Physics 105, Spring 2021, Reinsch

Homework Assignment 4

Due Thursday, February 18, 11:59 pm

Problem 1

In this problem we study a Hula Hoop (a hoop made of plastic tubing) of radius R. This R is the large radius of the hoop. The radius of the plastic tubing (which is typically about 1 centimeter) is taken to be negligibly small for this problem.

In a zero-g environment an astronaut can make the Hula Hoop move uniformly in a plane, which we define to be the xy plane. The xy coordinate system defines an inertial frame. The center of the hoop traces out a circular path of radius r_c. As a function of time, the center of the hoop is at $r_c(\cos(\omega t), \sin(\omega t))$, where ω is a constant.

A small particle of mass m can move around inside the tubing without friction. The motion of this particle does not affect the motion of the Hula Hoop. We will use an angle ϕ as the generalized coordinate for this 1 degree-of-freedom problem. The angle ϕ specifies the location of the particle relative to the center of the hoop, with the direction of the $+x$ axis as the reference direction.

Other than the constraint force, there are no forces acting on the particle.

(a) Calculate the Lagrangian and from it the equation of motion.

(b) We are interested in solutions of the form $\phi(t) = \omega t + \epsilon(t)$, where $\epsilon(t)$ is small. Find the frequency of small oscillations for $\epsilon(t)$.

Problem 2

In Example 7.4, z and ϕ are used as generalized coordinates.

(a) Write equations for x, y and z as functions of these generalized coordinates. The equation for z will just be $z = z$.

(b) Differentiate the formulas from part (a) with respect to time and write out the kinetic energy. The result should agree with the formula on page 257.

(c) The apparatus of Example 7.4 is now placed inside a railroad car. The train moves along straight tracks in the y direction and accelerates with constant acceleration a. Using the same generalized coordinates as above (defined relative to the apparatus), write out the kinetic energy relative to the inertial frame of the tracks. At $t = 0$ the train is at rest.

Problem 3

Taylor, Problem 7.36

Problem 4

Taylor, Problem 7.40

Problem 5

Taylor, Problem 7.43. Please let us know if you would like help with Python programming. Also note that the Physics Department has a dedicated tutor for Python and a tutor for Physics 105.

`https://physics.berkeley.edu/academics/tutoring`

Problem 6

Taylor, Problem 7.46