The Impact of Unstable Preferences on Consumption

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December 14, 2022

Introduction

Recent research has indicated, contrary to traditional economic theory, that consumers' preferences may not be stable over time. For example, Jacobs (2016) gives the following scenario, "For example, if an individual's marginal rate of substitution between ice-cream and winter boots and thus her chosen consumption bundle changes from January to July, though prices and incomes remain constant, this will hardly be to the surprise of anyone" (pg. 123). Additionally, behavioral economics findings indicate that the context that choices are made in may influence preferences, such as Tversky's and Kahneman's (1981) finding that preferences may be reversed based on framing effects. Achar et al. (2016) find that, "Incidental sadness leads to more hedonic consumption because consumers are trying to regulate their feelings" (p.168). Therefore, preferences do appear to vary across time and across environments, and they may depend on a consumers' emotional state. The purpose of this analysis will be to extend the idea of unstable preferences to a consumer's decision of how to allocate their income between two goods. This will be accomplished by introducing uncertainty into the utility function through probabilities, along with a parameter θ to measure the degree to which a consumer values a good depending on the context. In this paper, θ will represent the consumer's emotional state that causes additional utility from consuming a good. However, θ can be used to represent any situation that would cause additional utility from the consumption of a good. After solving an optimization problem and performing comparative statics on the optimal bundle of goods, I find that the consumer chooses to consume more of a good if the utility of consuming the good in one of two outcomes increases. Additionally, I find that the consumer will consume less of that same good if the probability of the outcome in which the consumer receives less utility from that good increases. Lastly, the consumer will choose to consume more of a good if the minimum necessary amount of that good purchased increases.

The Model

The utility function used in this scenario is a Stone Geary utility function of the form:

$$U(x, y) = \alpha \ln(x - a) + \beta \ln(y - b) \tag{1}$$

with two goods, x and y, where a represents the minimum amount of good x that must be purchased and b represents the minimum amount of good y that must be purchased (Finnoff, ECON 5390 Exam 2, 2022). Both α and are positive parameters. Next, assume that the consumer values good x less with probability p and values good x more with probability $(1 - p)$. Let the parameter θ measure the increased utility received from good x when the outcome with probability $(1-p)$ occurs. The utility function therefore becomes:

$$U(x,y) = p(\alpha \ln(x-a) + \beta \ln(y-b)) + (1-p)(\theta\alpha \ln(x-a) + \beta \ln(y-b)) \qquad (2)$$

where θ is a positive parameter measuring the increased utility from good x. Equation (2) assumes that a consumer values one good at two different levels based on two random outcomes. One possible example to reflect this scenario is that the consumer's utility received from good x depends on their emotional state. If the consumer is feeling relatively happy, they may receive more utility from good x, measured by θ, as reflected by the $(1-p)$ outcome. The underlying assumption, therefore, is that the consumer's emotional state is a random event reflected by probabilities. Generalizing this assumption, the utility model in (2) assumes that the consumer does not know when they will value good x more or how likely they will value good x more. The last assumption of (2) is that the two goods, x and y, are consumed at the same point in time. Therefore, the two outcomes, p and $(1-p)$, reflect that the consumer may receive the utility $\alpha \ln(x-a) + \beta \ln(y-b)$ or $\theta\alpha \ln(x-a) + \beta \ln(y-b)$, as opposed to simply applying the probability term only to $\alpha \ln(x-a)$.

Analysis: Changes in the Additional Utility From a Good

The utility maximization problem therefore becomes:

$$\max_{x,y} \quad U(x,y) = p[\alpha \ln{(x-a)} + \beta \ln{(y-b)}] + (1-p)[\theta\alpha \ln{(x-a)} + \beta \ln{(y-b)}] \quad (3)$$

$$s.t. M = wx + qy \quad (4)$$

Equation (4) represents the budget constraint that the consumer faces, where M is the consumer's income, w is the price of good x, and q is the price of good y. The Lagrangian of equations (3) and (4) is:

$$\mathcal{L} = p[\alpha \ln{(x-a)} + \beta \ln{(y-b)}] + (1-p)[\theta\alpha \ln{(x-a)} + \beta \ln{(y-b)}] + \lambda[M - wx - qy] \quad (5)$$

