

CS61C Fall 2018 GS Worksheet 1a Solution(Num Rep)

1. Number Representation (Spring 2017 MT1 #1)

Consider a base-5 number format where the digits can take values 0, 1, 2, 3, 4:

- a. What is the largest number that can be represented as an unsigned base-5 number of N digits? Express your answer in terms of N.

$$5^N - 1$$

- b. Convert 623_{10} to a number in base 5:

$$4443$$

2. Another Number Representation

- a. Convert the following 8-bit two's complement numbers from hexadecimal to decimal:

$$0x80 = -128$$

$$0xF4 = -12$$

$$0x0E = 14$$

- b. Assume that the most significant bit (MSB) of x is a 0. We store the result of flipping x's bits into y. Interpreted in the following number representations, how large is the magnitude of y relative to the magnitude of x? Circle ONE choice per row.

Unsigned	$ y < x $	$ y = x $	$ y > x $	Can't Tell
One's Complement	$ y < x $	$ y = x $	$ y > x $	Can't Tell
Two's Complement	$ y < x $	$ y = x $	$ y > x $	Can't Tell
Sign and Magnitude	$ y < x $	$ y = x $	$ y > x $	Can't Tell

- In unsigned, a number with the MSB of 1 is always greater than one with a MSB of 0.
- In one's complement, flipping all of the bits is the negation procedure, so the magnitude will be the same.
- In two's complement, y is a negative number. Its magnitude can be found by applying the negation procedure, which is flipping the bits and then adding 1, resulting in a larger magnitude than x.
- In sign and magnitude, the 2nd MSB bit will determine the relative magnitudes of x and y, so you can't tell for certain.

3 Back to the Base-ics (Fall 2017 MT1 #1)

a) Show how the binary string 0b1011 0110 can be interpreted and displayed as the following types:

Hexadecimal: 0x_____B6_____

Unsigned Decimal: _____182_____

Two's Complement Decimal: _____-74_____

b) What is the minimum number of bits needed to represent all the unsigned integer values that a three-digit base-7 number could encode? Your answer should be a simplified decimal value.

Powers of 7 are shown below for reference:

$$7^1 = 7$$

$$7^2 = 49$$

$$7^3 = 343$$

$$7^4 = 2401$$

$$7^5 = 16807$$

_____9_____

c) What bias should be added for a biased three-digit base-7 number to yield an equal number of positive and negative numbers? Your answer should be a simplified decimal value.

_____ -171 _____

d) Convert the unsigned number 0xDF to its base-7 equivalent (i.e. the base-7 number with the same decimal value). What is the resulting number? The prefix 0s is for base-7.

4. Num Rep!

Answer the following questions about number representation:

(a) Unsigned Base 4

- (i) What is the range that a 4 digit unsigned base 4 number can represent? Write the bounds in decimal.

Solution: 0000(base 4) ~ 3333 (base 4) = 0 ~ 255

- (ii) Convert 107(base 10) to unsigned base 4.

Solution: $107(\text{base } 10) = 64 + 16 * 2 + 4 * 2 + 3 = 1223(\text{base } 4)$

(b) Signed Base 4

- (i) Suppose we wanted to use a bias in order to represent negative numbers in base 4. If we are working with a 4 digit base 4 number, what should we choose as our bias?

(Our bias should create equal amounts of negative and positive numbers for our range. If this is not possible, select a bias that will result in 1 more negative number than positive numbers).

Express your answer in decimal.

Solution: $255/2 = 127$. So the bias is -128 to favor negative numbers.

- (ii) Suppose rather than using a bias notation, we decide to do the following. For each base 4 number, we will reserve the most significant digit to strictly be used as a sign bit. A digit value of 1 will indicate a negative number, and a digit value of 0 will indicate a positive number. Any other values will result in an invalid number.

For instance: 0003₄ = +3 1003₄ = -3 2003₄ = Invalid

How many valid representation can we represent with a 4 digit base 4 number using this scheme?

Solution: $2 \cdot 4 \cdot 4 \cdot 4 = 128$