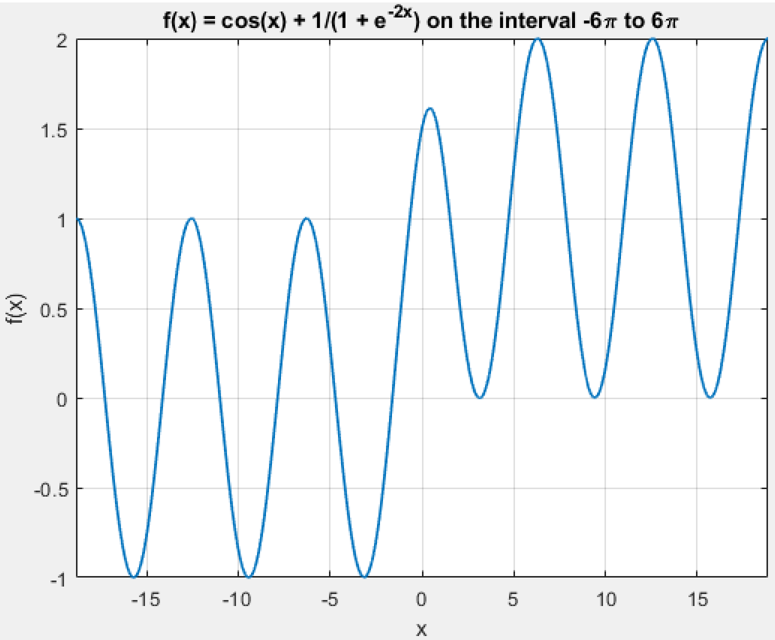
MACM 316: CA2 **AUTHOR: Andy Liu** **STUDENT ID: 301472847**

First graph is part a), second is part b), third is a demonstrative sketch for part e), where purple is g(x) and black is y = x

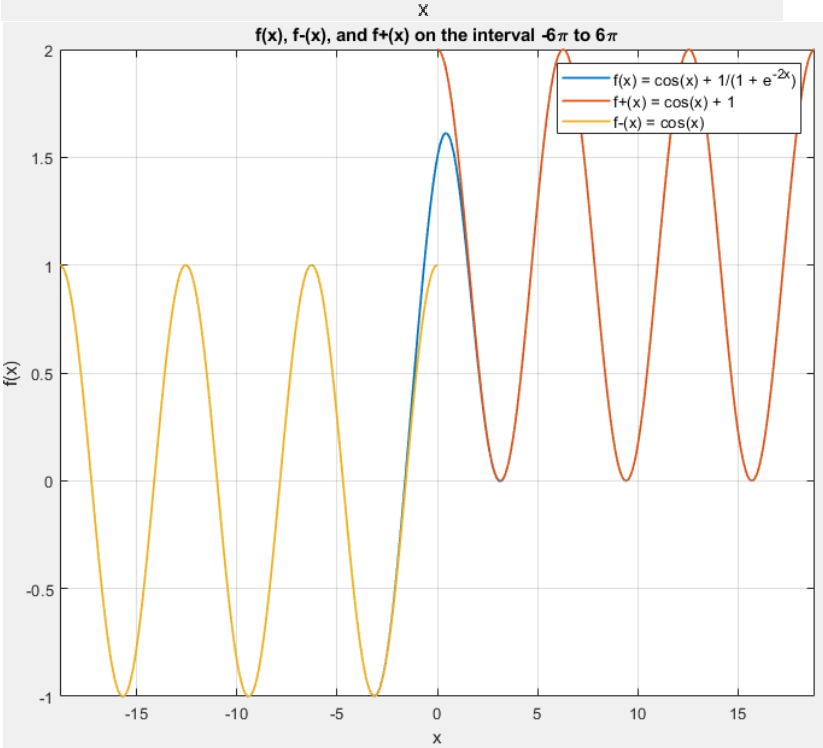


a) The graph oscillates similarly to the cos(x) function and fluctuates between f(x) = -1 and +1 on the negative interval and f(x) = 0 and 2 on the positive interval. There are 8 roots on the interval. The roots are roughly x = -17.28, -14.14, -11.00, -7.854, -4.712, -1.609, 3.076, 3.199.

b) Limit as x approaches positive infinity = 1, negative infinity = 0. Simpler limit functions:

$$f_-(x) = \cos(x)\,,\ and\ f_+(x) = \cos(x) + 1$$

all roots of f-(x) = cos (x) is $\frac{\boldsymbol{\pi}}{\mathbf{2}} + \boldsymbol{\pi n}$, where n is any integer. All roots of f+(x) = cos (x) + 1 are $\boldsymbol{\pi} + \mathbf{2}\boldsymbol{\pi n}$



c) first: $x^* = -1.60928, and\ x_- = -1.57080$

second: $x^* = -4.71231, and\ x_- = -4.71239$

third: $x^* = -7.85398, and\ x_- = -7.85398$

$$first\ error = |-1.60928 - (-1.57080)| = 0.03848$$

There are around 1-2 significant digits which means the error is high and the first x_- does not approximate x^* well. However, the accuracy increases in the next two approximations and are around 5-6 digits.