The corresponding first-order equations for equation (5) are:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = M - wx - qy = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{p\alpha}{x-a} + \frac{\theta\alpha(1-p)}{x-a} - w\lambda = 0 \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{p\beta}{y-b} + \frac{\beta(1-p)}{y-b} - q\lambda = 0 \quad (8)$$

Equation (6) states that the consumer will exhaust all of their income between goods x and y. Equation (7) states that the marginal utility of consuming good x must be balanced by the expected marginal utility of the income that is lost by purchasing good x. Equivalently, equation (7) can be interpreted as the consumer purchasing good x until the expected marginal utility of consuming good x balances with the marginal opportunity cost of spending on good x. Equation (8) states that the expected marginal utility of consuming good y must be balanced by the marginal utility of income lost by purchasing good y. Or, equivalently, the consumer should keep purchasing good y until the expected marginal utility from consuming good y balances with the marginal opportunity cost of spending on good y.

Next, a bordered Hessian can be created to analyze the second-order conditions:

$$[H^B] = \begin{bmatrix} 0 & -w & -q \\ -w & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(a-p)}{(x-a)^2} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2}] \end{bmatrix}$$

To find a maximum, the determinant of the bordered hessian should be positive.

$$|H^B| = -w(-1)^3 det \begin{bmatrix} -w & 0 \\ -q & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix} - q(-1)^4 det \begin{bmatrix} -w & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(a-p)}{(x-a)^2} \\ -q & 0 \end{bmatrix}$$

Simplifying this determinant results in:

$$\begin{aligned} & w[\frac{wp\beta}{(y-b)^2} + \frac{w(1-p)\beta}{(y-b)^2}] - q[-(\frac{qp\alpha}{(x-a)^2} + \frac{q\theta\alpha(1-p)}{(x-a)^2})] \\ &= \frac{w^2 p\beta + w^2(1-p)\beta}{(y-b)^2} + \frac{q^2 p\alpha + q^2\theta\alpha(a-p)}{(x-a)^2} \end{aligned} \tag{9}$$

By the assumptions of the problem, equation (9) is positive and therefore the determinant of the bordered Hessian is positive. Next, the Implicit Function rule (Finnoff, ECON 5390 Handout 7, 2022) can be used to determine how the optimal consumption of good x and good y changes with a change in the extra utility from good x, θ. Thus, by the Implicit Function rule:

$$\begin{bmatrix} 0 & -w & -q \\ -w & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(a-p)}{(x-a)^2} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix} \begin{bmatrix} \frac{\partial\lambda^*}{\partial\theta} \\ \frac{\partial x^*}{\partial\theta} \\ \frac{\partial y^*}{\partial\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{\alpha(1-p)}{x-a} \\ 0 \end{bmatrix} \tag{10}$$

Equation 10 can be solved by using Cramer's Rule (Sydsaeter et al., 2016, pg. 653). By Cramer's Rule, $\frac{\partial\lambda^*}{\partial x^*}$ is given by:

$$\frac{det[H^B_{\lambda^*}]}{det[H^B]} \tag{11}$$

where $H^B_{\lambda^*}$ is the modified bordered Hessian with respect to x^*. Notice that $H^B_{\lambda^*}$ is equivalent to:

$$H^B_{\lambda^*} = \begin{bmatrix} 0 & -w & -q \\ -\frac{\alpha(1-p)}{x-a} & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(a-p)}{(x-a)^2} & 0 \\ 0 & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix} \tag{12}$$

Taking the determinant of equation (12) results in:

$$-w(-1)^3 det \begin{bmatrix} -\frac{\alpha(1-p)}{x-a} & 0 \\ 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix} - q(-1)^4 det \begin{bmatrix} -\frac{\alpha(1-p)}{x-a} & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} \\ 0 & 0 \end{bmatrix}$$

$$= w[\frac{\alpha(1-p)}{x-a}(\frac{p\beta}{(y-b)^2}) + \frac{\alpha(1-p)}{x-a}(\frac{(1-p)\beta}{(y-b)^2})] \tag{13}$$

