WRITEUP

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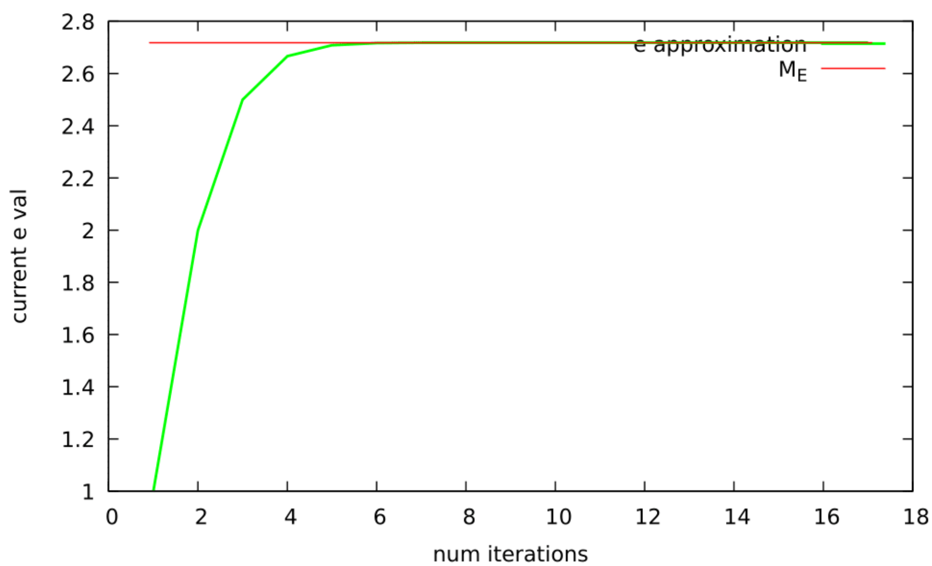
1 E Approximation

The E approximation uses the Taylor Series approximation to get the value of e, which is around 2.718281828459045 or at least the e approximation from the math library. This is calculated using the formula...

$$\sum_{k=0}^{\infty} \frac{1}{k!}$$

This formula can be then plotted on a graph alongside the e approximation by the math library utilizing gnuplot. This is shown here...



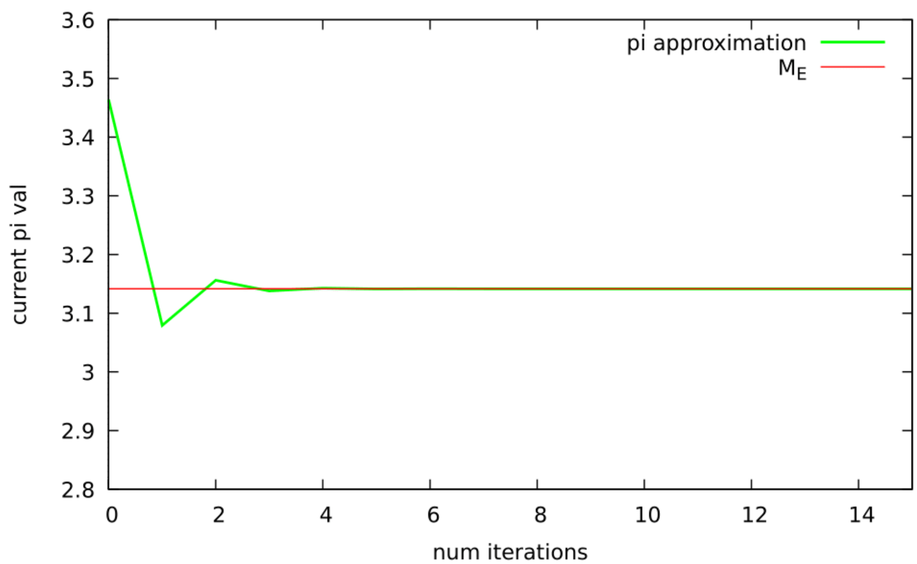
This graph shows in green the e approximation done by the Taylor series and in red the math libraries approximation for e. As you can see after about 6 iterations the two lines merge around the approximate value of e. Illustrating the fast convergence of the Taylor series.

2 Madhava Pi Approximation

The madhava.c file approximates the value of pi, which is around 3.141592653589793, or at least that's as accurate as C can get, using the Madhava formula. The formula is unique compared to the others because of how it converges to pi. Instead of convering in a logarithmic fashion it starts above the value of pi, then jumps below it and keeps repeating the same motion and slowly zeroes in on the value. The formula for the madhava series is...

$$\sqrt{12}\sum_{k=0}^{n}\frac{(-3)^{-k}}{2k+1}$$

This formula is calculated and every iteration is then plotted, using gnuplot, on a graph alongside the pi approximation formula using the math library in C. This can be seen in the graph below.



