Physics 105 - HW Ø solutions

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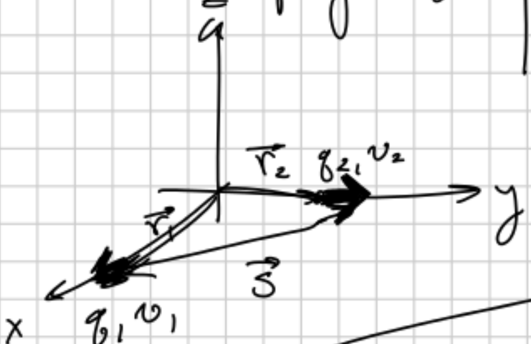
Fall

Note Title

1.) Taylor 1.32

Fig 1.8



$$|\vec{F}_{el}| = q_1 |\vec{E}(\vec{r}_1)| = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{s^2}$$

$$|\vec{F}_{mag}| = q_1 |\vec{v}_1 \times \vec{B}_1| = \frac{\mu_0 q_1 q_2}{4\pi} \frac{v_1 v_2}{s^2} \sin\varphi \sin\theta$$

angles between $(\vec{v}_2, \vec{s}), (\vec{v}_1, \vec{B})$

$$\frac{q_1 q_2}{s^2} = F_{el}(4\pi\varepsilon_0) = F_{mag} \frac{4\pi}{\mu_0} \frac{1}{v_1 v_2 \sin\theta \sin\varphi}$$

$$F_{mag} = F_{el} \frac{v_1 v_2 \sin\varphi \sin\theta}{\mu_0 \varepsilon_0} = F_{el} \frac{v_1 v_2}{c^2} |\sin\varphi \sin\theta|$$

$$|\sin\varphi \sin\theta| \leq 1 \implies \boxed{F_{mag} \leq \frac{v_1 v_2}{c^2} F_{el}}$$

In[1]:=
```
r2 = v0^2 t^2 + (1/4) g^2 t^4 - v0 g t^3 s (*  This is "r squared."   The variable 's' is Sin[θ] *)
```
Out[1]= $\frac{g^2 t^4}{4} - g\, s\, t^3\, v0 + t^2\, v0^2$

In[2]:=
```
r2dot = D[r2, t] (*  this is the time-derivative of r2  *)
```
Out[2]= $g^2 t^3 - 3\, g\, s\, t^2\, v0 + 2\, t\, v0^2$

In[3]:=
```
quad = Expand[r2dot / t] (*  this is a quadratic polynomial in t.  We don't want this to go negative  *)
```
Out[3]= $g^2 t^2 - 3\, g\, s\, t\, v0 + 2\, v0^2$

In[4]:=
```
tMin = t /. First@Solve[D[quad, t] == 0, t] (*  this is the value of t that minimizes 'quad'  *)
```
Out[4]= $\frac{3\, s\, v0}{2\, g}$

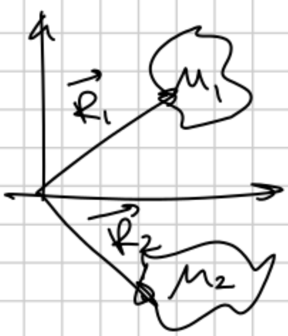
In[5]:=
```
quadMin = quad /. t → tMin    (*  this is the minimal value of 'quad'  *)
```
Out[5]= $2\, v0^2 - \frac{9\, s^2\, v0^2}{4}$

In[6]:=
```
sLimit = s /. First@Solve[Sqrt[2] v0 == 3 s v0 / 2, s]    (*  this is the limiting value of 's'  *)
```
Out[6]= $\frac{2\sqrt{2}}{3}$

In[7]:=
```
N@ArcSin[sLimit] / Degree (* this is the limiting angle, in degrees  *)
```
Out[7]= 70.5288

In[8]:=
```
(* an alternative approach *)
Minimize[quad, t]
```
Out[8]= $\left\{ \begin{cases} 2\, v0^2 & g == 0 \\ \frac{1}{4}\left(8\, v0^2 - 9\, s^2\, v0^2\right) & \text{True} \end{cases},\ \left\{ t \to \begin{cases} \frac{3\, s\, v0}{2\, g} & g > 0 \,||\, g < 0 \\ 0 & \text{True} \end{cases} \right\} \right\}$

Taylor 3.20



$$M\vec{R} = \sum_{j}^{N} m_j \vec{r}_j = \int \vec{r}\, dm$$

For each M_1 & M_2

$$\text{Total } M_T\vec{R}_T = \int \vec{r}_T\, dm_T = \int \vec{r}_1\, dm_1 + \int \vec{r}_2\, dm_2$$

$$(M_1+M_2)\vec{R}_T = M_1\vec{R}_1 + M_2\vec{R}_2 \,, \quad \vec{R}_T = \frac{M_1R_1 + M_2R_2}{M_1+M_2}$$