d) first: $x^* = 3.07643, and\ x_+ = 3.14159$

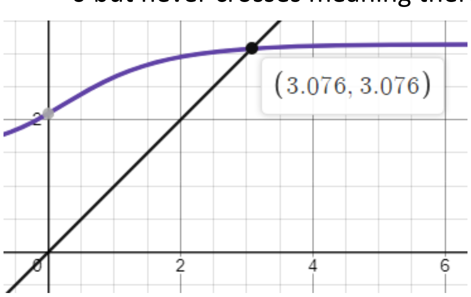
second: $x^* = 3.19924, and\ x_+ = 9.42478$

third: $x^* = 10.0000, and\ x_+ = 15.7080$

fourth: $x^* = 16.0000, and\ x_+ = 21.9911$

$$first\ error = |3.07643 - 3.14159| = 0.06516$$

I chose the initial brackets by looking at the graph and estimating. The first estimate once again isn't very accurate with only 1-2 accurate significant digits. However since f(x) has an extra root between x = [3,4], the roots no longer line up as x increases. After the first two positive roots in f(x), however, the function gets close to y = 0 but never crosses meaning there are no more roots. The function f+(x) continues to have roots, however, meaning f+(x) does not accurately approximate f(x).



e) The root found using the fixed-point method is $x^* = 3.07642$ so therefore, finding a fixed point of g(x) is the same as finding a root of f(x) = 0. The error is 0.00001 meaning 5 accurate significant figures. However, this fixed-point is calculated from both an initial guess of x = -1.5 and x = 3.0 which is unexpected. This is because y = x and x = g(x) only intersects at x = 3.07642 meaning it will only converge at x = 3.07642. The real problem with using our specific g(x) to find all roots is that it is not unique and only has 1 intersection with y = x. No matter our initial guess, the fixed-point method always converges to the single fixed point and **can't** find **all roots** of f(x).

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% Computing Assignment #2
% Author: Andy Liu
% ID: 301472847
x_og = linspace(-6*pi, 6*pi, 10000);
y_og = cos(x_og) + 1./(1 + exp(-2*x_og));
plot(x_og, y_og, 'DisplayName', 'f(x) = cos(x) + 1/(1 + e^{-2x})','LineWidth', 1.3);
hold on;
title('f(x), f-(x), and f+(x) on the interval -6\pi to 6\pi');
xlabel('x');
ylabel('f(x)');
grid on;
xlim([-6*pi, 6*pi]);

syms x_syms;
pos_limit = limit(1./(1 + exp(-2*x_syms)), x_syms, Inf);
neg_limit = limit(1./(1 + exp(-2*x_syms)), x_syms, -Inf);
disp(['Limit as x approaches positive infinity: ', char(pos_limit)]);
disp('Use f+(x) = cos(x) + 1 for x > 0 to approximate f for large positive values of x');
disp(['Limit as x approaches negative infinity: ', char(neg_limit)]);
disp('Use f-(x) = cos(x) for x < 0 to approximate f for large negative values of x');
disp('  ');

x_pos = linspace(0, 6*pi, 10000);
y_pos = cos(x_pos) + 1;
plot(x_pos, y_pos, 'DisplayName', 'f+(x) = cos(x) + 1', 'LineWidth', 1.3);
x_neg = linspace(-6*pi, 0, 10000);
y_neg = cos(x_neg);
plot(x_neg, y_neg, 'DisplayName', 'f-(x) = cos(x)', 'LineWidth', 1.3);
legend('show', 'FontSize', 10);
hold off;

func_y = @(x) cos(x) + 1./(1 + exp(-2*x));
[r, niter, rlist] = bisect2(func_y, [-4, 0], 0.000001);
[rneg, niterneg, rlist] = bisect2(func_y, [-5, -3], 0.000001);
[rneg1, niterneg1, rlist] = bisect2(func_y, [-10, -6], 0.000001);
disp(['Negative Root 1: ', num2str(r), ', Iterations: ', num2str(niter)]);
disp(['Negative Root 2: ', num2str(rneg), ', Iterations: ', num2str(niterneg)]);
disp(['Negative Root 3: ', num2str(rneg1), ', Iterations: ', num2str(niterneg1)]);
disp('  ');

[r1, niter1, rlist1] = bisect2(func_y, [3, 3.15], 0.000001);
[r2, niter2, rlist2] = bisect2(func_y, [3.15, 3.25], 0.000001);
[r3, niter3, rlist3] = bisect2(func_y, [9, 10], 0.000001);
[r4, niter4, rlist4] = bisect2(func_y, [15, 16], 0.000001);
disp(['Positive Root 1: ', num2str(r1), ', Iterations: ', num2str(niter1)]);
disp(['Positive Root 2: ', num2str(r2), ', Iterations: ', num2str(niter2)]);
disp(['Positive Root 3: ', num2str(r3), ', Iterations: ', num2str(niter3)]);
disp(['Positive Root 4: ', num2str(r4), ', Iterations: ', num2str(niter4)]);
disp('  ');

fp_func = @(x) acos(-1./(1 + exp(-2*x)));
[xfinal, fpiter, xlist] = fixedpt( fp_func, -1.5, 0.000001 );
[xfinal1, fpiter1, xlist1] = fixedpt( fp_func, 3.0, 0.000001 );
disp(['Fixed Point, Intial Guess x = -1.5 (): ', num2str(xfinal), ', Iterations: ', num2str(fpiter)]);
disp(['Fixed Point Root, Intial Guess x = 3.0 (): ', num2str(xfinal1), ', Iterations: ', num2str(fpiter1)]);
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