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MUSCULAR SYNERGISM—II. A MINIMUM-FATIGUE CRITERION FOR LOAD SHARING BETWEEN SYNERGISTIC MUSCLES

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Abstract—A new physiological criterion for muscular load sharing is developed. The criterion is based on the assumption that the endurance time of muscular contractions is maximized, hence muscular fatigue is minimized. The optimization problem is cast in the form of a linearly constrained, non-linear MINIMAX optimization.

The new method predicts that: (1) there is synergistic muscle action, (2) muscle force increases non-linearly with external force (load), (3) relatively more force is allocated to muscles that have a large maximum force (large muscles), (4) relatively more force is allocated to muscles with a high percentage of slow-twitch fibers (muscles that are fatigue-resistant), (5) the load sharing does *not* depend on the moment arm of the muscles (although the absolute force levels do depend on this variable).

The predicted load sharing between two cat muscles during standing and walking is in good agreement with direct force measurement data from the literature.

NOMENCLATURE

A_i	physiological cross-sectional area (cm^2)
a_i	muscle parameter (s)
b_i	moment arm (cm)
F_i	muscle force (N)
F_{imax}	maximum muscle force (N)
F_{ext}	external force (N)
F_{gas}	gastrocnemius force (N)
F_{sol}	soleus force (N)
i	muscle number
j	muscle number
k	muscle number
M	resultant moment (Ncm)
n	total number of muscles
p_i	muscle parameter
S_i	percentage of slow-twitch fibers
T_i	endurance time of individual muscle (s)
T^*	endurance time of activity (s)

INTRODUCTION

Body posture and body motion involve a co-ordinated mechanical action of skeletal muscles. The major joints in the body are crossed by several muscles which are active simultaneously during most activities (synergistic muscle action). The mechanical analysis of an activity with synergism results in an indeterminate problem: there are more unknown muscle forces than equilibrium equations. This problem can be solved by ignoring muscles and/or grouping muscles of similar function until the number of unknown forces equals the number of equations. However, these anatomical simplifications may induce considerable error and the mechanical action of individual muscles is obscured.

For specific purposes (in orthopaedics, rehabilitation, ergonomics, etc.) it is necessary to know the forces in individual muscles. In such applications the solution to the indeterminate problem can be obtained by formulating an objective function and utilizing an optimization technique. The optimization approach is based on the assumptions that the load sharing between the muscles is more or less unique during learned motor activities, and that the neural control of muscle action is governed by certain physiological criteria that guarantee 'efficient' muscle actions. The objective function to be optimized corresponds to these physiological criteria. Since the physiological criteria are presently unknown, objective functions in

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the literature have been chosen more or less for their simplicity and computational tractability (e.g. minimization of the sum of muscle forces or the sum of squared muscle forces). Although these 'MINISUM' objective functions are intuitively 'not unreasonable', the results are mixed (see part I of this paper, Dul *et al.*, 1984).

In the present paper, a physiological meaningful criterion for muscular load sharing is developed. The criterion is based on the hypothesis that muscular fatigue is minimized during learned endurance activities (static and dynamic). An endurance type of activity (i.e. constrained sitting posture, walking) involves sustained or repetitive muscular contractions. These contractions are fatiguing, and after a specific period of time (endurance time) the required mechanical output cannot be maintained anymore. It is assumed that the neuromuscular system anticipates this by selecting a load sharing between the muscles such that endurance time of the activity is maximized, hence muscular fatigue is minimized. This concept may be less useful for other types of activity (i.e. running, weight lifting) where quick contractions are involved.

In this paper, the minimum-fatigue criterion is described and validated. The criterion is used to calculate forces in individual human leg muscles during static-isometric knee flexion. The results are compared with the results reported in part I of this paper, in which MINISUM criteria were used for analysis of the same activity. The use of the minimum-fatigue criterion also results in a prediction of the time that the activity can be sustained. For validation purposes, this prediction is compared with experimental data from the literature. The minimum-fatigue criterion is then used to predict the load sharing between two cat muscles (soleus and medial gastrocnemius) during standing and walking. For these activities data from direct force measurements are available (Walmsley *et al.*, 1978). The agreement between the predictions of the minimum-fatigue criterion and the experimental data is remarkable.

METHODS

Formulation of the minimum-fatigue criterion

For our purpose we consider muscular fatigue to be a continuous process during muscular contractions that culminates in failure to maintain the required mechanical output. The endurance time is the maximum duration that an initially relaxed muscle can maintain the required output. This definition applies to both an individual muscle (output is the muscle force) and to a group of synergistic muscles performing the contraction (output is the resultant moment). In the present paper 'endurance time' refers to the endurance time of an individual muscle. The minimum of the individual muscle endurance times is denoted as the 'activity endurance time'. The definitions apply to both static and dynamic contractions.