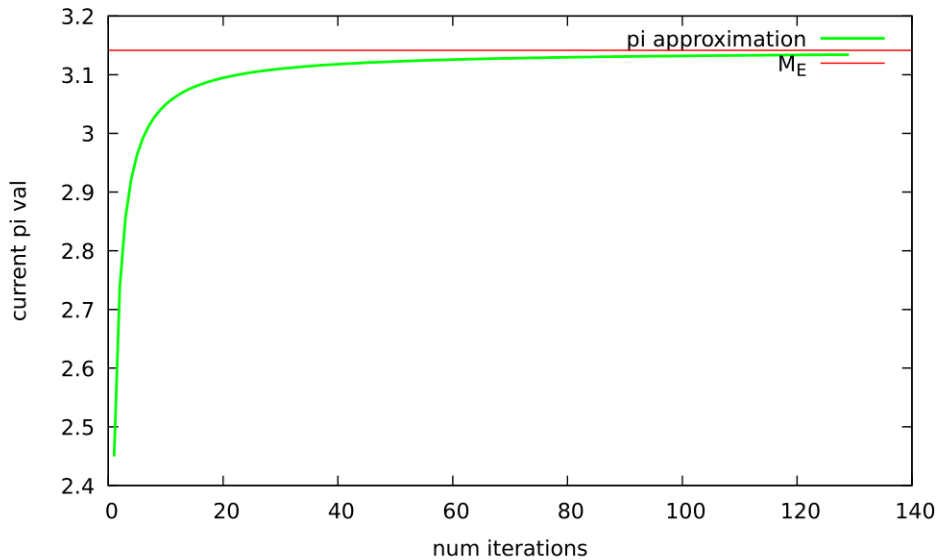
As you can see the green graph starts quite a bit above the red math library pi approximation, but only after 3 iterations it has already converged onto the red line. This is one of the fastest approximations of pi.

3 Euler Pi Approximation

The euler.c file uses the formula derived from Euler's solution to the Basel problem to approximate the value of pi. It does this utilizing this formula...

$$\sqrt{6\sum_{k=1}^{n}\frac{1}{k^2}}$$

Due to the nature of this formula the more iterations it goes through the slower it increments toward the approximate value of pi. This makes it quite a slow converging formula as you can see in the graph down below.



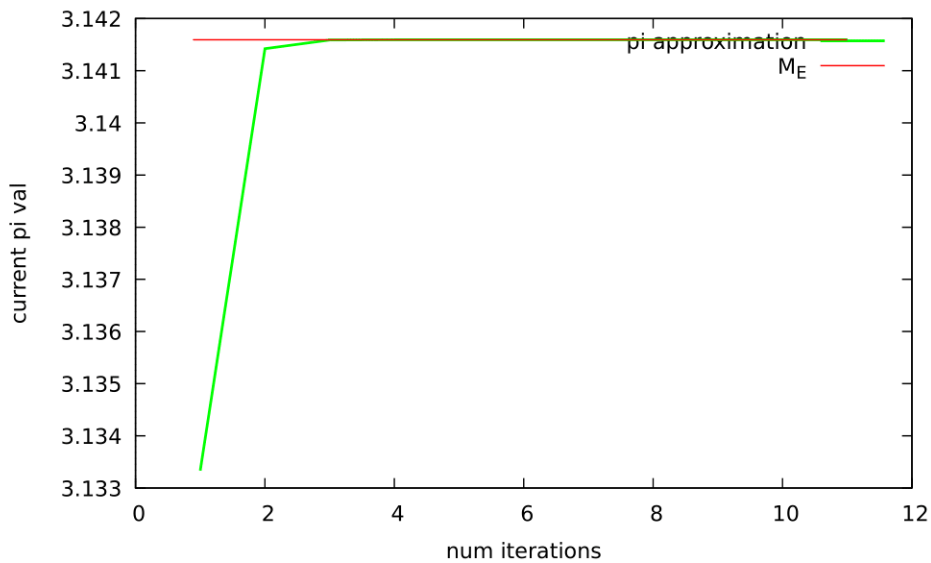
Even in over 120 iterations the Euler pi approximation doesn't fully reach the math libraries pi approximation, compared to a formula like Madhava's Euler's formula is severely outdated.

4 B.B.P Approximation of PI

The bbp.c file otherwise known as Bailey-Borwein-Plouffe approximates the value of pi using the Bailey-Borwein-Plouffe formula. This formula takes a different approach comparing to the other formulas, as you can see in the formula down below it uses multiple terms to converge in on the value of pi.

$$\sum_{k=0}^{n} 16^{-k}\left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6}\right)$$

Because of how the formula is structured, after the first iteration, it jumps very quickly to a value near of the math library approximate value of pi. This is the fastest formula as it converges with the red line in only 2 iterations.



The correctness of the formula is seen by how in only one iteration it lands so near what the true value of pi is while other formulas had to converge towards it.

