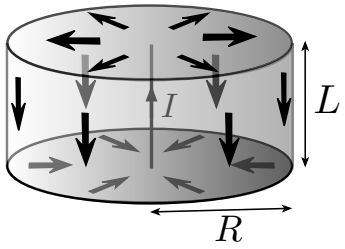
Problem Set 9

Physics 110A, UC Berkeley, Spring 2021

Due Monday, 4/12, at 11:59PM

Problem 1

On the right shows a system where a current I flowing upward along the central wire, spreading uniformly at the upper cap in the radial direction, flowing down vertically on the cylindrical wall, and then flowing radially inward back to the central wire. The radius and height of the of system are R and L respectively.



(a) Argue that the magnetic field everywhere has the form of $\boldsymbol{B} = B(s, z)\,\hat{\boldsymbol{\phi}}$.

(b) Find $B(s, z)$ at every point in the space.

(c) Check that the boundary condition $\boldsymbol{B}_{\text{above}} - \boldsymbol{B}_{\text{below}} = \mu_0(\boldsymbol{K} \times \hat{\boldsymbol{n}})$ is satisfied at the cylindrical wall and the upper cap.

Problem 2

Show that in a current-free volume of space, a static magnetic field can never have a local maximum by considering

$$\oint_{\mathcal{S}} \nabla(\boldsymbol{B} \cdot \boldsymbol{B}) \cdot d\boldsymbol{a},$$

where $\mathcal{S}$ is the surface of a small volume $\mathcal{V}$ containing a point P in space.

Problem 3

A uniformly charged solid sphere of radius R carries a total charge Q, and is spinning with an angular velocity ω about the z-axis.

(a) What is the magnetic dipole moment of the sphere?

(b) At the large distance limit $r \gg R$, find the approximate vector potential at a point (r, θ).

(c) Find the *exact* vector potential outside the sphere.

Below are selected optional problems from Griffiths. We do not collect your work, but you are encouraged to do as many practice problems as you can.

- Problem 5.4
- Problem 5.6
- Problem 5.7
- Problem 5.9
- Problem 5.13
- Problem 5.17
- Problem 5.21
- Problem 5.25
- Problem 5.37
- Problem 5.41
- Problem 5.50
- Problem 5.56
- Problem 5.59
- Problem 5.62