The endurance time decreases continuously with the magnitude of the required muscle force. The force–time relationship for individual muscles has only been measured in animals. During isometric contractions, Petrofsky and Lind (1979) measured this relationship for three cat muscles with different fiber type composition: the soleus, the medial gastrocnemius and the plantaris (Fig. 1). Muscle force ('strength') is given relative to the maximum muscle force ('maximum strength' or 'initial strength'). The estimated percentage of red, slow-twitch fibers in the three muscles is 98 %, 37 % and 26 % respectively (Ariano *et al.*, 1973; Close, 1972). For a given force level, a muscle with relatively more slow-twitch fibers has a greater endurance time and is thus more fatigue-resistant.

An activity is usually performed by several muscles

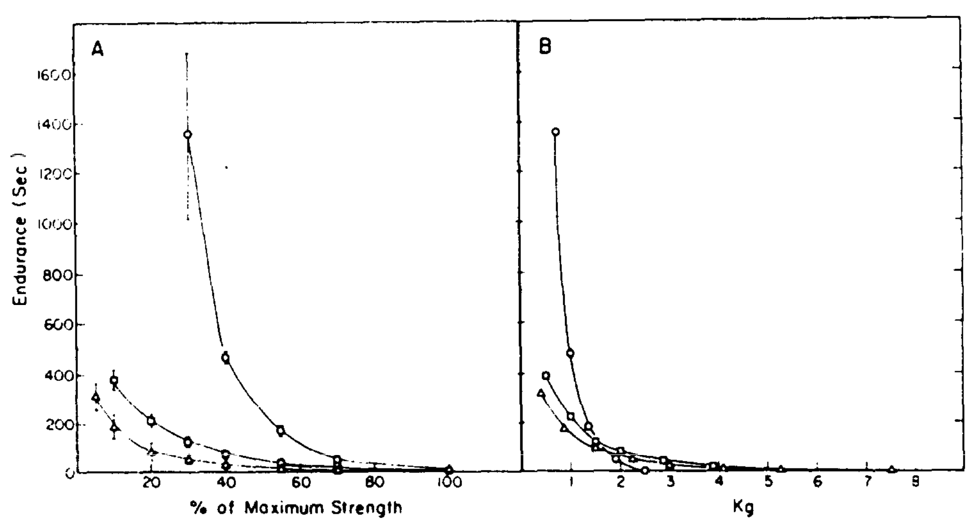

Fig. 1. Force–endurance time relationship for individual cat muscles. (○) soleus (98 % slow-twitch fibers); (□) medial gastrocnemius (37 % slow-twitch fibers); (△) plantaris (26 % slow twitch fibers); (From Petrofsky and Lind, 1979).

with different fiber type composition. The activity endurance time will be larger if relatively more force is allocated to muscles that have higher percentages of slow-twitch fibers. It is assumed that for endurance types of activity the load sharing among the muscles is such that the activity endurance time is maximized. This will be achieved when the endurance time of the muscle with the shortest endurance time is maximized. The minimum-fatigue criterion can be formulated mathematically as

maximize

$$\text{minimum } \{T_i\} \quad (i = 1, 2, \ldots n) \tag{1}$$

with

$$T_i = T_i(F_i, F_{imax}, S_i) \tag{2}$$

where: i = muscle number; n = total number of muscles; T_i = endurance time; F_i = muscle force; F_{imax} = maximum muscle force; S_i = percentage of slow-twitch fibers. The optimum value T^* is the maximum possible time for performance of the activity.

Force–endurance time relationship

According to Fig. 1, the endurance time (T_i) is a function of the muscle force (F_i), the maximum muscle force (F_{imax}) and the percentage of slow-twitch muscle fibers (S_i). These force–time relationships were obtained during isometric contractions. During dynamic contractions, the endurance time will be a function of at least the rate of contractions, and the velocity of shortening or lengthening. However, data on the relationship between endurance time and muscle force during dynamic activities are not available. Hagberg (1981) measured the relationship between the mean external force and the activity endurance time of human arm muscles (as a group) during dynamic (isokinetic) elbow flexion–extension. He found that this relationship was very similar to the external force–activity endurance time relationship obtained during static flexion–extension. Therefore, for the first approximation it could be assumed that during a dynamic activity the force–endurance time relationship for individual muscles is the same as this relationship during static activities.

The function $T_i = T_i(F_i, F_{imax}, S_i)$ is not known for individual human muscles but can be estimated using experimental data on cat and human muscles. It is assumed that the relationships of Fig. 1 are valid for all animal and human skeletal muscles that have corresponding percentages of slow-twitch fibers. This assumption is reasonable since the muscle force is scaled by dividing it by the maximum force, and the anatomy and physiology of animal and human muscle tissue is similar. The relationships can be quantified using a least squares curve fitting technique. The data points were measured manually from the figure (estimated maximum error for endurance time 10 s). Equation (3) was obtained to represent these points

$$T_i = a_i(F_i/F_{imax}\,100)^{p_i}. \tag{3}$$

This exponential form is equivalent to the linear form

$$\ln T_i = \ln a_i + p_i \ln (F_i/F_{imax}\,100) \tag{4}$$

a_i and p_i are muscle parameters depending on the percentage of slow-twitch fibers. a_i is the endurance time for a muscle force level of 1 % maximum muscle force. p_i indicates how much the endurance time decreases with muscle force (p_i is always negative). The result of the curve fitting is given in Table 1. The force–time relationship for a muscle with any given percentage of slow-twitch fibers can be found through interpolation. In Fig. 2, the muscle parameters $\ln a_i$ and $-p_i$ are plotted as a function of percentage of slow-twitch fibers (S_i). The linearity can be expressed by the following regression lines

