U-model-based sliding mode control for manipulator control systems

Changyi Lei*

Department of Engineering Mathematics,
University of Bristol,
Bristol, England, UK
Email: oa20932@bristol.ac.uk
*Corresponding author

Quanmin Zhu and Chenguang Yang

Department of Engineering Design and Mathematics,
University of the West of England,
Bristol, England, UK
Email: quan.zhu@uwe.ac.uk
Email: Charlie.Yang@uwe.ac.uk

Abstract: This paper investigates U-model-based Sliding Mode Control (SMC) of Multi-Input-Multi-Output (MIMO) uncertain manipulator systems with external disturbance. The proposed controller is composed of a U-model controller, and a sliding mode controller. The U-model controller is implemented to simplify the design procedure, which applies linear techniques to non-linear systems through cancelling the non-linearity and dynamics of original plant. And a sliding mode controller is designed based on Lyapunov theorem to improve the robustness of U-model-based control. The simulation on tracking task of a 2 Degree-of-Freedom (DoF) non-linear manipulator with disturbance demonstrates the enhanced robustness in U-control system operation.

Keywords: U-model; sliding mode control; manipulator; uncertain non-linear system.

Reference to this paper should be made as follows: Lei, C., Zhu, Q. and Yang, C. (2022) 'U-model-based sliding mode control for manipulator control systems', *Int. J. Cybernetics and Cyber-Physical Systems*, Vol. 1, No. 2, pp.137–156.

Biographical notes: Changyi Lei received his BSc degree in Mechanical Engineering from Beihang University, Beijing, China. Now, he is a MSc student in University of Bristol, majoring in Robotics.

Quanmin Zhu received his MSc degree from Harbin Institute of Technology, China in 1983 and PhD degree from University of Warwick, UK in 1989. Now, he is a Professor in Control Systems, with extensive research experiences in the field of non-linear system modelling, identification and control at University of the West of England. He has published over 300 papers on these topics. In the meantime, he is the Editor/Founder of *International Journal of Modelling, Identification and Control*, and an Editor of *International Journal of Computer Applications in Technology*.

Chenguang Yang received his PhD degree in Control Engineering from the National University of Singapore, Singapore in 2010, and Postdoctoral Training in Human Robotics from Imperial College London, UK. He is a Senior Member of IEEE. His research interests include human robot interaction and intelligent system design. He was awarded the EU Marie Curie International Incoming Fellowship, the UK EPSRC UKRI Innovation Fellowship and the Best Paper Award of *IEEE Transactions on Robotics* as well as over ten international conference best paper awards. He is a Co-Chair of the *Technical Committee on Bio-Mechatronics and Bio-Robotics Systems*, *IEEE Systems, Man and Cybernetics Society*; and a Co-Chair of the *Technical Committee on Collaborative Automation for Flexible Manufacturing*, *IEEE Robotics* and *Automation Society*. He serves as an Associate Editor of a number of IEEE transactions and other international leading journals.

1 Introduction

Robot manipulator is an indispensable and fundamental part for the whole industry and academia. It is broadly deployed in tasks that require accuracy, efficiency, repeatability, quality and reduction in human labour (Bamdad, 2013). For example, welding, automation assembly, cutting, packaging, painting and other industrial processes (Owen, 1992). However, considering that the manipulator dynamic system is a non-linear Multi-Input-Multi-Output (MIMO) system with coupling disturbances, the control problem becomes challenging and many state-of-the-art methods have been proposed (Huang et al., 2020). Mainstream control schemes for manipulator include PID (Hussein and Nemah, 2015), fuzzy logic (Bingul and Karahan, 2011), neural network (He et al., 2016), robust controller (Rigatos et al., 2017), etc. Although the above-mentioned methodologies have achieved certain success, some of them are tedious in design procedure (Alandoli and Lee, 2020), and there exists no widely accepted general framework for the control of manipulators.

U-model is a generic and systematic approach that converts system models into control-oriented time-varying expression, which was first proposed by Zhu and Guo (2002). It is a general model that can represent a wide range of linear, non-linear, polynomial and state-space models and simplifies the non-linear system design procedure by enabling linear system design techniques directly in non-linear systems (Zhu et al., 2016). U-model is distinguished from model-based and model-free frameworks. It utilises model-based methods to design the inverter of the dynamic plant, but the final closed-loop system performs independently from the plant. The advantages of U-model are:

- In designing a U-model controller, there is no need for making linearisation. It directly works on the non-linear plant itself.
- The whole design procedure is universal and effective for linear, non-linear, state-space and polynomial systems, making it a potent general methodology for controller design.
- Particularly, it greatly simplifies the controller design of non-linear systems, and builds a bridge between linear and non-linear systems.

The theoretical researches of U-model have been carried out in Discrete-Time (DT) system (Zhu and Guo, 2002; Zhu et al., 2016), Continuous-Time (CT) system (Zhu et al., 2019a) and state-space system (Li et al., 2020). Many controllers based on U-model were proposed to tackle challenges in practice, and achieved certain success. U-model and SMC were integrated for a helicopter pitch adjustment scenario (Wei et al., 2021). In Li et al. (2021), U-model and 2-DoF-Internal Mode Control (IMC) were combined to control Permanent Magnet Synchronous Motors (PMSM). Other controllers such as neural network (Zhu et al., 2019b), adaptive control (Zhang et al., 2018), Smith predictor (Geng et al., 2019), etc. were also explored to perform with U-model. Nevertheless, very little research of U-model is focused on MIMO systems. Besides, implementation of U-model requires precise system model, which is extremely difficult to obtain. In fact, robotic systems are constantly subject to disturbance and uncertainties, which will cause alteration of the dynamics.

Sliding Mode Control (SMC) is a special non-linear control scheme, which is distinguished from other controllers in terms of its discontinuity (Slotine and Li, 1991). Through constructing a switching function and using Lyapunov stability theorem, the SMC forces the state variables to slide on the sliding mode surface, with different control laws on different sides of the surface. The most remarkable characteristics of SMC are robustness, quick response and simple implementation (Shtessel et al., 2015). Current trends of SMC development lies in fuzzy controller, neural network, adaptive controller, terminal sliding mode, robust control, etc. In Taran and Pirmohammadi (2020), the authors proposed a fuzzy sliding mode control for the path control of a 2 DoF manipulator with imprecision and uncertainties. A novel motion control scheme based on Integral Terminal Sliding Mode (ITSM) and Deep Reinforcement Learning (DRL) was proposed in Sangiovanni et al. (2018). In Van et al. (2019), an adaptive technique was used to approximate the upper limit of the disturbance, which was fed into backstepping process for a non-linear fast terminal sliding mode controller design.

