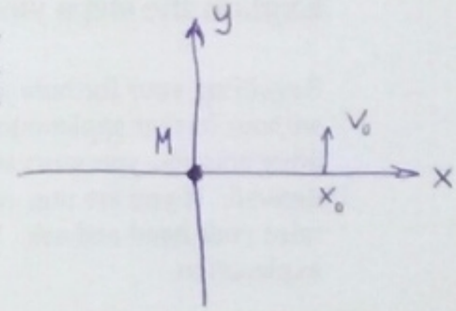
(a) $E = \frac{\gamma}{2c}(\varepsilon^2 - 1)$. If $\varepsilon = 0$, then $E = \frac{-\gamma}{2c}$



$$c = \frac{\ell^2}{GM\mu^2} = \frac{(\mu x_0 v_0)^2}{GM\mu^2} = \frac{x_0^2 v_0^2}{GM}$$

$$E = \frac{1}{2}\mu v_0^2 - \frac{\gamma}{x_0} = -\frac{\gamma}{2}\frac{GM}{x_0^2 v_0^2}$$

$$\mu v_0^4 - 2\frac{\gamma}{x_0}v_0^2 + \gamma GM/x_0^2 = 0$$

$$v_0^4 - 2\frac{GM}{x_0}v_0^2 + (GM/x_0)^2 = 0 \Rightarrow \left(v_0^2 - \frac{GM}{x_0}\right)^2 = 0$$

$$\boxed{v_0 = \sqrt{GM/x_0}}$$

(b) $U_{eff}(r) = \frac{\ell^2}{2\mu r^2} + U(r)$

$$= \frac{(\mu x_0 v_0)^2}{2\mu r^2} - \frac{GM\mu}{r} \Rightarrow \frac{U_{eff}}{\mu} = \frac{x_0^2 v_0^2}{2r^2} - \frac{GM}{r} = \frac{GMx_0}{2r^2} - \frac{GM}{r}$$

Thus $U_{eff}/(GM\mu) = \frac{x_0}{2r^2} - \frac{1}{r}$. $U'_{eff} = 0 \Rightarrow -\frac{x_0}{r^3} + \frac{1}{r^2} = 0$

$$\Rightarrow \boxed{r = x_0}$$

(c) $E = \frac{1}{2}\mu\left(\beta\sqrt{GM/x_0}\right)^2 - \frac{GM\mu}{x_0} = \frac{GM\mu}{x_0}\left(\frac{\beta^2}{2} - 1\right)$

$\ell = \mu x_0 \beta\sqrt{GM/x_0}$. $c = \beta^2 x_0$. $E = \frac{GM\mu}{2x_0\beta^2}(\varepsilon^2 - 1)$ equal

Thus $2\beta^2\left(\frac{\beta^2}{2} - 1\right) = \varepsilon^2 - 1$. $\beta^4 - 2\beta^2 + 1 = \varepsilon^2$. $\boxed{\varepsilon = \beta^2 - 1}$ (positive root)

(d) When $\phi = \frac{\pi}{2}$, we have $r = c = \boxed{\beta^2 x_0}$

(e)

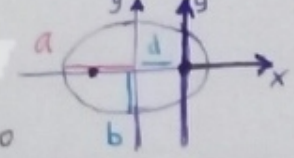


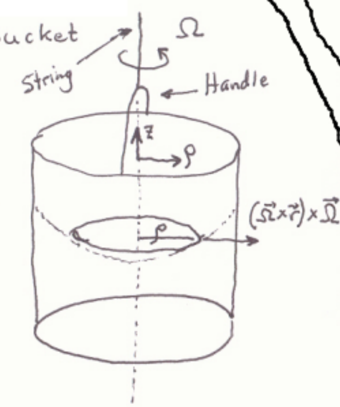
Fig 8.10

Answer $= a = \frac{c}{1-\varepsilon^2}$
$$= \frac{\beta^2 x_0}{\beta^2(2-\beta^2)} = \boxed{\frac{x_0}{2-\beta^2}}$$

Part (a)

Part (b) [See page 345]

Part (c) : Box in lower right



$\Omega =$ const.

Wait for water to become motionless in rotating frame.

Consider a small parcel of water at the surface. Let mass $= m$.

$$0 = m\ddot{\vec{r}} = \vec{F} + \vec{F}_{cf} \qquad (\vec{F}_{Cor} = 0)$$

$$= (\vec{F}_{ng} + \vec{F}_g) + m(\vec{\Omega}\times\vec{r})\times\vec{\Omega}$$

Thus

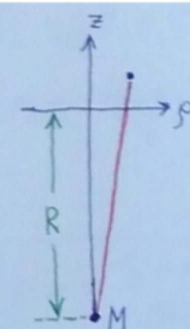
$$-\vec{F}_{ng} = \vec{F}_g + m\Omega^2\rho\hat{\rho}$$

As in discussion of tides, $\vec{F}_{ng}$ is perpendicular to surface.

Therefore the surface is an equipotential of U.

length of red line $= \sqrt{(z+R)^2+\rho^2}$

$$\boxed{U_{grav} = -\frac{GMm}{\sqrt{(z+R)^2+\rho^2}}}$$



$$U_{grav} \underbrace{- \tfrac{1}{2}m\Omega^2\rho^2}_{\text{This is } U_{cf}} = \text{const}$$

$$= \text{value at } \rho = 0$$

$$= -\frac{GMm}{z_0+R}$$

$$\frac{1}{m}U_{grav} = \frac{1}{2}\Omega^2\rho^2 - \frac{GM}{z_0+R}$$

$$\frac{-GM}{\frac{1}{2}\Omega^2\rho^2 - \frac{GM}{z_0+R}} = \sqrt{(z+R)^2+\rho^2} = \left[(z_0+R)^{-1} - \frac{\Omega^2\rho^2}{2GM}\right]^{-1}$$

$$\boxed{z(\rho) = \sqrt{\left[(z_0+R)^{-1} - \frac{\Omega^2\rho^2}{2GM}\right]^{-2} - \rho^2} \; -R}$$