$$-p_i = 0.25 + 0.036\,S_i \quad (r = 0.9982) \tag{5}$$

$$\ln a_i = 3.48 + 0.169\,S_i \quad (r = 0.9999). \tag{6}$$

These equations can be rewritten as

$$p_i = -0.25 - 0.036\,S_i \tag{7}$$

$$a_i = \mathrm{EXP}\,(3.48 + 0.169\,S_i). \tag{8}$$

When the percentage of slow-twitch fibers (S_i) and the

Table 1. Fitting data points from Fig. 1 to the curve $T_i = a_i(F_i/F_{imax}\,100)^{p_i}$

Muscle force (%) $(F_i/F_{imax})100$	Endurance time (s) Soleus (98%)	Medial gastrocnemius (37%)	Plantaris (26%)
5	—	—	310
10	—	380	185
20	—	220	80
30	1350	130	40
40	460	80	30
55	175	40	15
70	50	20	—
Slope (p_i)	−3.774	−1.484	−1.255
Intercept ($\ln a_i$)	20.083	9.652	7.955
Correlation coefficient (r)	−0.994	−0.969	−0.988

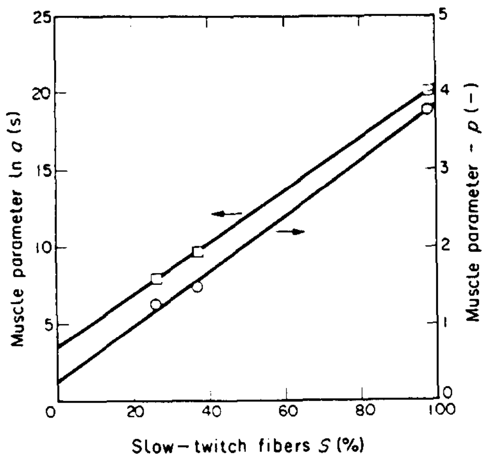

Fig. 2. The muscle parameters $\ln a_i$ and $-p_i$ as a function of percentage of slow-twitch fibers (S_i).

maximum force (F_{imax}) of a muscle are known, then the endurance time (T_i) is only a function of the muscle force (F_i) (equation 2). The maximum force in a muscle can be estimated from literature data. It is in general a function of the muscle size, length and velocity of shortening or lengthening.

Solution by MINIMAX optimization

The Minimum-Fatigue criterion is formulated as a non-linear MINIMAX optimization problem. Since T_i is a continuous decreasing function of F_i, maximization of the minimum of the set $\{T_i\}$ $(i = 1, 2, 3, \ldots n)$ is equivalent to minimization of the maximum of the set $\{1/T_i\}$. $F_i (i = 1, 2, 3, \ldots n)$ are the optimization parameters to be determined. Equality and inequality constraints are formulated as well. The equality constraints are the moment equilibrium equations (linear functions of F_i) obtained from a biomechanical model. The force equilibrium equations are incorporated in the moment equations. The number of moment equations depends on the number of joints and the degree of rotational freedom of each joint in the model. Other constraints that will be considered are the linear inequality constraints

$$0 \leqslant F_i \leqslant F_{imax} \quad (i = 1, 2, 3, \ldots n) \tag{9}$$

which indicate that muscles can only produce a tensile force and cannot exceed some maximum force. A solution technique for the linearly constrained MINIMAX problem has been given by Madsen and Schjaer-Jacobsen (1978). An interesting observation regarding the form of the solution for a problem with only one equality constraint (planar model of a single joint) is that until one of the equality constraints becomes active, the endurance times of all muscles are the same and are equal to the activity endurance time T^*. Based on this observation (which can be proven mathematically), it is easy to derive an expression for the load sharing between the muscles (see Appendix).

Static–isometric knee flexion

The minimum-fatigue criterion will be used to calculate forces in the human muscles that flex the knee. A two-dimensional model of the knee joint for static–isometric knee flexion was described in part I of this paper. An 'average' man is sitting with his thigh in the horizontal position. The lower leg presses against a resistance such that an external force is applied in the forward direction on the lower leg. The angle between the thigh and the lower leg is 140°. The activity that is analysed is the maintainance of leg posture while the external force is increasing quasi-statically. Several levels of external force between 30 and 275 N (corresponding to levels of knee flexor effort between zero and maximum) were analyzed. The moment equation of the planar model contains three unknown muscle forces

$$\Sigma b_i F_i = M \quad (i = 1, 2, 3) \tag{10}$$

$i = 1$: long hamstrings; $i = 2$: short hamstrings; $i = 3$: gastrocnemius. b_i is the moment arm of the i-th muscle. The left hand side of equation (10) is the total muscle moment produced by the three muscles. M is the resultant moment of the external force and the weight of the lower leg. The moments are taken about the knee joint center, a point on the line of action of the resultant joint contact force.

