Physics 105, Spring 2021, Reinsch

Homework Assignment 1

Due Thursday, January 28, 11:59 pm

Problem 1

Taylor, Problem 1.47, page 40

Problem 2

(a) Taylor, Problem 2.7, page 73

(b) Taylor, Problem 2.8, page 73

Problem 3

(a) Taylor, Problem 4.8, page 151

(b) We assume the sphere is glued to a horizontal table at a point called P. When the puck strikes the table, how far is it from P?

Problem 4

This is a modified form of Problem 5.6 on page 208.

(a) A mass on the end of a spring oscillates with angular frequency ω. At $t = 0$, its position is $x_o > 0$ and I give it a kick so that it moves back toward the origin and executes simple harmonic motion with amplitude $5x_o/4$. Find its position as a function of time in the form (III) of Problem 5.5.

(b) Find the position as a function of time in the form (II) of Problem 5.5.

Problem 5

You do not have to solve this problem. No extra credit will be given for solving this problem. Because of the pandemic's effect on students, we are reducing the workload. You should read through this problem, and think about how you might approach it. We will publish the solution when we publish the solutions to the other problems.

Supereggs were very popular several decades ago. In this problem we will study the ability of a superegg to stand upright on a flat surface. See

`https://en.wikipedia.org/wiki/Superegg`

[The related idea of a Superellipse has enduring cultural significance because of its incorporation into the design of architectural wonders such as the Estadio Azteca and the Sergels Torg.]

In cylindrical coordinates the upper half of a superegg can be described using the formula

$$\left(\frac{\rho}{r}\right)^p + \left(\frac{z}{h}\right)^p = 1 \tag{1}$$

where r and h are positive constant distance parameters and p is a pure number. At the end of the calculation we will plug in $p = 5/2$. Your answer should include the result you got before plugging in this number as well as the result you get when you plug in the number.

(a) We think of the formula above as defining a function $z(\rho)$ implicitly, that is, we do not solve the equation above for $z(\rho)$. By repeatedly differentiating the equation with respect to ρ show that z'' goes to zero as we approach the axis of symmetry. Note that p is greater than 2.

(b) For this part of the problem, we do solve the equation above for $z(\rho)$. Use the binomial series from the inside front cover of the textbook to calculate how z depends on ρ to leading non-constant order. From your result, it should again be clear that z'' goes to zero as we approach the axis of symmetry.

(c) Now we imagine a superegg at rest in an upright position on a table. If the superegg is tipped to the side slightly, the center of mass moves up. This effect is responsible for the stability of the superegg, and we will study it in detail. For this calculation we will work with positive values of ρ and z because of the fractional exponents involved. Start by drawing a two-dimensional sketch of the curve defined above (an approximate sketch for $p = 5/2$ and $r/h = 3/4$), with the ρ axis pointing to the right and the z axis pointing up. Mark a point (ρ_o, z_o) on the curve near the z axis and draw a line tangent to the curve there. Mark the point on the line that is closest to the origin; then calculate the distance from this point to the origin as a function of ρ_o exactly. We call this $b(\rho_o)$. [We still have not plugged in $p = 5/2$.]

(d) The next step is to work out a formula that shows the behavior of $b(\rho_o)$ for small values of ρ_o. The series expansion for $b(\rho_o)$ is not a familiar Taylor series with integer powers of ρ_o. It will be necessary to use the binomial series. Calculate the series for $b(\rho_o)$, going just to the leading term that depends on ρ_o. For example (for $p = 5/2$) you will drop a ρ_o^3 term because it is smaller than a $\rho_o^{5/2}$ term.