In summary, U-model has the potential to generate a simple and general framework for controller design, but it lacks robustness. SMC is a robust controller, which can be integrated with U-model to improve its robustness. Currently, a combination of U-model and SMC for MIMO non-linear system is still challenging, although some work has been carried out separately or only on Single-Input-Single-Output (SISO) systems. Therefore, this paper proposes a combined controller that takes the advantages of both MIMO U-model and SMC. The main contributions of this paper are:

- Extending U-model to MIMO systems, compared with previous research which mainly focus on SISO systems (Wei et al., 2021; Zhang et al., 2018; Geng et al., 2019; Wei et al., 2020).
- Proposing a general framework for non-linear system controller design with robustness based on U-model and SMC that can potentially be extended to control other uncertain robotic systems.

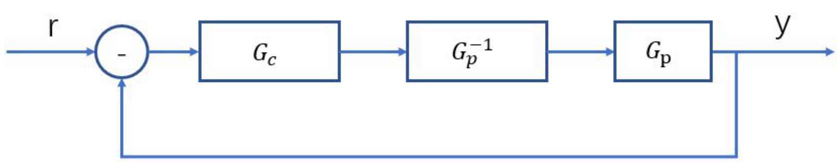
The rest of the paper is structured as follows. Section 2 describes general structure and idea of U-model, including the MIMO U-model expression. Specifically, U-model expression of continuous-time non-linear system in state-space is described. Section 3 presents the detailed design procedure based on dynamic model of a 2 DoF manipulator with disturbance. The complete framework of the proposed controller is shown, where a U-model controller as well as SMC is implemented. Section 4 presents the results and discussion of the trajectory tracking simulation. The performance of proposed methodology is compared to pure U-model controller. Section 5 draws the conclusion and puts forward future work.

2 U-model structure

2.1 General Framework of U-model controller

U-model is a model-independent methodology that has the potential to simplify non-linear system design and build a connection between linear and non-linear system (Xu et al., 2013). It consists of three parts: linear invariant controller G_c, dynamic inversion of plant G_p^{-1} and plant model G_p. Figure 1 shows a conceptual sketch of U-model, with r being the reference signal and y being the actual output. This close-loop transfer function is derived: $G = G_c / (1 + G_c)$. Therefore, by assigning G using linear system design procedures, U-model customises close-loop transient response regardless of original plant G_p, as long as G_p^{-1} exists and requires no more repetition in design procedure.

Figure 1 Conceptual sketch of U-model-based control



2.2 MIMO U-model expression for non-linear state-space CT system

A SISO CT polynomial dynamic system can be expressed as (Zhu et al., 2019a):

$$y^{(M)}(t) = \sum_{j=0}^{J} \lambda_j \left(Y_{M-1}, U_{N-1}, \Theta, t\right)\left(u^{(N)}(t)\right)^j, M > N \tag{1}$$

where $y(t) \in \mathbb{R}$ is the output, $u(t) \in \mathbb{R}$ is the input at time $t \in \mathbb{R}^+$. $y^{(M)}(t)$ and $u^{(N)}(t)$ are the M^{th} and N^{th} orders of derivative of output and input respectively. $\lambda_j(t) \in \mathbb{R}$ is a time-varying parameter that absorbs $Y_{M-1} = \left[y^{(M-1)}(t), \ldots, y(t)\right] \in \mathbb{R}^M$, $U_{N-1} = \left[u^{(N-1)}(t), \ldots, u(t)\right] \in \mathbb{R}^N$ and Θ. Θ contains all scalar coefficients. Throughout the study, it is assumed that the polynomial systems are strictly proper $(M > N)$, which guarantees the causality of the systems. Accordingly, for linear polynomial systems, $M > N$ indicates when equation (1) is converted to its Laplace transform, the denominator Laplace polynomial has higher order than the numerator in the resultant transfer function.

Take the following model as an example:

$$\ddot{y} = \dot{y} - 2y + u^2 + 0.5u \tag{2}$$

Accordingly, the continuous-time U-model realisation is:

$$\ddot{y} = \lambda_0 + \lambda_1 u + \lambda_2 u^2 \tag{3}$$

where $\lambda_0 = \dot{y} - 2y, \lambda_1 = 0.5, \lambda_2 = 1$.

Extend (1) to MIMO expression:

$$Y^{(M)}(t) = \sum_{j=0}^{J} \Lambda_j(t) \left(U^{(N)}(t)\right)^j \tag{4}$$

where $Y^{(M)} = \left[y_1^{(m_1)}(t), y_2^{(m_2)}(t), ..., y_a^{(m_a)}(t) \right]^T$ is a vector containing all outputs, $U^{(N)} = \left[\left(u_1^{(n_1)}(t)\right)^{j_1}, \left(u_2^{(n_2)}(t)\right)^{j_2}, ..., \left(u_b^{(n_b)}(t)\right)^{j_b} \right]^T$ is an input vector with the power j of all inputs. m_* is the order of derivative of y_* that is directly related to u, when u_* has the derivative order n_*. $\Lambda_j(t) \in \mathbb{R}^{a \times b}$ is a time-varying matrix instead of a scalar λ_j. For simplicity, all state variables are omitted without further declaration.

Consider a generalised MIMO continuous-time state-space model expression:

$$\begin{cases} \dot{X}(t) = F(X(t), U(t)) \\ Y(t) = H(X(t)) \end{cases} \tag{5}$$

where $Y \in \mathbb{R}^a$ is the output, $U \in \mathbb{R}^b$ is the input and $X \in \mathbb{R}^n$. F is a smooth function that maps control inputs to state variables, and H maps the states smoothly to output. Convert it to a multi-layer state-space expression in form of U-model (Wei et al., 2021).

$$\begin{cases} \dot{x}_1 = \sum_{i=0}^{n} \lambda_{1i} f_{1i}\left(x_2\right) \\ \dot{x}_2 = \sum_{i=0}^{n} \lambda_{2i} f_{2i}\left(x_3\right) \\ \quad \cdots \\ \dot{x}_n = \sum_{i=0}^{n} \lambda_{ni} f_{ni}\left(u_1, u_2, ..., u_b\right) \\ y_1 = h_1\left(x_1, x_2, ..., x_n\right) \\ y_2 = h_2\left(x_1, x_2, ..., x_n\right) \\ \quad \cdots \\ y_a = h_a\left(x_1, x_2, ..., x_n\right) \end{cases} \tag{6}$$

where λ_{mi} and $f_{mi}, 0 \le i \le n, 0 \le m \le n$ are time-varying parameters, and $h_l, 1 \le l \le a$ is smooth mapping from state vector to specific output. Take the following system as an example:

$$\begin{cases} \dot{x}_1 = x_1 + x_1 x_2 \\ \dot{x}_2 = -x_1 + u_1 + x_1 u_2 \\ y_1 = 2x_1 \\ y_2 = 3x_2 \end{cases} \tag{7}$$