The percentage of slow-twitch fibers in the three knee flexors were estimated from data given by Johnson *et al.* (1973)

$$\begin{aligned} \text{long hamstrings: } & S_1 = 67\,\% \\ \text{short hamstrings: } & S_2 = 67\,\% \\ \text{gastrocnemius: } & S_3 = 48\,\%. \end{aligned} \tag{11}$$

When these values are substituted in equations (7) and (8), then the muscle parameters p_i and a_i can be computed. Table 2 gives the data required for the computations. The minimum-fatigue criterion for

Table 2. Knee muscle data for computations

	Long Hamstrings $i = 1$	Short hamstrings $i = 2$	Gastrocnemius $i = 3$
Moment arm b_i (cm)	4.42	3.16	2.83
Maximum force F_{imax} (N)	765	156	1050
Slow-twitch fibers S_i (%)	67	67	48
Muscle parameter p_i	−2.66	−2.66	−1.98
Muscle parameter a_i (s)	2684486	2684486	108229

static-isometric knee flexion can now be formulated as

$$\text{minimize maximum } \{1/T_1, 1/T_2, 1/T_3\}$$

with: $T_1 = 601728470\ (F_1)^{-2.66}$

$T_2 = 8761381\ (F_2)^{-2.66}$

$T_3 = 11384094\ (F_3)^{-1.98}$

subject to: $4.42F_1 + 3.16F_2 + 2.83F_3 = \dot{M}$

$0 \leqslant F_1 \leqslant 765$

$0 \leqslant F_2 \leqslant 156$

$0 \leqslant F_3 \leqslant 1050.$

Solutions were obtained for different values of M (corresponding to different levels of external force) by the procedure outlined in the Appendix, by a general non-linear programming method (Townsend and Lam, 1975) and by the method of Madsen and Schjaer-Jacobsen (1978). All three approaches gave the same results in every case.

RESULTS AND DISCUSSION

For a planar model of a single joint, the load sharing between two muscles as predicted with the minimum-fatigue criterion is (equation (8) of the Appendix):

$$F_i = \frac{F_{imax}}{100}\left(\frac{a_j}{a_i}\right)^{1/p_i}\left(\frac{100}{F_{jmax}}\right)^{p_j/p_i}(F_j)^{p_j/p_i} \qquad (12)$$

a_i, a_j and p_i, p_j are muscle parameters depending on the percentage of slow-twitch fibers in the i-th and j-th muscle respectively (equations 7 and 8). Equation (12) is in general a non-linear equation: the load sharing between muscles with different percentages of slow-twitch fibers is non-linear. It is based on the maximum force and the percentage of slow-twitch fibers of the muscles. A remarkable result is that it does not depend on the moment arm of the muscles. Also a muscle with a small moment arm will give a contribution to the total required moment about the joint, as long as this will not decrease the activity endurance time. Although the load *sharing* between the muscles is independent of the moment arm, the absolute force levels do depend on this variable (see equation (10) in the Appendix).

When the percentages of slow-twitch fibers in the i-th and j-th muscle are equal, then $p_i = p_j$ and $a_i = a_j$, and equation (12) reduces to:

$$F_i / F_j = F_{imax} / F_{jmax}. \qquad (13)$$

Notice that equation (13) is a result of the optimization, whereas it was a constraint imposed *a priori* in the optimization problem described by Patriarco *et al.* (1981).

The predictions of the Minimum-Fatigue criterion can be compared with the predictions of non-linear criteria from the literature. The criteria are of the MINISUM form $\Sigma(x_i)^p$, where $p \neq 1$ and $x_i = F_i$, F_i/F_{imax} or F_i/A_i. As given in part I of this paper, the predicted load sharing between muscle i and j is then

for $x_i = F_i$

$$F_i/F_j = (b_i/b_j)^{1/(p-1)} \qquad (14a)$$

for $x_i = F_i/A_i$

$$F_i/F_j = (b_i/b_j)^{1/(p-1)}(A_i/A_j)^{p/(p-1)} \qquad (14b)$$

for $x_i = F_i/F_{imax}$

$$F_i/F_j = (b_i/b_j)^{1/(p-1)}(F_{imax}/F_{jmax})^{p/(p-1)} \qquad (14c)$$

b_i and b_j are the moment arms of the i-th and j-th muscle, A_i and A_j are the physiological cross-sectional areas, and F_{imax} and F_{imax} the maximum muscle forces. Now the predicted load sharing is linear for all values of $p \neq 1$, and it depends on the moment arms.

Human knee flexion

The muscle forces F_i and activity endurance time T^* for human knee flexion predicted with the minimum-fatigue criterion are given in Fig. 3 as a function of the external force (load).

