of a solid sphere with radius R and uniform charge density ρ is given by

$$\boldsymbol{E} = \begin{cases} \dfrac{\rho \boldsymbol{r}}{3\epsilon_0} & (r < R) \\ \\ \dfrac{kQ}{r^2}\,\hat{\boldsymbol{r}} & (r > R). \end{cases} \tag{1}$$

where Q is the total charge of the sphere. The magnetic field of an infinitely long thick cable with radius a is given by

$$\boldsymbol{B} = \begin{cases} \dfrac{\mu_0 J s}{2}\,\hat{\boldsymbol{\phi}} & (s < a) \\ \\ \dfrac{\mu_0 I}{2\pi s}\,\hat{\boldsymbol{\phi}} & (s > a) \end{cases} \tag{2}$$

where the net current I flows in the $+z$-direction. Note that the $\boldsymbol{E}$-field and $\boldsymbol{B}$-field are expressed in spherical and cylindrical coordinate respectively.

(a) Calculate the divergence and curl of $\boldsymbol{E}$ with spherical coordinate.

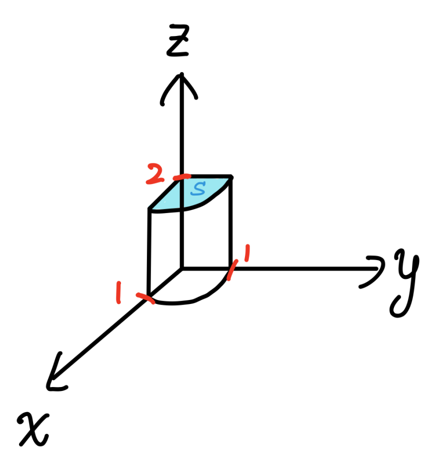
(b) Calculate the divergence and curl of $\boldsymbol{B}$ with cylindrical coordinate.

Independent from the previous part, consider a vector field $\boldsymbol{V} = s(2+\cos^2\phi)\hat{\boldsymbol{s}} + s\sin\phi\cos\phi\,\hat{\boldsymbol{\phi}} + 3z\,\hat{\boldsymbol{z}}$.

(c) Calculate the divergence and curl of the vector $\boldsymbol{V}$.

(d) Verify that the divergence theorem holds true using the quarter-cylinder of radius 1 and height 2 shown in the figure below.

(e) Verify that the Stoke's theorem holds true using the surface S shown in the figure below.



Problem 2

Show that the vector field

$$\boldsymbol{E} = \frac{K}{r^3}\left(2\cos\theta\,\hat{\boldsymbol{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right), \tag{3}$$

where K is a constant, can be written as a gradient of some scalar function $V(\boldsymbol{r})$ using Helmholtz theorem, and find the scalar function $V(\boldsymbol{r})$.

Problem 3

Show the following integral theorems:

(a) $\displaystyle\int_{\mathcal{V}} (\nabla T)\, d\tau = \oint_{\mathcal{S}} T\, d\boldsymbol{a}$

(b) $\displaystyle\int_{\mathcal{V}} (\nabla \times \boldsymbol{V})\, d\tau = -\oint_{\mathcal{S}} \boldsymbol{V} \times\, d\boldsymbol{a}$

(c) $\displaystyle\int_{\mathcal{V}} (T\,\nabla^2 U - U\,\nabla^2 T)\, d\tau = \oint_{\mathcal{S}} (T\,\nabla U - U\,\nabla T)\cdot\, d\boldsymbol{a}$

Here $\mathcal{V}$ is a three-dimensional region in the 3D flat space and $\mathcal{S}$ is its boundary. T, U are scalar fields, while $\boldsymbol{V}$ is a vector field. For (a), you can divergence theorem but with the vector field to be $\boldsymbol{c}T$ where $\boldsymbol{c}$ is a constant vector field. For (a), you can again consider divergence theorem but with the vector field to be $\boldsymbol{V} \times \boldsymbol{c}$ where again $\boldsymbol{c}$ is a constant vector field.

Below are selected optional problems from Griffiths. We do not collect your work, but you are encouraged to do as many practice problems as you can.

- Problem 1.29
- Problem 1.31
- Problem 1.33
- Problem 1.34
- Problem 1.36
- Problem 1.40
- Problem 1.45
- Problem 1.47
- Problem 1.49
- Problem 1.50
- Problem 1.62