Convert it to U-model expression based on the absorbing rule:

$$\begin{cases} \dot{x}_1 = \lambda_{10} + \lambda_{11} f_{11}\left(x_2\right) \\ \dot{x}_2 = \lambda_{20} + \lambda_{21} f_{21}\left(u_1, u_2\right) \\ y_1 = h_1\left(x_1, x_2\right) \\ y_2 = h_2\left(x_1, x_2\right) \end{cases} \tag{8}$$

where $\lambda_{10} = x_1, \lambda_{11} = x_1, \lambda_{20} = -x_1, \lambda_{21} = 1, f_{11}\left(x_2\right) = x_2, f_{21}(u) = u_1 + x_1 u_2,$ $h_1\left(x_1, x_2\right) = 2x_1, h_2\left(x_1, x_2\right) = 3x_2$.

2.3 MIMO U-model dynamic inversion for non-linear state-space CT system

In summary, the U-model-based dynamic inversion is undertook by calculating the following equation:

$$G_p^{-1} \Leftrightarrow U \in Y_d^{(M)}(t) - \sum_{j=0}^{J} \Lambda_j(t) \left(U^{(N)}(t)\right)^j = \mathbb{O}^a \tag{9}$$

where $Y_d(t)$ is the desired output vector and $\mathbb{O}^a$ is a $a \times 1$ null vector. The prerequisites for the solution to exist are external stability and non-minimum phase of the system. Reconsidering (6), apply derivative to the $p-th\left(0 \le p \le a\right)$ output y_p with respect to x_i :

$$\dot{y}_p = \sum_{i=0}^{n} h_p'(x_i) \dot{x}_i \tag{10}$$

Replace $\dot{x}_i$ with polynomial equation and we have:

$$\dot{y}_p = \sum_{i=0}^{n} \sum_{j=0}^{n} h_p'\left(x_i\right) \lambda_{ij} f_{ij}\left(x_j\right) \tag{11}$$

Repeating the above derivative and replacement procedures for m_p times, until getting a direct relationship between y and u . Rearrange the equation in a general expression:

$$y_p^{(m_p)} = \sum_{i=1}^{n-1} P_{ip}\left(X, t\right) \dot{x}_i + P_{np}\left(X, t\right) \dot{x}_n \tag{12}$$

where X is the vector of state variables, and P is a function of X. Note that (12) may contain multiple inputs, so that an equation set with a number of equations with the form of (12) should be solved to get the desired control vector $U \in \mathbb{R}^a$.

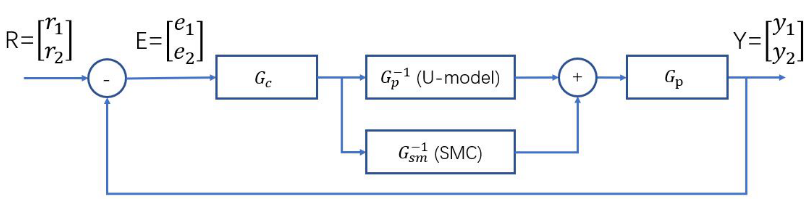
3 Controller design

The design procedure of U-model controller and sliding mode controller are articulated in this section, which will be used to fulfil the tracking task of a 2 DoF manipulator dynamic system with disturbance.

3.1 Overview of controller framework

Figure 2 exhibits the structure of U-model-based SMC controller. R is the reference signal input vector, Y is the output vector and E is the error vector. Similarly, r_*, e_*, y_* represent reference signal, error and output of each DoF. G_c is called invariant controller, G_p^{-1} is the dynamic inverter of U-model, G_{sm}^{-1} is the compensated inverter based on SMC, and G_p is the actual plant. The output error E is transferred through G_c and then fed into U-model and SMC inverter, respectively. The output of those two inverters are then summed up to produce desired control. It is expected to satisfy $\left(G_p^{-1} + G_{sm}^{-1}\right) G_p = \mathbb{I}_2$ through manipulation, so that the non-linearity of original plant can be cancelled, which is the core idea of U-model control. The U-model inverter and SMC inverter are designed separately, which will be discussed in the following sections.

Figure 2 Structure of U-model-based SMC



3.2 Target plant G_p

The general dynamic model of robot manipulators with disturbance is (He et al., 2017)

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + d(t), \tag{13}$$

where $\tau \in \mathbb{R}^n$ is control torque, and $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ represent joint angle vector, velocity and acceleration respectively. $M(q) \in \mathbb{R}^{n \times n}$ is inertial matrix of the manipulator, which describes the torque generated by angular acceleration due to the links' masses motion or distribution relative to each other. $C(q,\dot{q}) \in \mathbb{R}^{n \times n}$ describes the torque generated due to the links' engaging in rotational motion and damping components. $G(q) \in \mathbb{R}^n$ is contributed by the links' masses affected by gravity and elasticity of the links if they are not rigid (Craig, 1989). Denote overall disturbance as $d(t) \in \mathbb{R}^n$.

Rewrite (13) in state-space, and get the following equation:

$$G_p = \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = M^{-1}\left[(\tau - C\left(x_1, x_2\right)x_2 - G\left(x_1\right) - d(t)\right] \end{cases} \tag{14}$$

in which x_1 represents joint value q, x_2 is joint velocity $\dot{q}$ and $\dot{x}_2$ is angular acceleration. Equation (14) is the state-space dynamic model of a manipulator with disturbance. For simplicity, all time variables t are omitted if without being emphasised.

3.3 U-model Inverter G_p^{-1}

The U-model controller of a 2-DoF manipulator dynamic system is designed on the basis of the nominal model without disturbance. The whole design procedures include that of an invariant controller and dynamic inverter.