Muscle forces. For all levels of effort (external force between 30 N and 215 N), there is synergistic muscle action with a non-linear relationship between the muscle force and the external force. The force in the short hamstrings is always less than the force in the other muscles (except for values of the external force close to 30 N). This is because this muscle has a small maximum force. When the external force is less than 100 N (approximately 40% of maximum force), the force in the long hamstrings is greater than the force in the gastrocnemius. This is because this muscle has a higher percentage of slow-twitch fibers than the gastrocnemius. Above 100 N external force, the force in the gastrocnemius is greater than the force in the long hamstrings. This is because the gastrocnemius has a greater maximum force than the long hamstrings.

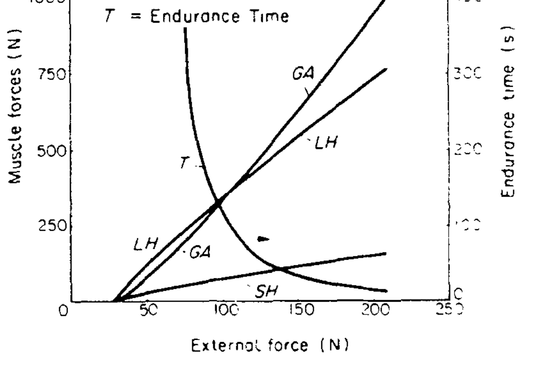

Fig. 3. Muscle forces and activity endurance time for human static-isometric knee flexion as a function of external force as predicted with the minimum-fatigue criterion.

The load sharing between the three muscles is given by equation (12), which for human knee flexion reduces to

$$F_2 = 0.2039 F_1 \quad (15)$$

$$F_3 = 0.1348(F_1)^{1.34}. \quad (16)$$

F_1 is related to the external force as follows (see equation (10) in the Appendix)

$$5.06F_1 + 0.38(F_1)^{1.34} = 37F_{ext} - 1050. \quad (17)$$

The forces in the long and short hamstrings (F_1 and F_2) are linearly related according to their maximum forces (equation 15) since these muscles have the same percentage of slow-twitch fibers.

For external forces below 210 N, no muscle has reached its maximum force. At 210 N, the long and short hamstrings reach their maximum force simultaneously because their forces are always related. At 215, the gastrocnemius has also reached its maximum force such that the external force cannot be increased any further.

The results of the minimum-fatigue criterion can be compared with the results of non-linear MINISUM criteria from the literature. Pedotti *et al.* (1978) defined a quadratic criterion consisting of the sum of quadratic ratios of muscle force to maximum muscle force ($\Sigma(F_i/F_{imax})^2$; $p = 2$, $x_i = F_i/F_{imax}$ in equation (14)). Crowninshield and Brand (1981) defined a cubic criterion consisting of the sum of cubic muscle stresses ($\Sigma(F_i/A_i)^3$; $p = 3$, $x_i = F_i/A_i$). According to our previous investigation, these two non-linear criteria are physiologically more consistent than other criteria from the literature since they predict synergistic muscle action and that there is relatively more force in large muscles. Figure 4a compares the muscle force predictions of these non-linear criteria with the predictions of the minimum-fatigue criterion. Figure 4b shows the predictions for the load sharing between the long hamstrings and the short hamstrings, and between the long hamstrings and the gastrocnemius. Figure 4b also includes predictions of another quadratic criterion used by Pedotti *et al.* (1978), namely ΣF_i^2. The predictions are obtained from equations (12) and (14). All criteria are used in combination with the maximum muscle force constraint.

From Fig. 4 it turns out that, although the general pattern of load sharing is similar for the different criteria, the predicted magnitude of the muscle force is not the same. The quadratic criterion and the cubic criterion predict that there is linear synergism, whereas the minimum-fatigue criterion predicts non-linear synergism. The non-linearity would be more pronounced if the differences between the percentages of slow-twitch fibers of the muscles were larger. All criteria predict that there is relatively more force in large muscles (i.e. muscles with large maximum force, or muscles with large physiological cross-sectional area). For the quadratic and cubic criteria, relatively more force is also allocated to muscles that have large moment arms. In the quadratic criteria there is more emphasis on the moment arm than in the cubic criterion (linear vs square root, see equation (14)). The load sharing predicted with the minimum-fatigue criterion does not depend on moment arm; relatively more force is allocated to muscles with a high percentage of slow-twitch fibers. Fiber type composition is not a parameter in the quadratic and cubic optimization.

Unfortunately, the predictions cannot be compared with measured data, since human muscle forces cannot be measured *in vivo*. Furthermore, EMG-measurements will not provide a reliable quantitative measure of muscle force that would be useful here for validation purposes. However, since the minimum-fatigue optimization also predicts the activity endurance time, this result can be compared with measured data.

Endurance time. Figure 3 shows that the activity endurance time of the muscle group for knee flexion as predicted with the minimum-fatigue criterion decreases exponentially with the external force. Below 210 N, the endurance times of all individual muscles are the same and equal to the activity endurance time. Above 210 N, the force in the two hamstrings is maximum and their endurance times are equal and constant (13 s), and always greater than the activity endurance time. The activity endurance time is now determined by the gastrocnemius. At 215 N, this muscle also reaches its maximum force. The activity endurance time is then 11 s. Below 210 N, all muscles fail to maintain the required force simultaneously, whereas above 210 N only the gastrocnemius fails to do so. The gastrocnemius has a lower percentage of slow-twitch fibers than the two hamstrings which have the same percentage of these fibers.