By the assumptions of the utility function (2), equation (13) is positive. Thus, $\frac{\partial\lambda^*}{\partial\theta}$ is equivalent to:

$$\frac{\partial\lambda^*}{\partial\theta} = \frac{w[\frac{\alpha(1-p)}{x-a}(\frac{p\beta}{(y-b)^2}) + \frac{\alpha(1-p)}{x-a}(\frac{(1-p)\beta}{(y-b)^2})]}{\frac{w^2p\beta+w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha+q^2\theta\alpha(a-p)}{(x-a)^2}} > 0 \tag{14}$$

The numerator of equation (14) was found to be positive by the requirements of a the second-order conditions for a maximum. Therefore, $\frac{\partial \lambda^*}{\partial \theta} > 0$. This implies that as the additional utility from consuming good x increases, the additional utility from increasing the consumer's budget, M, is positive. Or, in other words, the marginal utility of an additional dollar is positive as θ increases. This process can be repeated to find $\frac{\partial x^*}{\partial \theta}$.

$$\begin{aligned} det[H^B_{x^*}] &= det \begin{bmatrix} 0 & 0 & -q \\ -w & -\frac{\alpha(1-p)}{x-a} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix} \\ &= -q(-1)^4 det \begin{bmatrix} -w & -\frac{\alpha(1-p)}{x-a} \\ -q & 0 \end{bmatrix} \\ &= \frac{q^2\alpha(1-p)}{x-a} \end{aligned} \tag{15}$$

From the assumptions of equation (2), equation (15) is positive. Therefore, $\frac{\partial x^*}{\partial \theta}$ is equivalent to:

$$\frac{\partial x^*}{\partial \theta} = \frac{\frac{q^2\alpha(1-p)}{x-a}}{\frac{w^2p\beta + w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha + q^2\theta\alpha(a-p)}{(x-a)^2}} > 0 \tag{16}$$

Equation (16) states that as the additional utility received from consuming good x increases, the consumer will purchase more of good x. Assuming θ represents the consumer's emotional state, if the consumer's emotional state is stronger, the consumer will purchase more of good x. Lastly, I repeat this process for $\frac{\partial y^*}{\partial \theta}$.

$$det[H^B_{y^*}] = \begin{bmatrix} 0 & -w & 0 \\ -w & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} & -\frac{\alpha(1-p)}{x-a} \\ -q & 0 & 0 \end{bmatrix}$$

$$-w(-1)^3 det \begin{bmatrix} -w & -\frac{\alpha(1-p)}{x-a} \\ -q & 0 \end{bmatrix}$$

$$= \frac{-wq\alpha(1-p)}{x-a} \tag{17}$$

Since $w, q, \alpha(1-p)$ and $(x-a)$ are all positive parameters, equation (17) is less than zero. Therefore, $\frac{\partial y^*}{\partial \theta}$ is equivalent to:

$$\frac{\partial y^*}{\partial \theta} = \frac{\frac{-wq\alpha(1-p)}{x-a}}{\frac{w^2p\beta + w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha + q^2\theta\alpha(a-p)}{(x-a)^2}} < 0 \tag{18}$$

Equation (18) states that as the additional utility received from consuming good x increases, the consumer will consume less of good y. Or, in other words, as the emotional state that causes the consumer to consume more of good x increases, the consumer will consume less of good y.