3.3.1 Invariant controller design

When controlling a robot manipulator, it is essential for robots to reach the target location smoothly and as quickly as possible without unnecessary oscillation. In control engineering, critical damping system is preferred since it reaches the equilibrium as quickly as possible without causing oscillation or overshoot. Consequently, a critically damped second order transfer function is assigned as the desired system response. For simplicity and without loss of generality, the ideal close-loop transfer function is specified as a 2×2 diagonal matrix:

$$G(s) = diag\left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}, \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}\right) \tag{15}$$

where the damping ratio ξ and natural frequency ω_n are selected according to specific requirements, and $diag$ represents diagonal matrix.. Accordingly, the invariant controller can be derived as

$$G_c(s) = diag\left(\frac{G(s)}{1 - G(s)}, \frac{G(s)}{1 - G(s)}\right) = diag\left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s}, \frac{\omega_n^2}{s^2 + 2\xi\omega_n s}\right) \tag{16}$$

3.3.2 Ideal inverter G_p^{-1} design

From (14), we can see that the derivative of x_2 is directly related to control torque τ, so it can be used to design ideal inverter G_p^{-1} directly. Rewrite (14) into U-model expression with $d(t)$ being zero vector:

$$\begin{cases} u = \tau \\ \dot{y} = \dot{x}_1 = x_2 \\ \ddot{y} = \dot{x}_2 = \lambda_0 + \lambda_1 u \\ \lambda_0 = -M^{-1}\left(Cx_2 + G\right) \\ \lambda_1 = M^{-1} \end{cases} \tag{17}$$

Therefore, the inverter G_p^{-1} is designed as:

$$u = M\ddot{y} + Cx_2 + G \tag{18}$$

Here, $\ddot{y}$ should be replaced by $\ddot{x}_d$, which represents referenced angular acceleration. Therefore, we have:

$$u = M\ddot{x}_d + Cx_2 + G \tag{19}$$

3.4 Compensated inverter G_{sm}^{-1} – sliding mode control

Assumption 1: *Assume disturbance variable $d(t)$ is unknown and bounded, which satisfies the following condition*

$$\| d(t) \| < D \tag{20}$$

where $D > 0$ represents the upper bound of disturbance.

Remark 1: *In practice, although disturbance seems irregular and impossible to be precisely modelled, it is dependent on the states of the system. For example, viscous friction is proportional to velocity of the robot. The states of a robot are usually bounded, and therefore it is reasonable to assume disturbance is bounded. Similar assumptions can be seen in other relevant researches. In Bai et al. (2020), it is assumed that the unknown dynamics of a non-strict feedback system is bounded. Similarly, in He et al. (2017), the authors assumed that the disturbance exerted by human and environment is bounded.*

With this assumption, we are able to design the compensated inverter based on backstepping and sliding mode control. Consider (14), we design the first virtual control variable

$$z_1 = x_1 - z_d \tag{21}$$

where x_1 is the angle vector of the joints, and z_d is the reference angle vector. The physical meaning of z_1 is the angular error. Therefore, it is easily seen that

$$\dot{z}_1 = x_2 - \dot{z}_d \tag{22}$$

in which $\dot{z}_d$ is the reference angular velocity. Assign partial Lyapunov scalar function

$$V_1 = \frac{1}{2} z_1^T z_1 \tag{23}$$

Obviously, V_1 is semi-positive definite. Inspect the derivative of V_1:

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T \left(x_2 - \dot{z}_d \right) \tag{24}$$

Borrowing ideas from Lyapunov stability theorem and backstepping, design the second virtual variable

$$z_2 = x_2 - \dot{z}_d + c_1 z_1, \tag{25}$$

where c_1 is a positive scalar. z_2 means a weighted sum of angular error and angular velocity error, with c_1 adjusting the relative importance of those two errors. Integrate (24) into (25):

$$\dot{V}_1 = z_1^T \dot{z}_1 = z_1^T \left(x_2 - \dot{z}_d \right) = -c_1 z_1^T z_1 + z_1^T z_2 \tag{26}$$

From linear algebra, $z_1^T z_1$ is semi-positive definite, so if z_2 manages to approach 0, then $\dot{V}_1 \le 0$, and therefore the system can be stable according to Lyapunov stability theorem. Now, we should design a controller that ensures the convergence of z_2. Rearrange (14) into system with respect to virtual variables z_1, z_2:

$$\begin{cases} \dot{z}_1 = z_2 - c_1 z_1 \\ \dot{z}_2 = M^{-1}\left(\tau + c_1 C z_1 - C\dot{z}_d - C z_2 - G - d \right) + c_1 \dot{z}_1 - \ddot{z}_d \end{cases} \tag{27}$$

Assign second Lyapunov function

$$V_2 = V_1 + \frac{1}{2} z_2^T z_2. \tag{28}$$

Inspect the derivative of V_2:

$$\dot{V}_2 = -c_1 z_1^T z_1 + z_1^T z_2 + z_2^T [M^{-1}(\tau + c_1 C z_1 - C\dot{z}_d - C z_2 - G - d) + c_1 \dot{z}_1 - \ddot{z}_d] \tag{29}$$

Accordingly, using isokinetic reaching law (Wang et al., 2019), design compensated inverter:

$$\begin{cases} u_{eq} = C\left(-c_1 z_1 \dot{z}_d + z_2 \right) + G + M\left(-c_1 \dot{z}_1 + \ddot{z}_d - c_2 z_2 - z_1 \right) \\ u_{sw} = -\eta \odot \operatorname{sgn}\left(z_2 \right) \\ u = u_{eq} + u_{sw} \end{cases} \tag{30}$$

where $\eta > D$ is a positive scalar, and $\odot$ means Hadamard product. The larger it is, the quicker the system converges to equilibrium, and also higher frequency of chattering. u_{eq} represents equivalent control in sliding mode surface, and u_{sw} is switching control. To alleviate the chattering problem, sgn function is substituted by a saturation function

$$sat(x) = \frac{x}{|x| + \varepsilon} \tag{31}$$

where $\varepsilon > 0$ is a positive time-invariant scalar that adjusts the smoothness of the function.

3.4.1 Stability analysis for compensated inverter G_{sm}^{-1}

Theorem 1: *For a class of second-order dynamic systems (14), using controller presented in (30), the system can converge to equilibrium.*

Proof: Integrating (30) into (27):

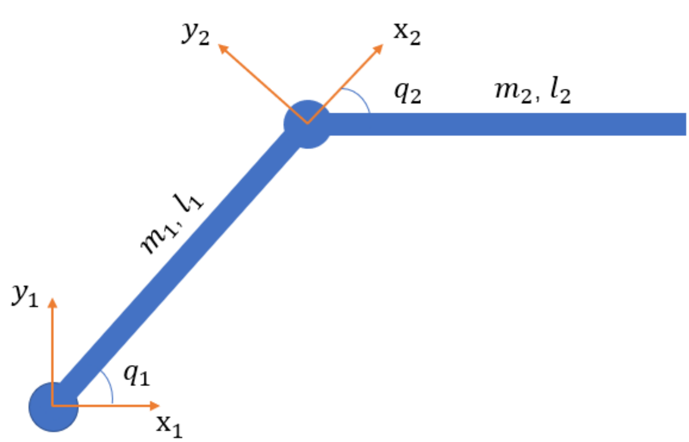
$$\dot{V}_2 = -c_1 z_1^T z_1 - c_2 z_2^T z_2 - z_2^T M^{-1}\left(\eta \odot sgn\left(z_2\right) - d\right) \tag{32}$$

From the physical meaning and characteristic of mass matrix M, it is positive definite, so that M^{-1} is also positive definite. Based on assumption 1, $z_2^T M^{-1}\left(\eta \odot sgn(z_2) - d\right)$ is constantly larger or equal to 0. So $\dot{V}_2 \leq 0$, which satisfies Lyapunov stability theorem, and that the system (27) can be stable under the controller designed. This means that z_1, z_2 will converge to equilibrium. Considering that z_1 is angular error and z_2 is a weighted sum of angular error and angular velocity error, stability of (27) means that the system (14) will follow the desired trajectory despite disturbance.