The predicted activity endurance time is compared with measured data. Figure 5 shows measurements for different muscle groups during static-isometric contractions. The regression lines are relationships between external force and activity endurance time according to the 'Law of Monod and Rohmert' (Rohmert, 1960; Monod, 1972). This 'law' states that the relationship between the ratio of external force to maximum external force and the activity endurance time is approximately the same for all human muscle groups. It turns out that the activity endurance time predicted by the minimum-fatigue optimization for different external force levels during static-isometric knee flexion lies virtually on the solid regression line.

Cat standing and walking

As noted above, the prediction of human muscle forces cannot be compared with measured data. However, Walmsley *et al.* (1978) performed a direct muscle force measurement in cats. They measured the force in the soleus and medial gastrocnemius (plantarflexors of the ankle) with chronically implanted transducers. The percentage of slow-twitch fibers in the cat soleus and medial gastrocnemius is 98 % and 37 %

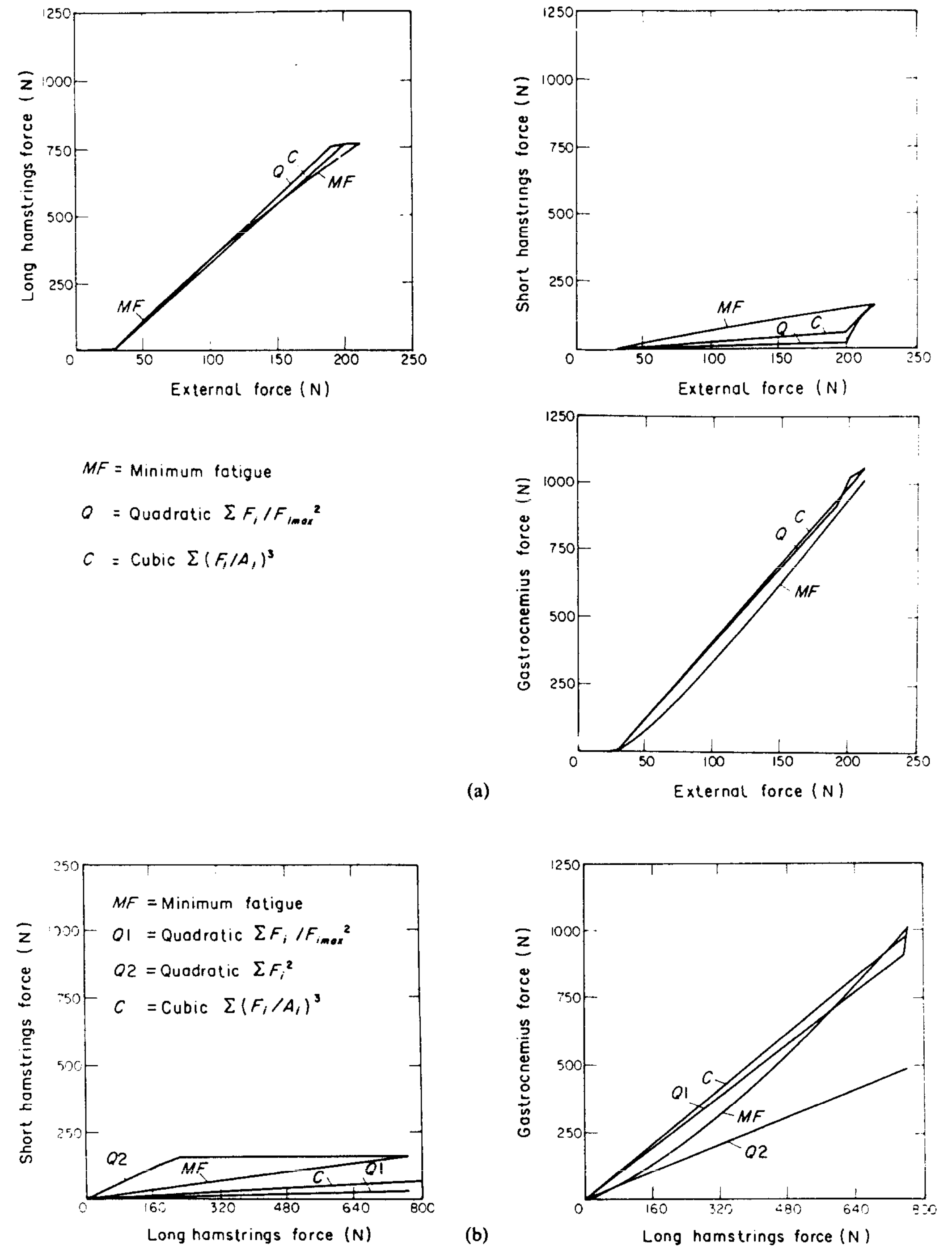

Fig. 4. (a) Muscle forces during human knee flexion as predicted with different non-linear criteria. (b) Muscular load sharing during human knee flexion as predicted with different non-linear criteria.

