MIDTERM EXAM 2, Physics 105, Spring 2021, Reinsch

Write this information in the upper left on the first page of your solutions:

NAME:

STUDENT ID NUMBER:

SIGNATURE:

- The exam is open-book, open-notes, and you may look at all files on our bCourses site. You are not allowed to use the internet or consult with anyone except the instructors.
- No Mathematica, Wolfram Alpha, or any other computer help. You may use a calculator (a small device that computes numerical results like 1 / 7 = 0.1428571428571429).
- We will not be doing Zoom monitoring or any sort of video proctoring.
- Explain how you got your answers. Put a box around your answers.
- At the end of the exam you must create a pdf file that contains all of your work. After you upload it you must download it to verify that it is legible and contains all of your work.

Problem 1

A star is at rest at the origin of an xy coordinate system. A small test particle is initially at the location $(x,y) = (x_0,0)$, where x_0 is a positive constant. Its initial velocity points in the y direction and has magnitude v_0. The mass of the particle is negligible compared to that of the star.

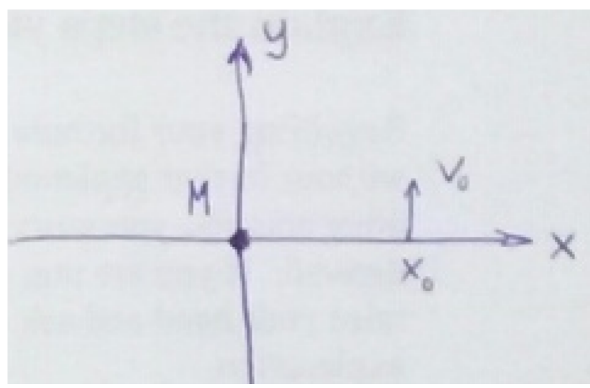
(a) For what value of v_0 would we get a circular orbit? Calculate this by first writing the relationship between the energy E and the eccentricity from the Chapter 8 Summary and then setting the eccentricity to zero.

(b) For the value of v_0 that you calculated in part (a), write out the effective potential energy for the Equivalent One-Dimensional Problem of Chapter 8, and show that it has a minimum at x_0.

(c) For the rest of this problem we will assume that the initial value for v_0 is actually β times the value you found in part (a). Here β is an arbitrary number between 1 and the square root of 2. What is the eccentricity of the orbit?

(d) When the particle first crosses the y axis, how far is it from the star?

(e) When the velocity vector points in the $-x$ direction, how far is the particle from the star?



Problem 2

A bucket of water is rotating about a vertical axis through its center, with constant angular velocity Ω. We wait for the water to reach equilibrium in the rotating frame (this means the water is motionless in the rotating frame). We will calculate the shape of the surface of the water by studying the problem in the rotating frame.

(a) We consider a small parcel of water (mass m) at the surface. Your solution should involve the definition of potential energies for both the gravitational and centrifugal forces on the small parcel of mass m. Use cylindrical coordinates, with the z axis pointing up. The z axis is the axis of rotation. Use a constant z_0 for the z value of the surface of the water at the axis of rotation. We assume that the Earth's gravitational field is that of a point mass M located a distance R below the laboratory floor. The laboratory floor is at $z = 0$. (We ignore the rotation of the Earth about its axis). What is U_{grav}, the gravitational potential energy of m, expressed in the cylindrical coordinates we are using?

(b) Beginning with the formula for the centrifugal force in the Chapter 9 Summary, explain how to introduce a potential energy U_{cf} for this force, and derive an expression in cylindrical coordinates.

(c) As explained in Chapter 9, the surface of the water is an equipotential of $U_{grav} + U_{cf}$. Use this to calculate the function $z(\rho)$ that describes the surface of the water.