Analysis: Changes in the Probability of Each Outcome

Next, comparative statics for the impact of changes on λ^*, x^*, and y^* from changes in the probability of each outcome, p, can be examined. By the Implicit Function Rule, the following relation must hold:

$$[H^B] = \begin{bmatrix} 0 & -w & -q \\ -w & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(a-p)}{(x-a)^2} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2}] \end{bmatrix} \begin{bmatrix} \frac{\partial\lambda^*}{\partial p} \\ \frac{\partial x^*}{\partial p} \\ \frac{\partial y^*}{\partial p} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{\alpha}{x-a} + \frac{\theta\alpha}{x-a} \\ 0 \end{bmatrix} \tag{19}$$

First, Cramer's Rule can be used to find $\frac{\partial\lambda^*}{\partial p}$.

$$det[H^B_{\lambda^*}] = \begin{bmatrix} 0 & -w & -q \\ -\frac{\alpha}{x-a} + \frac{\theta\alpha}{x-a} & -\frac{-p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} & 0 \\ 0 & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix}$$

$$= -w(-1)^3 det \begin{bmatrix} -\frac{\alpha}{x-a} + \frac{\theta\alpha}{x-a} & 0 \\ 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix} - q(-1)^4 det \begin{bmatrix} \frac{\alpha}{x-a} - \frac{\theta\alpha}{x-a} & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} \\ 0 & 0 \end{bmatrix}$$

$$= w(-\frac{\alpha}{x-a} + \frac{\theta\alpha}{x-a})(-\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2})$$

$$= w[\frac{p\alpha\beta + \alpha\beta(1-p) - p\theta\alpha\beta - \theta\alpha\beta(1-p)}{(x-a)(y-b)^2}] \tag{20}$$

Since all of the parameters in (2) are positive, equation (20) is negative since $p\alpha\beta + \alpha\beta(1-p) < p\theta\alpha\beta + \theta\alpha\beta(1-p)$. Therefore, $\frac{\partial\lambda^*}{\partial p}$ is equivalent to:

$$\frac{\partial\lambda^*}{\partial p} = \frac{w[\frac{p\alpha\beta+\alpha\beta(1-p)-p\theta\alpha\beta-\theta\alpha\beta(1-p)}{(x-a)(y-b)^2}]}{\frac{w^2p\beta+w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha+q^2\theta\alpha(a-p)}{(x-a)^2}} < 0 \tag{21}$$

Equation (21) states that as the probability of the consumer receiving the utility $\alpha\ln(x-a)+\beta\ln(y-b)$ increases, the marginal utility of additional income decreases. This process can be repeated to find $\frac{\partial x^*}{\partial p}$.

$$det[H^B_{x^*}] = det\begin{bmatrix} 0 & 0 & -q \\ -w & -\frac{\alpha}{x-a}+\frac{\theta\alpha}{x-a} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2}-\frac{(1-p)\beta}{(y-b)^2} \end{bmatrix}$$

$$= -q(-1)^4 det\begin{bmatrix} -w & -\frac{\alpha}{x-a}+\frac{\theta\alpha}{x-a} \\ -q & 0 \end{bmatrix}$$

$$\frac{q^2\alpha}{x-a} - \frac{q^2\theta\alpha}{x-a} \tag{22}$$

Since $\frac{q^2\theta\alpha}{x-a} > \frac{q^2\alpha}{x-a}$, equation (22) is negative. Therefore, $\frac{\partial x^*}{\partial p}$ is equivalent to:

$$\frac{\partial x^*}{\partial p} = \frac{\frac{q^2\alpha}{x-a} - \frac{q^2\theta\alpha}{x-a}}{\frac{w^2p\beta+w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha+q^2\theta\alpha(a-p)}{(x-a)^2}} < 0 \tag{23}$$

Equation (23) states that as the probability of the consumer's receiving the utility $\alpha\ln(x-a)+\beta\ln(y-b)$ increases, the optimal consumption of good x decreases. Next, this process can be applied to find $\frac{\partial y^*}{\partial p}$.