respectively (see above). The soleus is a smaller muscle than the medial gastrocnemius. The average maximum force ('initial isometric strength') is 25 N for the soleus and 51 N for the medial gastrocnemius (Petrofsky and Lind, 1979). The moment arms of these muscles with respect to the ankle joint center are approximately the same, since these muscles have a common insertion at the hindfoot (Achilles tendon). Using equations (7), (8), (12) and (14) the load sharing between the two muscles can be predicted with the minimum-fatigue criterion and with different criteria from the literature. It is acceptable to base the predictions upon a one-joint optimization, although the gastrocnemius is a two-joint muscle acting at both knee and ankle. According to cat EMG data (Wetzel *et al.* 1973) the gastrocnemius acts in concert with the soleus muscle (which is a one-joint muscle acting at the ankle). Furthermore, Goslow *et al.* (1973) point out that 'while

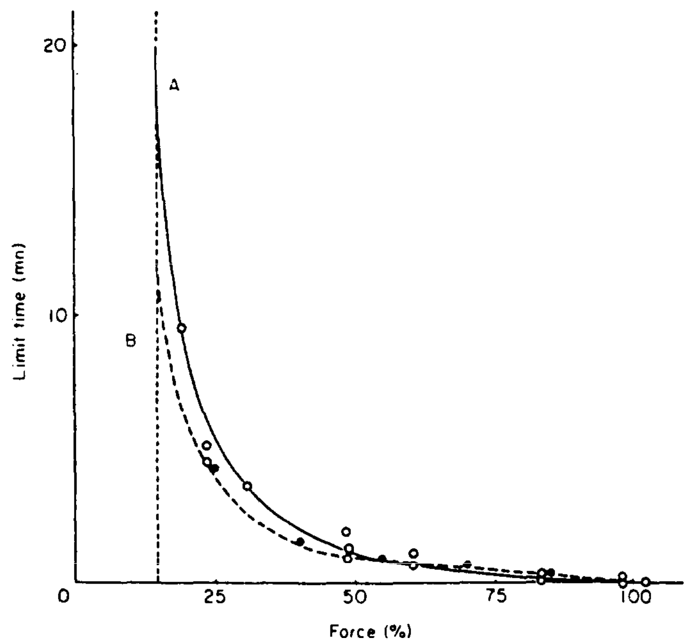

Fig. 5. Measured relationship between external froce and activity endurance time for different human muscle groups during static–isometric contractions. The relationship predicted with the Minimum-Fatigue criterion (not shown) coincides with the solid regression line (From Monod 1972).

the medial gastrocnemius is bifunctional and capable of both knee flexion and ankle extension, its reflex connections and electrical activity show it to be largely concerned with ankle extension during stepping'.

In Fig. 6, the predictions are compared with direct force measurement data. The force in the soleus is given as a function of the force in the medial gastrocnemius for several activities. The symbol (▲) relates to measured instantaneous forces during standing and (●) to measured peak forces during walking and other movements. The lines are the predictions from the minimum-fatigue criterion (equation 12), from several non-linear MINISUM criteria from the literature (equation 14) and, for comparison, for several linear criteria that have been used in the literature. According to our previous investigation the linear criteria predict that only the medial gastrocnemius is active, since this muscle is larger than the soleus (larger maximum force and larger cross-sectional area) and has a moment arm that, if at all different, is probably larger than that of the soleus.

The non-linear criteria from the literature predict that there is a linear relationship between the force in the soleus and the force in the gastrocnemius. The minimum-fatigue criterion predicts that these forces are non-linearly related according to the equation $F_{sol} = 5.08\ (F_{gas})^{0.42}$. (equation 8, Appendix)

It turns out that only the prediction from the

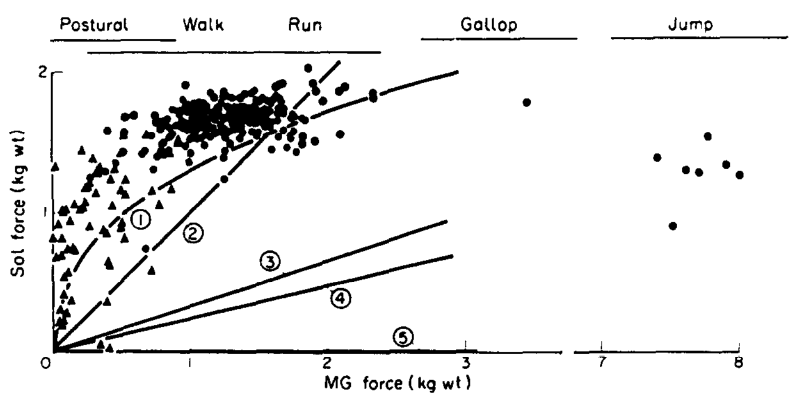

Fig. 6. Measured vs predicted load sharing between two cat muscles during standing and locomotion. (Measurements from Walmsley *et al.*, 1978). 1. Minimum-fatigue criterion; 2. Quadratic criterion: ΣF_i^2, 3. Quadratic criteria: $\Sigma(F_i/A_i)^2$, $\Sigma(F_i/F_{imax})^2$; 4. Cubic criterion: $\Sigma(F_i/A_i)^3$; 5. Linear criteria: ΣF_i, $\Sigma F_i/A_i$, $\Sigma F_i/F_{imax}$.

minimum-fatigue criterion is in good agreement with the measurements, not only for standing but also for walking and running.