4 Simulation

The simulation is implemented on trajectory tracking of a 2 DoF manipulator in MATLAB Simulink, of which the structure is shown in Figure 3. Note that $\left(x_1, y_1\right)$ is the base frame, which is fixed to the ground and describes global coordinates. The reference trajectory will be given in Cartesian space with respect to base frame, and the coordinates have to be converted to joint space via inverse kinematics.

Figure 3 Structural graph of 2 DoF manipulator



4.1 Dynamic model calculation

The manipulator is modelled as with two homogeneous links. According to Euler-LaGrange equation, the dynamic model of a 2 DoF manipulator is calculated, and the results of system matrices M, C, G are shown below:

$$M(q) = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \tag{33}$$

$$C(q,\dot{q}) = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \tag{34}$$

$$G(q) = \begin{pmatrix} G_{11} \\ G_{21} \end{pmatrix} \tag{35}$$

$$\begin{cases} M_{11} = \frac{1}{3}\left(m_1 l_1^2 + m_2 l_2^2\right) + m_2\left(l_1^2 + l_1 l_2 \cos q_2\right), 1ex \\ M_{12} = m_2\left(\frac{1}{3} l_2^2 + \frac{1}{2} l_1 l_2 \cos q_2\right), 1ex \\ M_{21} = m_2\left(\frac{1}{3} l_2^2 + \frac{1}{2} l_1 l_2 \cos q_2\right), 1ex \\ M_{22} = \frac{1}{3} m_2 l_2^2 \end{cases} \tag{36}$$

$$\begin{cases} C_{11} = h\dot{q}_2, 1ex \\ C_{12} = h\left(\dot{q}_1 + \dot{q}_2\right), 1ex \\ C_{21} = h\dot{q}_1, 1ex \\ C_{22} = 0, 1ex \\ h = -\frac{1}{2} l_1 l_2 m_2 \sin q_2 \end{cases} \tag{37}$$

$$\begin{cases} G_{11} = \frac{1}{2} m_1 g l_1 \cos q_1 + m_2 g\left[l_1 \cos q_1 + \frac{1}{2} l_2 \cos\left(q_1 + q_2\right)\right], 1ex \\ G_{21} = m_2 g\left[\frac{1}{2} l_2 \cos\left(q_1 + q_2\right)\right], 1ex \\ g = 9.8 \end{cases} \tag{38}$$

in which m_*, l_* are the mass and length of the links respectively, and $q_*, \dot{q}_*$ are the angle and angular velocity of each joint.

4.2 Parameters specification

The parameters of the manipulator are given in Table 1, and the parameters of the controllers are in Table 2. The end effector starts with $(2,0)$ in Cartesian space with respect to the base frame of the manipulator, and the velocity and acceleration are zero. The reference trajectory is a circle in Cartesian space defined as

$$\begin{cases} x_1 = 0.8 + \cos\left(0.4t\right) \\ y_1 = 0.8 + \sin\left(0.4t\right) \end{cases} \tag{39}$$

Besides, the reference acceleration and velocity can be obtained by derivative of x_d, y_d. The system model is added with uniform random values to both joints ranging from –0.2 N*m to 0.2 N*m to simulate disturbance $d(t)$, as will be shown in Figure 8.

Table 1 Parameters of manipulator

Parameters	*Link1*	*Link2*
Mass/kg	$m_1 = 1$	$m_2 = 1$
Length/m	$l_1 = 1$	$l_2 = 1$

Table 2 Parameter of controllers

Parameters	*Link1*	*Link2*
c_1	10	10
c_2	10	10
η	20	20
ε	0.01	0.01
ω_n	3	3
ξ	1	1

4.3 Results and discussion

Inverse kinematics is first calculated to derive desired angles in joint space, and then feed them to U-model-based sliding mode control system. The simulation lasts for 200 s and the sampling time is 0.001 s. The results are shown in Figures 4 to 12.

