The Last 7%

A study guide for every Advanced Functions unit taught by one Ms. Saleem

By Huy Truong

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(I'm joking but iT would be nice)
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Unit 1: The basics & review.

Relation vs. Function:

Relation:
When x value changes, y value will change (i.e x is the independent variable and y is the dependent variable)

Function:
A special relation, where for every 1 x value, there is only 1 y value.

**REMEMBER:* VERTICLE LINE TEST
If a vertical line drawn on a graph crosses x 2 values, it is a relation, not a function.

Properties of Functions:

Turning Point:
A point on a function where the function goes from increasing to decreasing or vice versa. Remember that the point that the graph changes direction (point of inflection) is not included in the increasing/decreasing intervals.

Absolute Max/Min:
The absolute highest/lowest point on the graph

Local Max/min:
The highest/lowest point on the graph that is within an interval/domain of a function.

Interval notation:
ex. xe(-∞,2)u(4,+∞)
This means x is all numbers within negative infinity to 2, union (or and) 4 to positive infinity

Set-builder notation:
ex. {xeR| 1 < x <4}
Like you are used to writing domain and range, says: x is a member of all real numbers where x is larger than 1 but less than 4.

Odd Functions:
Odd degree polynomials can have 1 to n x-intercepts, where n is the degree of the polynomial.
*Degree of a polynomial is the powers in a function totalled (eg. x^3+x^2, 5 is the degree)

The domain and range of a polynomial odd function are xeR and yeR

The odd degree polynomial has no absolute max or min points.

**REMEMBER: An odd function starts high, ends low or starts low, ends high.*
Odd degree polynomial functions can have between 0 to n-1 turning points, where n is the degree of the function.

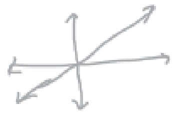
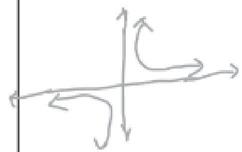
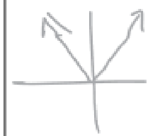
Odd degree polynomial functions can only have an even number of turning points.

Even Functions:
Even degree polynomial functions can have 0 to n x-intercepts, where n is the degree of the function

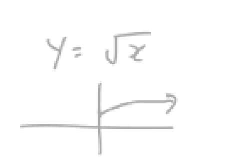
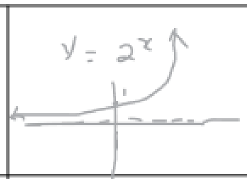
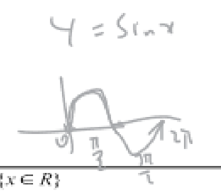
The domain of an even degree polynomial function is xeR
The range of an even degree polynomial is dependent on what the vertex is.

Even degree functions with a positive leading coefficient will have an absolute minimum, if it has a negative leading coefficient, it will have an absolute maximum.
**REMEMBER: The leading coefficient is the base of the exponent with the largest power.*

Even degree polynomial functions can have between 1 to n-1 turning points, where n is the degree of the polynomial. Even degree polynomial function can only have an odd number of turning points.
Properties of functions:

Parent Function	$f(x) = x$	$g(x) = x^2$	$h(x) = \frac{1}{x}$	$k(x) = \lvert x \rvert$
Sketch				
Domain	$\{x \in R\}$	$\{x \in R\}$	$\{x \in R \mid x \neq 0\}$	$\{x \in R\}$
Range	$\{y \in R\}$	$\{y \in R \mid y \geq 0\}$	$\{y \in R \mid x \neq 0\}$	$\{y \in R \mid y \geq 0\}$
Intervals of Increase	$x \in (-\infty, \infty)$	$x \in (0, \infty)$	None	$x \in (0, \infty)$
Intervals of Decrease	None	$x \in (-\infty, 0)$	$x \in (-\infty, 0) \cup (0, \infty)$	$x \in (-\infty, 0)$
Turning Points	None	(0,0)	None	(0,0)
Location of Discontinuities	None	None	x = 0	None
Asymptotes	None	None	x = 0, y = 0	None
Zeros	x = 0	x = 0	None	x = 0
y-intercepts	y = 0	y = 0	None	y = 0
Symmetry (even, odd or neither)	Odd	even	Odd	even
End Behaviours	$x \to \infty, y \to \infty$ $x \to -\infty, y \to -\infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to 0^+$ (above HA) $x \to -\infty, y \to 0^-$ (below HA)	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$