5 Viete Pi Approximation

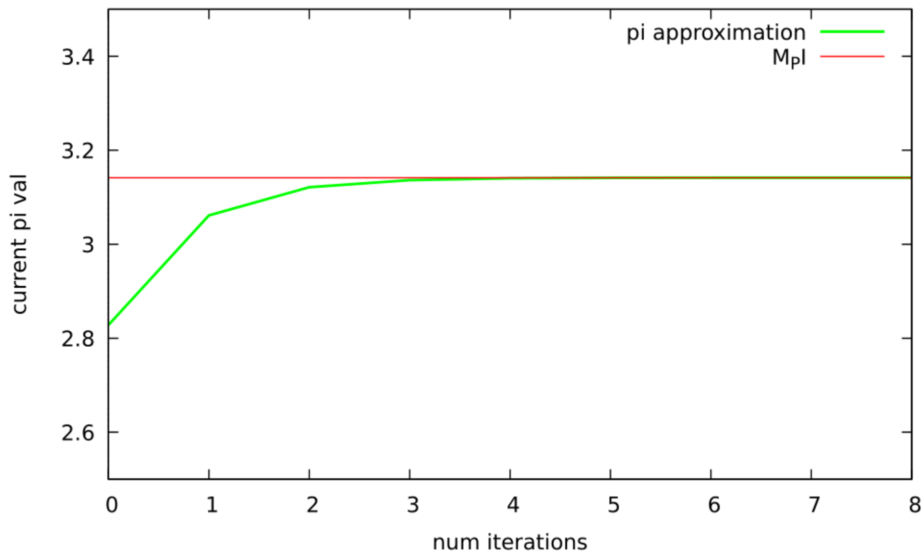
The file, viete.c, uses Viete's formula to approximate the value of pi. Or it approximates the value of $\frac{2}{\pi}$, and then through basic math we can get the value of pi. This is all done using the formula below.

$$\frac{2}{\pi} = \prod_{k=1}^{\infty} \frac{a_k}{2}$$

Where $a_1 = \sqrt{2}$ and $a_k = \sqrt{2 + a_{k-1}}$. This may look complicated but it can be written simply as.

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} x \frac{\sqrt{2+\sqrt{2}}}{2} x \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2}$$

This formula converges quite rapidly onto the value of pi using such simple methods as only $\sqrt{2}$ and dividing by 2. It's a very interesting formula with a very interesting graph aswell specifically the starting point which already less than 0.5 off from the approximate value of pi. This can be seen down below.



This graph converges so quickly, but the main spotlight is definitely on the starting point which is given to us by such simple methods.

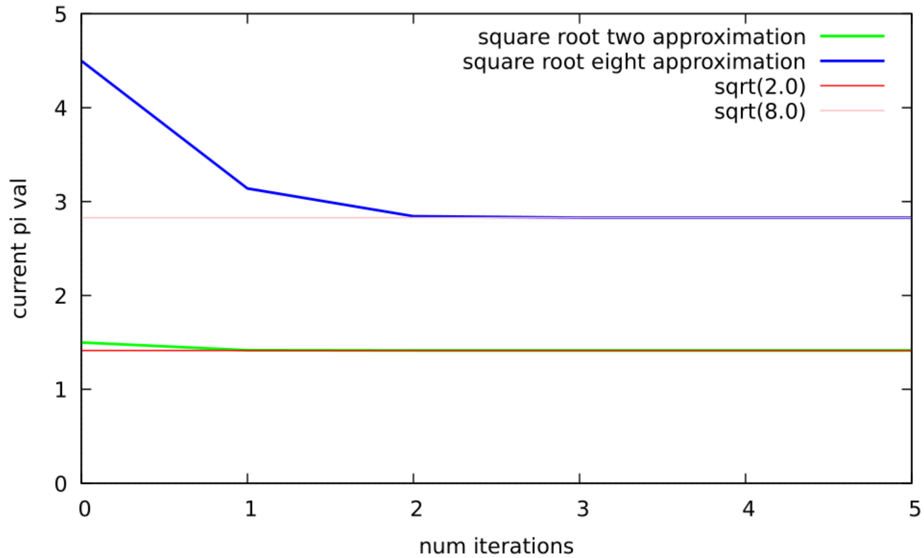
6 Newton's Square Root Approximation

The file newton.c approximates the square root of the value given to it, as a parameter, using the Newton-Raphson method. This method is a simple way to calculate the approximate formula for a square root and it utilizes the formula below.

$$y_k = 0.5 * (\frac{y_{k-1} + x}{y_{k-1}})$$

Where x is the value you are trying to find the square root of, and $y_1 = 1$

This formula allows you to approximate any square root, but it favors smaller values and as we increase the number we are trying to square root the longer it takes to converge to the approximate square root value. This can be seen in the graph below, where we compare the square root functions for the values 2 and 8.



These results show a peculiar conclusion to all these functions and their approximations. The values, $\pi, e, and\sqrt{x}$ are widely used throughout all of math and sciences, and these formula illustrate the numerous ways we are able to approximate them. Moreover, the graphs are great illustrations to show the rate of convergence for each of the formulas so we can truly see which ones work better than others. Overall the Madhava formula, Viete's formula, and BBP formula are definitely the best ones for approximating the value of pi, and each one of them have unique advantages and disadvantages that make them great.