$$det[H^B_{y^*}] = det\begin{bmatrix} 0 & -w & 0 \\ -w & -\frac{p\alpha}{(x-a)^2}-\frac{\theta\alpha(1-p)}{(x-a)^2} & -\frac{\alpha}{x-a}+\frac{\theta\alpha}{x-a} \\ -q & 0 & 0 \end{bmatrix}$$

$$= -w(-1)^3 det\begin{bmatrix} -w & -\frac{\alpha}{x-a}+\frac{\theta\alpha}{x-a} \\ -q & 0 \end{bmatrix}$$

$$w[-\frac{q\alpha}{x-a} + \frac{q\theta\alpha}{x-a}] \tag{24}$$

Equation (24) is positive since $\frac{q\theta\alpha}{x-a} > \frac{q\alpha}{x-a}$. Therefore, $\frac{\partial y^*}{\partial p}$ is equivalent to:

$$\frac{\partial y^*}{\partial p} = \frac{w[-\frac{q\alpha}{x-a} + \frac{q\theta\alpha}{x-a}]}{\frac{w^2p\beta+w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha+q^2\theta\alpha(a-p)}{(x-a)^2}} > 0 \tag{25}$$

Equation (25) states that as the probability of the consumer receiving the utility $\alpha \ln(x-a) + \beta \ln(y-b)$ increases, the optimal consumption of good y increases.

Analysis: Changes in the Minimum Consumption of Good x

Next, Cramer's rule can be used to determine how changes in the minimum necessary amount of good x, represented by the a parameter, influences λ^*, x^*, and y^*. By the Implicit Function Rule, the following relation will hold:

$$[H^B] = \begin{bmatrix} 0 & -w & -q \\ -w & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(a-p)}{(x-a)^2} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^*}{\partial a} \\ \frac{\partial x^*}{\partial a} \\ \frac{\partial y^*}{\partial a} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{p\alpha}{(x-a)^2} - \frac{\alpha\theta(1-p)}{(x-a)^2} \\ 0 \end{bmatrix} \tag{26}$$

Applying Cramer's Rule results in:

$$det[H^B_{lambda^*}] = det \begin{bmatrix} 0 & -w & -q \\ -\frac{p\alpha}{(x-a)^2} - \frac{\alpha\theta(1-p)}{(x-a)^2} & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} & 0 \\ 0 & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix}$$

$$= -w(-1)^3 det \begin{bmatrix} -\frac{p\alpha}{(x-a)^2} - \frac{\alpha\theta(1-p)}{(x-a)^2} & 0 \\ 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix} - q(-1)^4 det \begin{bmatrix} -\frac{p\alpha}{(x-a)^2} - \frac{\alpha\theta(1-p)}{(x-a)^2} & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} \\ 0 & 0 \end{bmatrix}$$

$$= w\left(\frac{\alpha p^2\beta + \alpha p(1-p)\beta + \alpha\theta(1-p)p\beta + \alpha\theta\beta(1-p)^2}{(x-a)^2(y-b)^2}\right) \tag{27}$$

Notice that equation (27) is positive since all of the parameters are positive. Therefore, $\frac{\partial \lambda^*}{\partial a}$ is equivalent to:

$$\frac{\partial \lambda^*}{\partial a} = \frac{w(\frac{\alpha p^2\beta+\alpha p(1-p)\beta+\alpha\theta(1-p)p\beta+\alpha\theta\beta(1-p)^2}{(x-a)^2(y-b)^2})}{\frac{w^2 p\beta+w^2(1-p)\beta}{(y-b)^2}+\frac{q^2 p\alpha+q^2\theta\alpha(a-p)}{(x-a)^2}} > 0 \tag{28}$$

Equation (28) states that the marginal utility of additional income increases as the minimum necessary amount of good x that must be purchased increases. Next, Cramer's rule can be applied to find $\frac{\partial x^*}{\partial a}$:

$$det[H^B_{x^*}] = det\begin{bmatrix} 0 & 0 & -q \\ -w & -\frac{p\alpha}{(x-a)^2}-\frac{\alpha\theta(1-p)}{(x-a)^2} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2}-\frac{(1-p)\beta}{(y-b)^2} \end{bmatrix}$$

$$-q(-1)^4 det\begin{bmatrix} -w & -\frac{p\alpha}{(x-a)^2}-\frac{\alpha\theta(1-p)}{(x-a)^2} \\ -q & 0 \end{bmatrix}$$

$$= \frac{pq^2\alpha+q^2\alpha\theta(1-p)}{(x-a)^2} \tag{29}$$

Notice that equation (29) is positive. Therefore, $\frac{\partial x^*}{\partial a}$ is equivalent to:

$$\frac{\partial x^*}{\partial a} = \frac{\frac{pq^2\alpha+q^2\alpha\theta(1-p)}{(x-a)^2}}{\frac{w^2 p\beta+w^2(1-p)\beta}{(y-b)^2}+\frac{q^2 p\alpha+q^2\theta\alpha(a-p)}{(x-a)^2}} > 0 \tag{30}$$