Parent Function	$m(x) = \sqrt{x}$	$p(x) = 2^x$	$r(x) = \sin x$	$q(x) = \cos x$

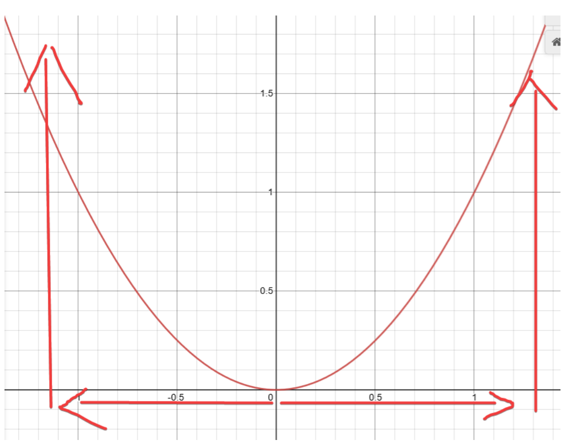
Sketch				
Domain	$\{x \in R \mid x \geq 0\}$	$\{x \in R\}$	$\{x \in R\}$	$\{x \in R\}$
Range	$\{y \in R \mid y \geq 0\}$	$\{y \in R \mid y > 0\}$	$\{y \in R \mid -1 \leq y \leq 1\}$	$\{y \in R \mid -1 \leq y \leq 1\}$
Intervals of Increase	$x \in (0, \infty)$	$x \in (-\infty, \infty)$	Restrict domain [0,360°] $x \in (0°, 90°) \cup (270°, 360°)$	Restrict domain [0,360°] $x \in (180°, 360°)$
Intervals of Decrease	None	None	Restrict domain [0,360°] $x \in (90°, 270°)$	Restrict domain [0,360°] $x \in (0°, 180°)$
Turning Points	None	None	Restrict domain [0,360°] (90°, 1), (270°, -1)	Restrict domain [0,360°] (0°, 1), (180°, -1)
Location of Discontinuities	None	None	None	None
Asymptotes	None	y = 0	None	None
Zeros	x=0	None	Restrict domain [0,360°] x = 0°, 180°, 360°	Restrict domain [0,360°] x = 90°, 270°
y-intercepts	y=0	(0, 1) or y = 1	(0, 0) or y = 0	(0, 1) or y = 1
Symmetry (even, odd or neither)	Neither	Neither	Odd	Even
End Behaviours	$x \to \infty, y \to \infty$ $x \to 0, y \to 0$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to 0^{+}$ (above HA)	Oscillating	Oscillating

End behaviours of polynomials:

An end behaviour is where the graph approaches when we move towards its right or left ends. For example, a parabola ($y=x^2$) will have these end behaviours:

as x > ∞, y > ∞
as x > -∞, y > ∞

This is because as we move along a parabola, if we move towards -x, y will move to +y, and the same for +x, y will also go up.



Absolute values:
Absolute value is the positive form of any value written as |x|
*|-x| = x

Inverse function:
The inverse fo a function is when you swap the x and y values.
MAPPING RULE: (x,y) ->> (y,x)
The domain of a parent function becomes the range of the inverse function
The range of a parent function becomes the domain of the inverse function.

To turn a base parent function into an inverse function, swap x and y in the function and then isolate for y.
Example”

4. a) Determine the equation of the inverse of $f(x) = 2(x+3)^2 - 8$.

1. Switch x and y.
2. Solve for y.

$$y = 2(x+3)^2 - 8$$
$$x = 2(y+3)^2 - 8$$
$$x + 8 = 2(y+3)^2$$
$$\frac{x+8}{2} = (y+3)^2$$
$$\pm\sqrt{\frac{x+8}{2}} = y + 3$$
$$\left.\begin{aligned} y &= \pm\sqrt{\frac{x+8}{2}} - 3 \\ y &= \pm\sqrt{\frac{x}{2} + 4} - 3 \end{aligned}\right\} \text{ Not a function.}$$

Since this is a not a function because it does not pass the vertical line test, you must restrict the domain the base parent function such that the inverse of it will pass the vertical line test.