Figure 4 Joint 1 output angle

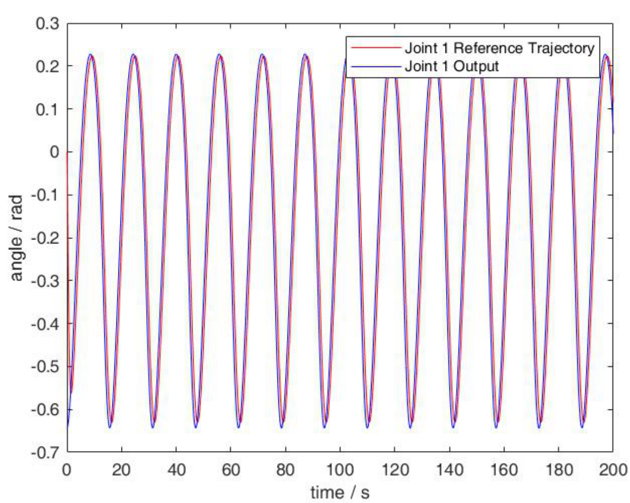


Figure 5 Joint 2 output angle

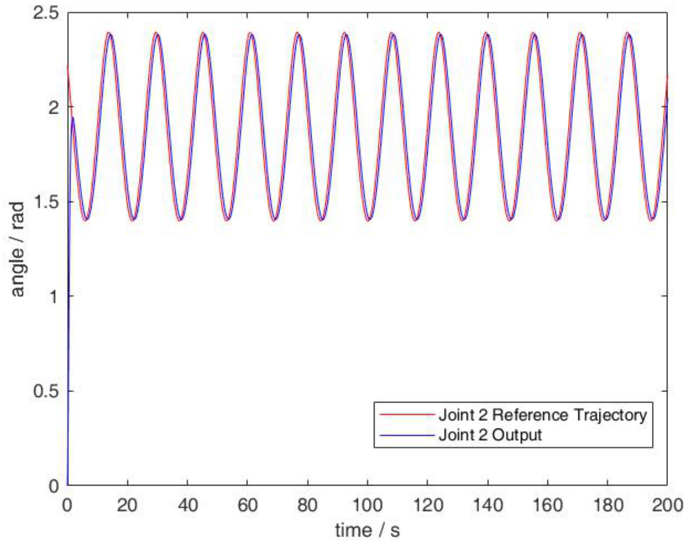


Figure 6 Joint 1 SMC control torque

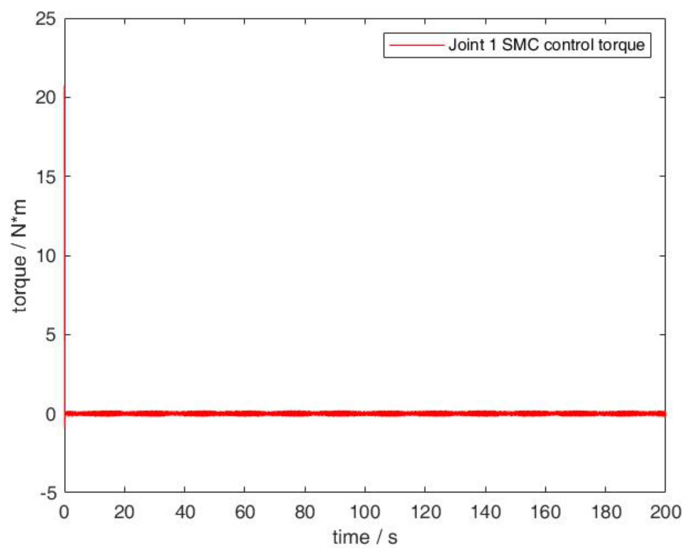


Figure 7 Joint 2 SMC control torque

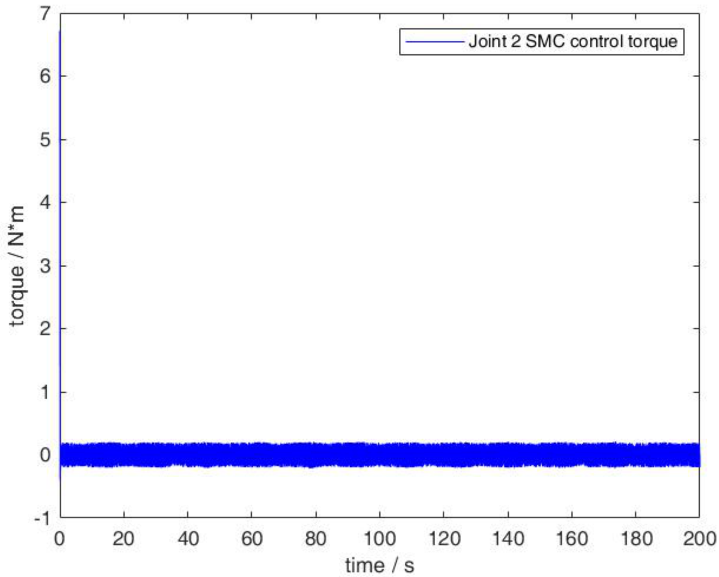


Figure 8 Noise

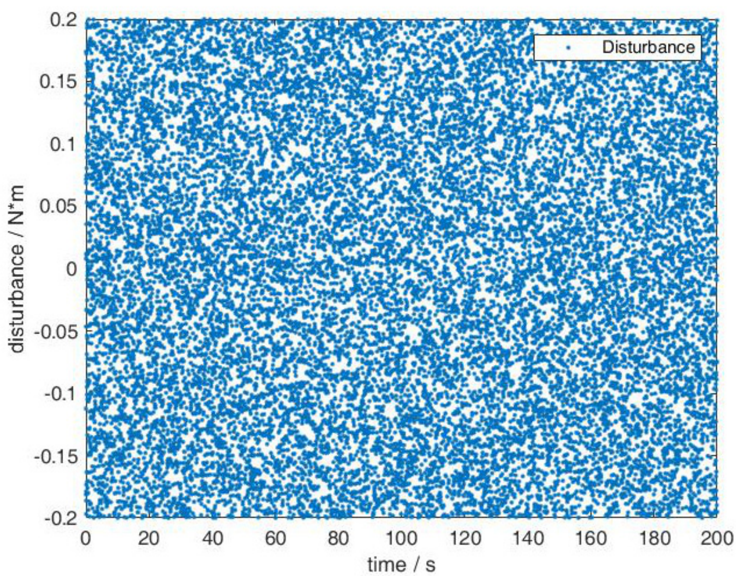


Figure 9 Sliding mode variable profile

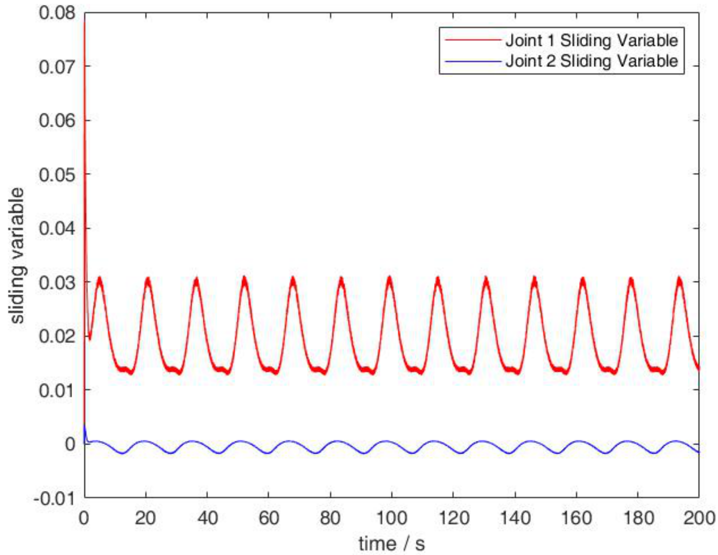


Figure 10 Joint 1 U-model control torque

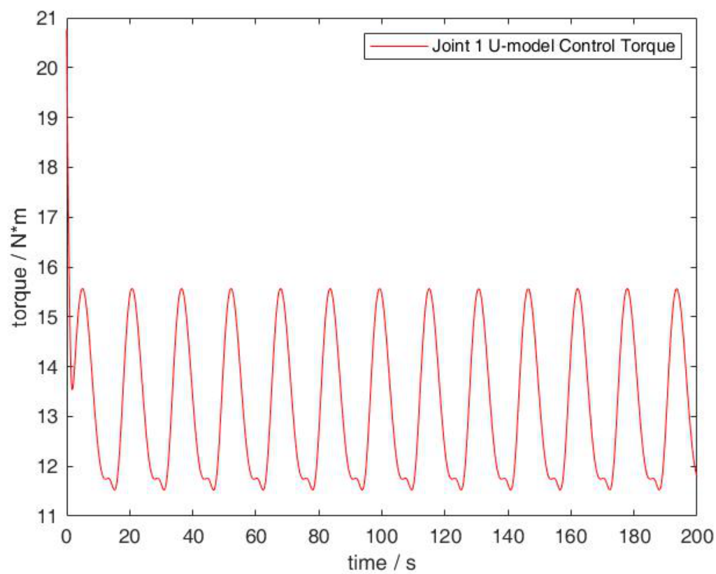


Figure 11 Joint 2 U-model control torque

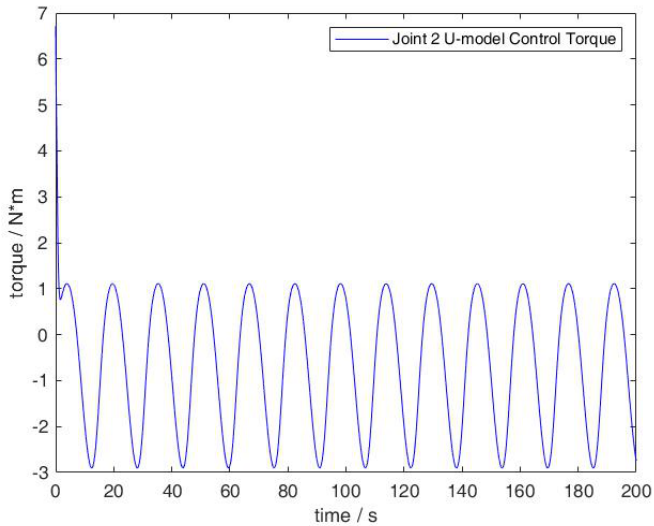


Figure 12 End effector trajectory for circle input

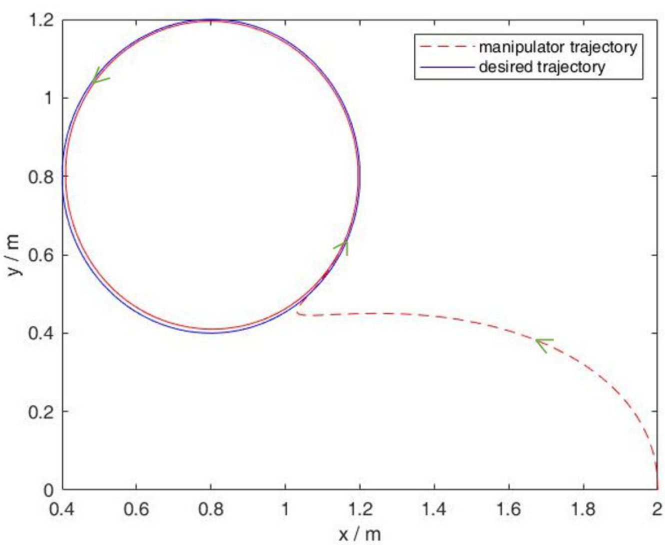


Figure 4 shows the reference angle and the real angle of the first joint. The blue line indicates the reference angle, which is well followed by the real output indicated by yellow line. Similarly, Figure 5 illustrates the output profile of the second joint. Figures 6 and 7 indicate the control output provided by the compensated inverter based on SMC. We can see that the output performs high frequency oscillation, which indicates the chattering of SMC. And that the chattering suppression is not ideal. Figure 8 shows the noise added to both joints, which are uniform random values ranging from –0.2 N*m to