Equation (30) states that as the minimum necessary amount of good x that much be purchased increases, the consumer will purchase more of good x. Lastly, Cramer's Rule can be used to find $\frac{\partial y^*}{\partial a}$:

$$det[H^B_{y^*}] = det\begin{bmatrix} 0 & -w & 0 \\ -w & -\frac{p\alpha}{(x-a)^2}-\frac{\theta\alpha(1-p)}{(x-a)^2} & -\frac{p\alpha}{(x-a)^2}-\frac{\alpha\theta(1-p)}{(x-a)^2} \\ -q & 0 & 0 \end{bmatrix}$$

$$= -w(-1)^3 det \begin{bmatrix} -w & -\frac{p\alpha}{(x-a)^2} - \frac{\alpha\theta(1-p)}{(x-a)^2} \\ -q & 0 \end{bmatrix}$$

$$= w(\frac{-qp\alpha - q\alpha\theta(1-p)}{(x-a)^2}) \tag{31}$$

Notice that equation (31) is negative. Therefore, $\frac{\partial y^*}{\partial a}$ is equivalent to:

$$\frac{\partial y^*}{\partial a} = \frac{w(\frac{-qp\alpha - q\alpha\theta(1-p)}{(x-a)^2})}{\frac{w^2 p\beta + w^2(1-p)\beta}{(y-b)^2} + \frac{q^2 p\alpha + q^2\theta\alpha(a-p)}{(x-a)^2}} \tag{32}$$

Equation (32) states that as the minimum necessary amount of good x that must be purchased increases, the consumer will spend less on good y.

Analysis: Changes in the Minimum Consumption of Good y

For the last of the comparative static analysis, the Implicit Function Theorem can be used to determine how changes in the minimum necessary amount of good y that must be purchased, represented by the parameter b, influences λ^*, x^*, and y^*. According to the Implicit Function Theorem, the following relation must hold:

$$[H^B] = \begin{bmatrix} 0 & -w & -q \\ -w & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(a-p)}{(x-a)^2} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2}] \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda^*}{\partial b} \\ \frac{\partial x^*}{\partial b} \\ \frac{\partial y^*}{\partial b} \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{p\alpha}{(x-a)^2} - \frac{\alpha\theta(1-p)}{(x-a)^2} \\ 0 \end{bmatrix} \tag{33}$$

Applying Cramer's Rule to find $\frac{\partial \lambda^*}{\partial b}$ results in:

$$[H^B_{\lambda^*}] = det\begin{bmatrix} 0 & -w & -q \\ 0 & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} & 0 \\ -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} & 0 & -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} \end{bmatrix}$$

$$= -w(-1)^3 det\begin{bmatrix} 0 & 0 \\ -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} & -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} \end{bmatrix} -q(-1)^4 det\begin{bmatrix} 0 & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} \\ -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} & 0 \end{bmatrix}$$

$$= -q(\frac{-p^2\beta\alpha - p\beta\theta\alpha(1-p) - \beta(1-p)p\alpha - \beta\theta\alpha(1-p)^2}{(y-b)^2(x-a)^2}) \tag{34}$$

By the assumptions of the utility function in (2), equation (34) is positive. Therefore, $\frac{\partial\lambda^*}{\partial b}$ is equivalent to:

$$\frac{\partial\lambda^*}{\partial b} = \frac{-q(\frac{-p^2\beta\alpha - p\beta\theta\alpha(1-p) - \beta(1-p)p\alpha - \beta\theta\alpha(1-p)^2}{(y-b)^2(x-a)^2})}{\frac{w^2p\beta + w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha + q^2\theta\alpha(a-p)}{(x-a)^2}} > 0 \tag{35}$$