Piecewise functions:
A function defined by multiple sub-functions.
This is hard to explain so watch these videos on it:

Evaluating piecewise functions:
https://www.youtube.com/watch?v=OYOXMyFKotc

Graphing piecewise functions:
https://www.youtube.com/watch?v=Uzw9tsGq2Pw

Combining functions:

We can combine two or more functions by addition, subtraction, multiplication and division.
Consider
$f(x) = x + 1$ and $g(x) = x$

Write the equation of:

a) $f(x) + g(x) = (f + g)(x) = x + 1 + x = 2x + 1$

b) $f(x) - g(x) = (f - g)(x) = x + 1 - x = 1$

c) $f(x)g(x) = (fg)(x)$ $(x+1)(x) = x^2 + x$

if two functions have domains that overlap (same x value points); they can be added subtracted or multiplied to create a new functio non that shared domain.

Example:
Let f = {,(-2,4),(1,3), (3,5),(4,4),(5,6)} and
g = {(-2,1), (0,2), (1,2), (4,2),(5,4)}.

Determine:

a) f + g = {(-2,5) (1,5) (4,6) (5,10)}

b) g - f = {(-2,-3), (1,-1) (4,-2) (5,-2)}

c) fg = {(-2,4) (1,6) (4,8) (5,24)}

a) f+g:
The highlighted points are the points that have a shared x value, so to create the new function for f+g, we keep the same x value, but add the y values in order of f+g,

The rest share the same concept, but instead of adding, we can subtract or multiply.

Average rate of change:
The average rate of change between 2 points on a graph can be calculated but taking the y values of the graph, subtracting them from each other, and dividing that by the x values subtracted by each other.

*y_2-y_1 / x_2-x_1 = AROC

The population of a town is modelled by $P(t) = 50t^2 + 1000t + 20000$ where $P(t)$ is the size of the population and t is the number of years since 2000.

a) What is the population of the town in the year 2000?.

b) Calculate the average rate of change in the population size for the time period 2005 to 2020.

a) $P(0) = 20000$

b) $AROC = \dfrac{P(20) - P(5)}{20-5}$

$= \dfrac{60000 - 26250}{15}$

$= 2250$ people/Year

Instantaneous rate of change:

Similar to the average rate of change, but instead we will the instant rate of change for 1 point. We can do this by subtracting and adding 0.001 to the point to get 2 points we now can use in this equation:

f(x+0.001) - f(x-0.001) / (x+0.001)-(x-0.001)

Example: Estimate the slope of tangent to $f(x) = x^2$ at $x = 5$ using centred approach

Instantaneous rate of change : $\dfrac{(5.001)^2 - (4.999)^2}{5.001 - 4.999}$

$= \dfrac{0.02}{0.002} = 10$

Composition of functions:

Let f(x) = x+1 and g(x) = 2x

f(g(x)) or fog(x) = (2x)+1

Unit 2: Polynomials

The symmetry of a function:
An even function is symmetric about the y axis. You can calculate algebraicly through seeing if f(-x) = f(x)

An odd function is rotationally symmetric about the origin. You can calculate it algebraicly through checking if f(-x) = -f(x)

Graphing polynomial functions (the quick way):
An official Huy-video: https://youtu.be/fSqWLo_YGtQ

Graphing using sign charts:
An official Huy-video: https://youtu.be/G1hCoUL74ps

Dividing polynomial functions:
An official Huy-video: https://youtu.be/Toexd-uUIJ4

The Remainder Theorem:
The remainder theorem states that if a division statement is set in the form f(x) = quotient/divisor and the divisor is set in x-k form, f(k) will equal the remainder.

Example 1 Use the remainder theorem to determine the remainder for each division.

a) $(3m^2+7m+1)\div(m+3)$

$P(-3) = 3(-3)^2+7(-3)+1$
$= 7 \rightarrow R$

division statement

f(x) = (m+3) (quoitent) + Remainder

if = -3 then the divisor will be equal to zero and the value of function will be equal to remainder.

b) $(8x^3+12x^2-4x+5)\div(2x+3)$

$P(-\frac{3}{2}) = 8(-\frac{3}{2})^3+12(-\frac{3}{2})^2-4(-\frac{3}{2})+5$
$= 11$ R

**NOTE THAT -K VALUE IS USED, This is because the statement x-k implies that it will be x-(-3) = x+3. SO REMEMBER TO FLIP THE SIGN*

The Factor Theorem:
The factor theorem is the theorem used to get the factors of a quotient.
**THE FACTOR OF A QUOTIENT IS A DIVISOR THAT DIVIDES INTO NO REMAINDERS*
An official Huy-video: https://youtu.be/sKkknGYIvf4

Finite Differences:
Finite differences are really long to explain, and basically never covered in this class so if you really want to know, ask Huy

Linear Inequalities:

Example 3 Solve the following inequality. Illustrate your solution on a number line.

