1)

```
public class FibonacciRecursivo {
    public static int fibonacci(int n) {
        if (n<=1)
            return n;
        else
            return fibonacci(n-1)+fibonacci(n-2);
    }
    public static void main(String[] args) {
        int n=10;
        System.out.println("Fibonacci("+n+")
                          ="+fibonacci(n));
    }
}
```

2) Recurrencia

$F(0)=0,\ F(1)=1$

$F(n)=F(n-1)+F(n-2)$ para $n\geq 2$

Recurrencia lineal de segundo orden con coeficientes constantes

3. Ecuación general

3.1. Suposición de solución

$F(n)=r^n$

$r^n=r^{n-1}+r^{n-2} \Rightarrow r^2=r+1$

3.2. Ecuación característica

$r^2-r-1=0$

$r_{1,2}=\frac{1\pm\sqrt{5}}{2}$

$\emptyset=\frac{1+\sqrt{5}}{2}$

$\psi=\frac{1-\sqrt{5}}{2}$

ESTILO

Solucion general = $F(n) = A\phi^n + B\psi^n$

4. $F(0) = 0 \Rightarrow A\phi^0 + B\psi^0 = A + B = 0$

$F(1) = 1 \Rightarrow A\phi + B\psi = 1$

Resolviendo

1. $A + B = 0 \Rightarrow B = -A$
2. $A\phi - A\psi = 1 \Rightarrow A(\phi - \psi) = 1 \Rightarrow A = \frac{1}{\phi - \psi}$

$\phi - \psi = \frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2} = \sqrt{5} \Rightarrow A = \frac{1}{\sqrt{5}}, B = -\frac{1}{\sqrt{5}}$

$F_n = \frac{1}{5}(\phi^n - \psi^n)$ Ecuacion cerrada (Binet)

Demostrar (por induccion)

$n \geq 0$

$F(n) = \frac{1}{\sqrt{5}}(\phi^n - \psi^n)$

$n = 0 \quad F(0) = \frac{1}{\sqrt{5}}(\phi^0 - \psi^0) = \frac{1}{\sqrt{5}}(1-1) = 0$

$n = 1 \quad F(1) = \frac{1}{\sqrt{5}}(\phi - \psi) = \frac{1}{\sqrt{5}}(\sqrt{5}) = 1$

Paso inductivo

$F(k+1) = F(k) + F(k-1)$

Forma cerrada

$F(k+1) = \frac{1}{\sqrt{5}}(\phi^{k+1} - \psi^{k+1})$ por definicion

$F(k) + F(k-1) = \frac{1}{\sqrt{5}}(\phi^k - \psi^k) + \frac{1}{\sqrt{5}}(\phi^{k-1} - \psi^{k-1})$

$$= \frac{1}{\sqrt{5}}(\theta^K + \theta^{K-1} - \nu^K - \nu^{K-1})$$

Factorizamos

$$\theta^K + \theta^{K-1} = \theta^{K-1}(\theta+1) = \theta^{K+1} \quad (\text{ya que } \theta^2 = \theta+1)$$

$$\nu^K + \nu^{K-1} = \nu^{K-1}(\nu+1) = \nu^{K+1}$$

$$F(K) + F(K-1) = \frac{1}{5}(\theta^{K+1} - \nu^{K+1}) = F(K+1)$$

Demostrado por inducción