The measurements show again that the fiber type composition appears to be important parameter in muscular load sharing. In most cases, the force in the soleus is greater than the force in the medial gastrocnemius, although the soleus has a smaller maximum force, and the moment arms are approximately the same.

CONCLUSIONS

From the direct force measurements in cats it can be concluded that the minimum-fatigue criterion for muscular load sharing is physiologically consistent. Hence, the use of this criterion to analyze human (endurance) activities presumably results in reasonable predictions of muscle forces (and consequently joint forces).

The minimum-fatigue criterion inherently predicts that there is non-linear load sharing between the muscles and that the load sharing depends on the external force. The relationship between individual muscle force and external force is non-linear as well. (This last result is important for the interpretation of the measured relationship between EMG and external force.) Further the new criterion predicts that there is synergistic muscle action with relatively more force in muscles with a large maximum force (large muscles) and in muscles with a large percentage of slow-twitch fibers (muscles that are fatigue-resistant). The load sharing does not depend on the moment arms of the muscles.

A characteristic feature of the minimum-fatigue criterion is that the optimum value of the objective function has a physiological meaning. It is the inverse of the time that the activity can be sustained. The predicted endurance time for static-isometric knee flexion is in good agreement with measured values from the literature. This is another indication for the validity of the minimum-fatigue criterion. The use of the minimum-fatigue criterion for the analysis of posture and movement provides a prediction of the time that a posture can be maintained or a movement can be performed until the muscles fail to produce the required mechanical output. At lower load levels all muscles will fail simultaneously; at higher loads some muscles will not fail. This type of prediction may be useful in fields like human factors, engineering and ergonomics.

The development and use of the minimum-fatigue criterion involves estimation of several muscle parameters (e.g. maximum force, percentage of slow-twitch fibers). The estimations can be readily updated as better data become available.

The use of the minimum-fatigue criterion requires the solution of a linearly constrained, non-linear MINIMAX optimization problem. We have presented solutions for simple situations (two dimensional model of a single joint). Solutions of more complex problems (three-dimensional, multiple joints) have been discussed elsewhere (Dul, 1983).

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APPENDIX. SOLUTION FOR THE SIMPLE MINIMAX PROBLEM

Minimize

$$\text{maximum}\left\{\frac{1}{T_i}\right\} \quad (i = 1, 2, \ldots n) \tag{1}$$

with

$$T_i = a_i\left(\frac{F_i}{F_{imax}}100\right)^{p_i} \quad (i = 1, 2, \ldots n) \tag{2}$$

for $\quad 0 \leqslant F_i \leqslant F_{imax}: T_i = T_j = T^* \quad (i, j = 1, 2, \ldots n) \qquad (3)$

substitute (2) in (3): $T^* = a_i\left(\dfrac{F_i}{F_{imax}}100\right)^{p_i} \quad (i = 1, 2, \ldots n) \qquad (4)$

for $i = j$

$$T^* = a_j\left(\frac{F_j}{F_{jmax}}100\right)^{p_j} \tag{5}$$

rewrite (4)

$$F_i = \frac{F_{imax}}{100}\left(\frac{T^*}{a_i}\right)^{1/p_i} \quad (i = 1, 2, \ldots n) \tag{6}$$

substitute (5) in (6)

$$F_i = \frac{F_{imax}}{100}\left(\frac{a_j\left(\frac{F_j}{F_{jmax}}100\right)^{p_j}}{a_i}\right)^{1/p_i} \quad (i = 1, 2, \ldots n) \tag{7}$$

rewrite (7)

$$F_i = \frac{F_{imax}}{100}\left(\frac{a_j}{a_i}\right)^{1/p_i}\left(\frac{100}{F_{jmax}}\right)^{p_j/p_i} \quad (F_j)^{p_j/p_i} \quad (i = 1, 2, \ldots n) \tag{8}$$

moment equation

$$\sum_{i=1}^{n} b_i F_i = M \tag{9}$$

substitute (8) in (9)

$$\sum_{i=1}^{n} b_i \frac{F_{imax}}{100}\left(\frac{a_j}{a_i}\right)^{1/p_i}\left(\frac{100}{F_{jmax}}\right)^{p_j/p_i}(F_j)^{p_j/p_i} = M. \tag{10}$$

Equation (10) is a non-linear equation with one unknown variable (F_j). The solution for F_j can be substituted in equation (8) to find F_i $(i = 1, 2, \ldots n)$ and in equation (5) to find the activity endurance time T^*. For $F_i = F_{imax}$ $(i = k)$, $T_i \neq T^*$ $(i = k)$, and $T_i = T^*$ $(i \neq k)$.