0.2 N*m. Figure 9 is the values of switching functions. As defined in (25), the sliding mode variables is a weighted sum of angular error and angular velocity error. It also shows chattering issue due to existence of SMC. Figures 10 and 11 are control output produced by pure U-model controller. It is worth noting that Figure 12 shows the trajectory of the manipulator end-effector in Cartesian space. The manipulator starts with (2, 0) and follows the circle afterwards despite disturbance. This reveals the satisfying capability of the proposed methodology in terms of suppressing disturbance.

In comparison with Figures 4 and 5, the outputs using pure U-model controller are shown in Figures 13 and 14, where the outputs are deviated from the reference signals due to disturbance. This in turn highlights the outstanding performance of proposed method in terms of robustness.

Figure 13 Joint 1 pure U-model output

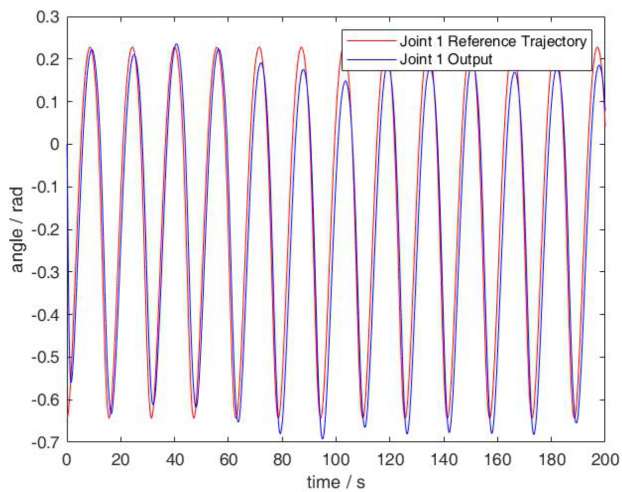
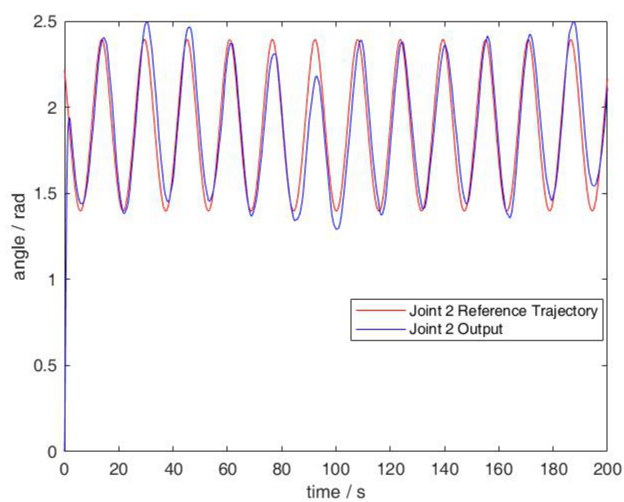


Figure 14 Joint 2 pure U-model output



5 Conclusions

An innovative controller for tracking issue of (uncertain) robotic systems composed of U-model and SMC is proposed in this paper. The proposed methodology successfully combines the simplicity in design process and robustness against disturbance. Its satisfying performance in simulation is demonstrated from the dynamic inversion of U-model that enables linear system design techniques despite of non-linearity, and the robustness of SMC. It can serve as a potent alternative to other non-linear system control methods, and could be potentially implemented on other robotic systems.