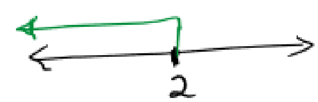
$$6 \times \left[\frac{3x-8}{2} + \frac{2}{3} \le \frac{4-3x}{6} \right]$$

$$3(3x-8)+4 \le 4-3x$$

$$9x-24+4 \le 4-3x$$

$$12x \le 24$$

$$x \le 2$$



Isolate for x in the context of an inequality, and find x.
REMEMBER THAT IF YOU MULTIPLY OR DIVIDE BY A NEGATIVE NUMBER, YOU MUST FLIP THE INEQUALITY.

Unit 3: Rational Functions:

Reciprocal Functions:
Reciprocal functions are polynomial functions under 1 as a fraction. i.e x^2 reciprocal is $1/x^2$

- All y-coordinates of the original function are the reciprocal of the y value of the original function.
- The graph of a reciprocal function will have a vertical asymptote at every zero/root
- If the original function is linear or quadratic, it will always have a horizontal asymptote at y=0
- Intervals of increase on the original function will become intervals of decrease, and vice-versa.
- If the range of the original function includes 1 and/or -1, the reciprocal function will intersect the original function at a point where the y-value is 1 or -1.
- If the original function has a local min or max, the reciprocal function will flip those, min will become max and vice-versa.

Quotients of Rational Polynomial Functions:

Horizontal Asymptotes:

- If the degree of rational polynomial's numerator (*remember degree is all powers totalled*) is less than the degree of the denominator, there will a horizontal asymptote at y=0

- If the degree of numerator is equal to the degree of the denominator, the horizontal asymptote is at the ratio of the leading coefficient. (*the leading coefficient is the whole number/base of the exponent, not the power*)

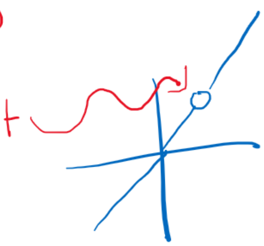
- If the degree of the numerator is larger than the degree of the denominator by 1, you have a slant asymptote, and you get it by dividing the rational function *(ignore the remainders)*

Graphing Rational Functions:

*If a rational function has the same factor in both the denominator and numerator, there will do a hole at that point. For example:

eg. $\dfrac{x^2(x+1)}{x(x+1)}$ $(x+1)$ $(x+1)$

Therefore, a hole at where $(x+1)=0$
which is -1
A hole is represented by a hallow dot



How to graph Rational Functions; an official Huy-video: https://youtu.be/w7Az5g4WLuM

Solving Rational Equations:

- The root of the equation $\frac{ax+b}{cx+d}=0$ is the zero (x-intercept) of the function $f(x)=\frac{ax+b}{cx+d}$. Note that the zeroes of a rational function are the zeroes of the function in the numerator. Reciprocal functions do not have zeroes. All functions of the form $f(x)=\frac{1}{g(x)}$ have the x-axis as a horizontal asymptote. They do not intersect the x-axis.
- You can solve a rational equation algebraically by multiplying each term in the equation by the lowest common denominator and solving the resulting polynomial equation.
- When using graphing technology to solve a rational equation, you can either determine the zeroes of the corresponding rational function, or determine the intersection of two functions.
- When solving contextual problems, it is important to check for inadmissible solutions that are outside the domain determined by the context.