Physics 105, Fall 2016

Solutions to

Week 1 Practice Problems Taylor 4.38(a) and 4.39

Thanks to David Gee for this solution to

Taylor problem 4.38(a)

Taylor 4.38(a)

Because of the constraint from the pendulum, the velocity is purely tangential, thus the speed of the mass is $\ell\dot{\phi}$. Using the given expression for the potential energy, the conservation of energy condition gives:

$$E = \frac{1}{2}m\ell^2\dot{\phi}^2 + mg\ell\,(1 - \cos\phi)$$

At the maximum angle Φ, $\dot{\phi} = 0$, thus $E = mg\ell\,(1 - \cos\Phi)$. From the half-angle formula, $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$. Solving for $\dot{\phi}$:

$$\dot{\phi} = \sqrt{\frac{4g}{\ell}\left(\sin^2\frac{\Phi}{2} - \sin^2\frac{\phi}{2}\right)}$$

By symmetry, the period is four times the time it takes to go from 0 to Φ, thus:

$$\begin{aligned}
\tau &= 4\int_0^{\Phi} \frac{d\phi}{\dot{\phi}} \\
&= 2\sqrt{\frac{\ell}{g}} \int_0^{\Phi} \frac{d\phi}{\sqrt{\sin^2\frac{\Phi}{2} - \sin^2\frac{\phi}{2}}}
\end{aligned}$$

In the limit of small oscillations, $0 \le \phi \le \Phi \ll 1$, thus $\sin\frac{\phi}{2} \approx \frac{\phi}{2}$ and $\sin\frac{\Phi}{2} \approx \frac{\Phi}{2}$. Thus the period of small oscillations is:

$$\begin{aligned}
\tau_0 &= 4\sqrt{\frac{\ell}{g}} \int_0^{\Phi} \frac{d\phi}{\sqrt{\Phi^2 - \phi^2}} \\
&= 4\sqrt{\frac{\ell}{g}} \left[\arcsin\frac{\phi}{\Phi}\right]_{\phi=0}^{\phi=\Phi} \\
&= 4\sqrt{\frac{\ell}{g}} \left[\frac{\pi}{2} - 0\right] \\
&= 2\pi\sqrt{\frac{\ell}{g}}
\end{aligned}$$

Substituting yields:

$$\tau = \tau_0 \frac{1}{\pi} \int_0^{\Phi} \frac{d\phi}{\sqrt{\sin^2\frac{\Phi}{2} - \sin^2\frac{\phi}{2}}}$$

To rewrite this, let $A = \sin\frac{\Phi}{2}$ and $Au = \sin\frac{\phi}{2}$. Then $A\,du = \frac{1}{2}\cos\frac{\phi}{2}\,d\phi = \frac{1}{2}\sqrt{1 - A^2u^2}\,d\phi$; the positive square root is chosen because $0 \le \phi \le \pi$. For the limits of integration, when $\phi = 0$, $u = 0$, and when $\phi = \Phi$, $u = 1$. Thus:

$$\begin{aligned}
\tau &= \tau_0 \frac{1}{\pi} \int_0^{1} \frac{1}{\sqrt{A^2 - A^2u^2}} \frac{2A\,du}{\sqrt{1 - A^2u^2}} \\
&= \tau_0 \frac{2}{\pi} \int_0^{1} \frac{du}{\sqrt{1 - u^2}\sqrt{1 - A^2u^2}}
\end{aligned}$$

Problem 4.39

Here are the two integrals we will need

```
Integrate[1 / Sqrt[1 - u^2], {u, 0, 1}]
```

$$\frac{\pi}{2}$$

```
Integrate[u^2 / Sqrt[1 - u^2], {u, 0, 1}]
```

$$\frac{\pi}{4}$$

The binomial expansion is on the inside front cover of the book.
Putting it all together, we get from Eq. (4.103)

```
τo (2 / π) Integrate[(1 + A^2 u^2 / 2) / Sqrt[1 - u^2], {u, 0, 1}]
```

$$\frac{1}{4}\left(4 + A^2\right)\tau_o$$

Then we use the definition of A to complete part (c) of the problem. Part (b) is gotten by setting A = 0.

```
A = Sin[Φ / 2]
```

Regarding the numerical question at the end of the problem,

```
N[1 + (1 / 4) Sin[45 Degree / 2]^2]
```

1.03661

This is close to the exact value 1.040 given at the end of the problem, and the difference is of order A^4, as we expect.