Some future work is suggested to further extend this research. Firstly, the proposed methodology is currently only verified in simulation. It should be tested on a real robot experiment to observe its performance in practice for more comprehensive analysis and comparison. Secondly, it is suggested to expand U-model to other MIMO systems, specifically over-actuated and under-actuated systems. Last but not least, the chattering issue of SMC needs to be further dealt with appropriately.

Acknowledgement

The authors would like to show their gratitude to the editors and the anonymous reviewers for their helpful comments and constructive suggestions with regard to the revision of the paper.

References

Alandoli, E.A. and Lee, T.S. (2020) 'A critical review of control techniques for flexible and rigid link manipulators', *Robotica*, Vol. 38, No. 12, pp.2239–2265.

Bai, W., Li, T. and Tong, S. (2020) 'NN reinforcement learning adaptive control for a class of nonstrict-feedback discrete-time systems', *IEEE Transactions on Cybernetics*, Vol. 50, No. 11, pp.4573–4584.

Bamdad, M. (2013) 'Analytical dynamic solution of a flexible cable-suspended manipulator', *Frontiers of Mechanical Engineering*, Vol. 8, No. 4, pp.350–359.

Bingul, Z. and Karahan, O. (2011) 'A fuzzy logic controller tuned' with pso for 2 dof robot trajectory control', *Expert Syst. Appl.*, Vol. 38, pp.1017–1031.

Craig, J.J. (1989) *Introduction to Robotics: Mechanics and Control*, Addison-Wesley Longman Publishing Co., Inc., 2nd ed., USA.

Geng, X., Zhu, Q., Liu, T. and Na, J. (2019) 'U-model based predictive control for nonlinear processes with input delay', *Journal of Process Control*, Vol. 75, pp.156–170.

He, W., David, A.O., Yin, Z. and Sun, C. (2016) 'Neural network control of a robotic manipulator with input deadzone and output constraint', *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, Vol. 46, No. 6, pp.759–770.

He, W., Huang, H. and Ge, S. (2017) 'Adaptive neural network control of a robotic manipulator with time-varying output constraints', *IEEE Transactions on Cybernetics*, Vol. 47, pp.3136–3147.

Huang, D., Yang, C., Ju, Z. and Dai, S-L. (2020) 'Disturbance observer enhanced variable gain controller for robot teleoperation with motion capture using wearable armbands', *Autonomous Robots*, Vol. 44, pp.1217–1231.

Hussein, M.T. and Nemah, M.N. (2015) 'Control of a two-link (rigid-flexible) manipulator', *Proceedings of the 3rd RSI International Conference on Robotics and Mechatronics (ICROM)*, pp.720–724.

Li, R., Zhu, Q., Kiely, J. and Zhang, W. (2020) 'Algorithms for u-model-based dynamic inversion (um-dynamic inversion) for continuous time control systems', *Complex.*, pp.1–14.

Li, R., Zhu, Q., Narayan, P., Yue, A., Yao, Y. and Deng, M. (2021) 'U-model-based two-degree-of-freedom internal model control of nonlinear dynamic systems', *Entropy*, Vol. 23, pp.1–22.

Owen, T. (1992) *Robots and Manufacturing Automation (Second Edition) by C. RAY Asfall John Wiley & Sons, 487pages incl. Index*, Robotica, Vol. 10, No. 6, pp.579–580.

Rigatos, G., Pierluigi, S. and Raffo, G.V. (2017) 'A nonlinear h-infinity control method for multi-dof robotic manipulators', *Nonlinear Dynamics*, Vol. 88, pp.1–20.

Sangiovanni, B., Incremona, G.P., Ferrara, A. and Piastra, M. (2018) 'Deep reinforcement learning based self-configuring integral sliding mode control scheme for robot manipulators', *IEEE Conference on Decision and Control (CDC)*, pp.5969–5974.

Shtessel, Y., Edwards, C., Fridman, L. and Levant, A. (2015) *Sliding Mode Control and Observation*, Birkhäuser Basel, 1st ed., Springer.

Slotine, J. and Li, W. (1991) *Applied Nonlinear Control*, Prentice Hall.

Taran, B. and Pirmohammadi, A. (2020) 'Designing an optimal fuzzy sliding mode control for a two-link robot', *Journal of The Brazilian Society of Mechanical Sciences and Engineering*, Vol. 42, pp.1–12.

Van, M., Mavrovouniotis, M. and Ge, S. (2019) 'An adaptive backstepping nonsingular fast terminal sliding mode control for robust fault tolerant control of robot manipulators', *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, Vol. 49, pp.1448–1458.

Wang, Y., Feng, Y., Zhang, X., Liang, J. and Cheng, X. (2019) 'New reaching law control for permanent magnet synchronous motor with extended disturbance observer', *IEEE Access*, pp.186296–186307.

Wei, W., Chen, N., Zhiyuan, Z., Liu, Z.W. and Zuo, M. (2020) 'U-model-based active disturbance rejection control for the dissolved oxygen in a wastewater treatment process', *Mathematical Problems in Engineering*, pp.1–14.

Wei, Z., Li, R. and Zhu, Q. (2021) 'Sliding mode plus u-control method for continuous time siso dynamic systems', *International Journal of Cybernetics and Cyber-Physical Systems*, Vol. 1, pp.68–92.

Xu, F., Zhu, Q., Zhao, D. and Li, S. (2013) 'U-model based design methods for nonlinear control systems: a survey of the development in the 1st decade', *Control and Decision*, Vol. 28, No. 7.

Zhang, J., Zhu, Q., Li, Y., Wu, X. and Gao, F. (2018) 'A general u-model based adaptive control design procedure for nonlinear non-affine systems', *Proceedings of the 37th Chinese Control Conference (CCC)*, pp.2860–2864.

Zhu, Q. and Guo, L. (2002) 'A pole placement controller for non-linear dynamic plants', *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, Vol. 216, pp.467–476.

Zhu, Q., Zhang, W., Na, J. and Sun, B. (2019a) 'U-model based control design framework for continuous-time systems. In *2019 Chinese Control Conference (CCC)*, pp.106–111.

Zhu, Q., Zhang, W., Zhang, J. and Sun, B. (2019b) 'U-neural network-enhanced control of nonlinear dynamic systems', *Neurocomputing*, Vol. 352, pp.12–21.

Zhu, Q., Zhao, D. and Zhang, J. (2016) 'A general u-block model-based design procedure for nonlinear polynomial control systems', *International Journal of Systems Science*, Vol. 47, pp.3465–3475.