Equation (35) states that as the minimum necessary amount of good y that must be purchased increases, the marginal utility of income is positive. Next, using Cramer's rule to find $\frac{\partial x^*}{\partial b}$ results in:

$$[H^B_{x^*}] = det\begin{bmatrix} 0 & 0 & -q \\ -w & 0 & 0 \\ -q & -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} & -\frac{p\beta}{(y-b)^2} - \frac{(1-p)\beta}{(y-b)^2} \end{bmatrix}$$

$$= -q(-1)^4 det\begin{bmatrix} -w & 0 \\ -q & -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} \end{bmatrix}$$

$$= -q(\frac{wp\beta + w\beta(1-p)}{(y-b)^2}) \tag{36}$$

By the assumptions of the utility function (2), equation (36) is negative. Therefore:

$$\frac{\partial x^*}{\partial b} = \frac{-q(\frac{wp\beta + w\beta(1-p)}{(y-b)^2})}{\frac{w^2p\beta + w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha + q^2\theta\alpha(a-p)}{(x-a)^2}} < 0 \tag{37}$$

Equation (37) states that as the minimum necessary amount of good y that must be purchased increases, the consumer will choose to purchase less of good x. Lastly, using Cramer's Rule to find $\frac{\partial y^*}{\partial b}$ results in:

$$[H^B_{y^*}] = det \begin{bmatrix} 0 & -w & 0 \\ -w & -\frac{p\alpha}{(x-a)^2} - \frac{\theta\alpha(1-p)}{(x-a)^2} & 0 \\ -q & 0 & -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} \end{bmatrix}$$

$$= -w(-1)^3 det \begin{bmatrix} -w & 0 \\ -q & -\frac{p\beta}{(y-b)^2} - \frac{\beta(1-p)}{(y-b)^2} \end{bmatrix}$$

$$= w(\frac{wp\beta + w\beta(1-p)}{(y-b)^2}) \tag{38}$$

Notice that equation (38) is positive. Therefore, $\frac{\partial y^*}{\partial b}$ is equivalent to:

$$\frac{\partial y^*}{\partial b} = \frac{w(\frac{wp\beta + w\beta(1-p)}{(y-b)^2})}{\frac{w^2p\beta + w^2(1-p)\beta}{(y-b)^2} + \frac{q^2p\alpha + q^2\theta\alpha(a-p)}{(x-a)^2}} > 0 \tag{39}$$

Equation (39) states that as the minimum amount of good y that must be purchased increases, the consumer will purchase more of good y.

Summary and Conslusions

From this analysis, it appears that as the potential for the consumer to receive higher utility from good x, represented by θ, increases, the consumer will choose to purchase more of good x. If θ is used to represent the consumer's emotions, then as the consumer's emotional state increases, they would choose to purchase more of good x. More generally, if this consumer values good x on random days more than others, they will choose to purchase more of good x. The opposite holds for good y: if the consumer values good x more on random days, they will choose to purchase less of good y. Additionally, as the probability of receiving more utility from good x increases, the consumer will choose to purchase more of good x and less of good y. Therefore, in the model presented in this paper, consumers will tend to purchase more of a certain good if the probability that they receive higher utility from that good increases. For example, if a consumer knows that they tend to enjoy coffee more when they are happy, though they can't predict when they will be happy, they will choose to spend more of their budget on coffee. From the Stone-Geary utility function in (2), as the minimum necessary amount of a good that must be purchased increases, the consumer will choose to purchase more of that good. Overall, this model extends the expected utility theory to unstable (and in this case random) preferences. Future research could apply this additionally parameter to other functional forms of utility functions to determine if there are similar results. Additionally, Lowenstein's (1987) findings that consumers value delayed consumption could be substituted in this model for the probability terms. In this case, the consumer derives utility from the anticipation of consuming the good, and thus a negative "discount factor" that measures delayed utility could be introduced. Overall, models that incorporate unstable preferences over time allow economists to increase the realism in their models, thus increasing the ability to accurately describe the world around us.

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