Problem Set 12

Physics 110A, UC Berkeley, Spring 2021

Due Tuesday, 5/4, at 11:59PM

Problem 1

Find the inductance and capacitance per unit length for a pair of parallel straight wires at a distance d from each other. The currents in the two wires are in opposite directions. For simplicity, assume that the wires are thin tubes of radius a ($a \ll d$), and the current distributes uniformly over the surface of the tube. You can think that the two wires are connected with each other at infinity to form a closed loop.

Problem 2

An annular disk of thickness t, inner radius R_1, outer radius R_2 and conductivity σ is placed in a uniform constant magnetic field $\boldsymbol{B}$ that points perpendicularly to the disk. A *net radial* current I_0 flows from the inner edge to the outer edge of the disk. Recall that if n is the number of electrons per volume, the volume current density $\boldsymbol{J}$ can be written by $\boldsymbol{J} = ne\,\boldsymbol{v}$, where e and $\boldsymbol{v}$ are the electric charge and the drift velocity of the electrons respectively.

(a) Use motional EMF to show that there is is net circular current, i.e. the net current flowing across a radial line between R_1 to R_2,
$$I = \frac{\sigma B I_0}{2\pi ne} \ln \frac{R_2}{R_1}.$$

(b) Show that every circle concentric with the annular disk is an equipotential curve. Note that this just requires a short argument.

Problem 3

A non-conducting ring of radius R and mass M has a net charge Q uniformly distributed on it. The ring is dropped from rest from a height $z = h$ and falls to the ground ($z = 0$) through a non-uniform magnetic field $\boldsymbol{B}(\boldsymbol{r})$ that possesses azimuthal symmetry. The plane of the ring remains horizontal during its fall. The ring is small enough such that the magnetic field through the ring is approximately uniform spatially, and is concentric with the symmetry axis of the field.

(a) Show that the torque exerted on the ring is
$$\boldsymbol{N} = -\frac{QR^2}{2}\frac{dB_z}{dt}\,\hat{\boldsymbol{z}}.$$

(b) Using the torque, find the final angular velocity and final angular kinetic energy of the ring when it falls to the ground. Express your answer in terms of Q, R, M, $B_z(h)$, $B_z(0)$

(c) As the ring rotates, it effectively carries a current, and thus possesses a magnetic dipole moment $\boldsymbol{m}$. Find the dipole moment $\boldsymbol{m}$ in terms of Q and R when the ring has an angular velocity $\omega\,\hat{\boldsymbol{z}}$.

(d) Recall that the magnetic force on a magnetic dipole due to a non-uniform magnetic field is $\boldsymbol{F} = m_k \nabla B_k$, i.e. $F_i = m_k \partial_i B_k$, where we have used the Einstein summation convention. Find the work done by the magnetic force as the rings falls vertically to the ground.

Comments: The work you calculated in part (d) should be negative but has the same absolute value as the kinetic energy in part (b). What this means is that the electromagnetic force does zero net work. What happened is that the gravitational potential energy transfers into the kinetic energy of the ring, while the electromagnetic force facilitates the transfer of some translational kinetic energy into rotational kinetic energy without doing a net work.

Below are selected optional problems from Griffiths. We do not collect your work, but you are encouraged to do as many practice problems as you can.

- Problem 7.34
- Problem 7.37
- Problem 7.39
- Problem 7.40
- Problem 7.43
- Problem 7.44
- Problem 7.47
- Problem 7.49
- Problem 7.53
- Problem 7.54