Example 1 Solve each equation algebraically.

a) $x\left[\frac{12}{x}+x=8\right]$; $x\neq 0$

$12+x^2=8x$

$x^2-8x+12=0$

$(x-6)(x-2)=0$

$x=6$ or $x=2$

b) $(2x+3)(2x-3)\left[\frac{2x}{2x+3}-\frac{2x}{2x-3}=1\right]$; $x\neq \pm\frac{3}{2}$

$2x(2x-3)-2x(2x+3)=(2x+3)(2x-3)$

$\cancel{4x^2}-6x-\cancel{4x^2}-6x=4x^2-9$

$4x^2+12x-9=0$

$x=\frac{-12\pm\sqrt{12^2-4(4)(-9)}}{2(4)}$

$=\frac{-12\pm\sqrt{288}}{8}=\frac{-12\pm12\sqrt{2}}{8}$

$\boxed{x=\frac{-3\pm3\sqrt{2}}{2}}$

Example 2 Solve each equation. Round your answers to two decimal places, if necessary.

Example 1 Solve each inequality algebraically. Write the solution using interval notation.

a) $\frac{3x+1}{2x-4}<0$

$\frac{3x+1}{2(x-2)}<0$

x int: $-\frac{1}{3}$

VA: $x=2$

		$-1/3$		2	
$3x+1$	$-$	0	$+$		$+$
$2x-4$	$-$		$-$	0	$+$
quotient	$+$		$-$		$+$

$x\in\left(-\frac{1}{3},2\right)$

b) $\frac{x+1}{x-2}\geq\frac{x+7}{x+1}$

$\frac{x+1}{x-2}-\frac{x+7}{x+1}\geq 0$

$\frac{(x+1)(x+1)-(x+7)(x-2)}{(x-2)(x+1)}\geq 0$

$\frac{\cancel{x^2}+2x+1-(\cancel{x^2}+5x+4)}{(x-2)(x+1)}\geq 0$

$\frac{-3(x-5)}{(x-2)(x+1)}\geq 0$

x int: 5

VA: $x=2,-1$

VA VA

(+) -1 (−) 2 (+) 5 (−)

$x\in(-\infty,-1)\cup(2,5]$

Example 2 Solve each inequality. Write the solution using set notation.

Unit 4 Exponential & Logarithmic Functions

Exponent Rules For $a \neq 0, b \neq 0$	
Product Rule	$a^x \times a^y = a^{x+y}$
Quotient Rule	$a^x \div a^y = a^{x-y}$
Power Rule	$\left(a^x\right)^y = a^{xy}$
Power of a Product Rule	$\left(ab\right)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Graphing Exponential Functions:

$f(x) = ab^{k(x-d)} + c$

mapping rule: $(x, y) \rightarrow \left(\frac{1}{k}x + d, ay + c\right)$

H.A: Y=C

To graph an exponential function, map out the points of the original function and apply the mapping rule.

Applications of exponential functions

$f(x) = ab^x$

x = time

a = initial value

$b = 1 + r$ for exponential growth

$b = 1 - r$ expo. decay

r = rate

Doubling:

$f(x) = a(2)^{\frac{x}{D}}$ D = doubling period

Half life:

$f(x) = a\left(\frac{1}{2}\right)^{\frac{x}{H}}$ H = half life

Finance:

$A = P(1 + i)^n$

A = final value

P = Principal / initial value

i = interests per compounding period

n = no. of compounding periods

anual comp. period: i, n

Semi-anual comp. period: $\frac{i}{2}, 2n$

Monthly: $\frac{i}{12}, 12n$

weekly: $\frac{i}{52}, 52n$

For finance, replace the i and n values for the desired compounding period, annually, semi-annually, monthly or weekly.

Graphing Logarithmic Functions:

$y = a\log_{10}[k(x-d)] + c$

mapping rule: $(x, y) \rightarrow (\frac{1}{k}x + d, ay + c)$

* is log has no base, assume base 10

Logarithmic Application:

- *Logarithms and Chemistry*

Chemists define the acidity of a liquid on a pH scale,

$pH = -\log[H^+]$ where $[H^+]$ is the concentration of the hydrogen ion in moles per litre.

A liquid with a pH lower than 7 is called an acid. A substance with a pH greater than 7 is called a base. Chemists calculate the pH of a substance to an accuracy of two decimal places.

Example 1 A soft drink has a pH of approximately 3. What is the concentration of hydronium ions in a soft drink?

$3 = -\log[H+]$

$-3 = \log_{10}[H+]$

$10^{-3} = H$

Logarithms and Earthquakes

The formula Richter used to define the magnitude of an earthquake is

$M = \log\left(\frac{I}{I_0}\right)$ where I is the intensity of the earthquake being measured,

I_0 is the intensity of a reference earthquake,

and M is the Richter number used to measure the intensity of earthquakes.

Earthquakes below magnitude 4 usually cause no damage, and quakes below 2 cannot be felt. A magnitude 6 earthquake is strong, while one of magnitude 7 or higher causes major damage.

Example 2 An Alaskan earthquake was 4 times more intense than a San Francisco earthquake that had a magnitude of 3.4 on the Richter scale. What was the magnitude of the Alaskan earthquake on the same scale?

Logarithms and Sound

The formula used to compare sound is

$$L = 10\log\left(\frac{I}{I_0}\right)$$

where I is the intensity of the sound being measured,

I_0 is the intensity of a sound at the threshold of hearing,

and L is the loudness measured in decibels.

Evaluating Logarithms:

Logarithmic Properties	
Product Rule	$\log_a(xy) = \log_a x + \log_a y$
Quotient Rule	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
Power Rule	$\log_a x^p = p\log_a x$
Change of Base Rule	$\log_a x = \frac{\log_b x}{\log_b a}$
Equality Rule	If $\log_a x = \log_a y$ then $x = y$

Example of Logarithm and Exponential Question:

$\log_a b \rightarrow a^x = b$

ex. $\log_{10}(100)$

$10^x = 100$

$10^x = 10^2$

$x = 2$

$5^{2x} + 4(5^x) - 12 = 0$

let $5^x = w$

$w^2 + 4w - 12 = 0$

$(w-2)(w+6) = 0$

$w = 2$ or $w = -6$

$5^x = 2$ $\quad$ $5^x = -6$

$x\log(5) = \log(2)$

$x = \frac{\log(2)}{\log(5)}$

$\boxed{x = 0.4307}$

$x\log(5) = \underline{\log(-6)}$

no negative logs

so no solution

Unit 5: Trigonometry:

- To convert from degree measure to radians, multiply by $\frac{\pi}{180°}$.
- To convert from radians to degrees, multiply by $\frac{180°}{\pi}$.

Trigonometry will be using mainly ***RADIANS***

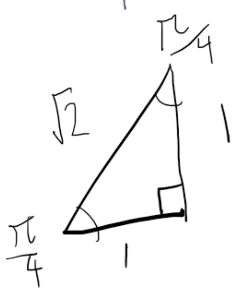
Important Notes:

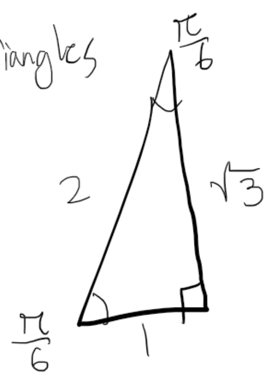
α = Related acute angle

θ = Standard angle

CAA: co-related acute angle

Special triangles





Important Trigonometric Identities:

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Addition and Subtraction Formulas

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Related & Corelated Angles

S	A
T	C

Ex:
Sin150 = Sin30

Related angles

$\frac{\pi}{2}$

$\sin(\pi-\theta) = \sin\theta$

$\cos(\pi-\theta) = -\cos\theta$

$\tan(\pi-\theta) = -\tan\theta$

$\sin\theta$

$\cos\theta$

$\tan\theta$

π θ $0^{r}/2\pi$

$\sin(\pi+\theta) = -\sin\theta$

$\cos(\pi+\theta) = -\cos\theta$

$\tan(\pi+\theta) = \tan\theta$

$\sin(2\pi-\theta) = -\sin\theta$ $\sin(-\theta) = -\sin\theta$

$\cos(2\pi-\theta) = \cos\theta$ $\cos(-\theta) = \cos\theta$

$\tan(2\pi-\theta) = -\tan\theta$ $\tan(-\theta) = -\tan\theta$

$\frac{3\pi}{2}$

$\frac{\pi}{2}$

$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

[anything with the supplimentry angles switch]

$\frac{\pi}{2}-\theta$

π θ $0^{r}/2\pi$

$\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta$

$\cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta$

$\tan\left(\frac{3\pi}{2}-\theta\right) = \cot\theta$

$\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$

$\cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$

$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$

$\frac{3\pi}{2}$