* Computer Vision (Final Exam) __/__/__

1) $\parallel$ proj. eqns:

$$x = \alpha X + x_0$$
$$y = \alpha(\cos(\theta)Y - \sin(\theta)Z) + y_0$$

To show: $\begin{bmatrix} x \\ y \end{bmatrix} = \alpha P R_x(\theta) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

$R_x(\theta) \rightarrow 3\times3$ (rotation over X)

$P \rightarrow 2\times3$ (ortho. proj)

$\alpha \rightarrow$ scaling factor

To find: α, x_0, y_0 world point $(0,0,0)$ projects onto $(0,0)$

$(1,0,0) \rightarrow (3,0)$

Ans: Rotation matrix:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Orthogonal Projection:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Applying these transformations:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \alpha P R_x(\theta) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$x = \alpha (X) + x_0$$
$$y = \alpha (Y\cos\theta - Z\sin\theta) + y_0$$

from,

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \alpha \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \alpha \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$= \alpha \begin{bmatrix} X \\ Y\cos\theta - Z\sin\theta \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

Now, to find α, x_0, y_0

pt 1

$(0,0,0) \rightarrow (0,0)$

$0 = \alpha(0) + x_0$

$0 = \alpha(0\cos\theta - 0\sin\theta) + y_0$

$\therefore x_0 = 0, \; y_0 = 0$

pt 2

$(1,0,0) \rightarrow (3,0)$

$3 = \alpha(1) + x_0$ here $x_0 = 0$

$0 = \alpha(0\cos\theta - 0\sin\theta) + y_0$ here $y_0 = 0$

$\therefore \alpha = 3$

$\therefore \alpha = 3, \; x_0 = 0, \; y_0 = 0$

2)

a)

Ans: Due to consistent perspective, shadows, and lighting captured in the painting, a subtle optical illusion occurs in your brain as you move around.

This illusion gives the impression that the eyes are tracking your movements, even though the image remains static.

b)

Ans: These objects are so far away that their relative positions do not significantly change as we move horizontally or vertically. The distance is effectively constant thus they appear stationary.

Since our eyes are not fixed in a single orientation, these objects do change with rotation. As we rotate, the direction in which we look changes. This change results in the apparent movement or rotation.

3) Mirror images: comparing fundamental matrix

Ans: Let X be a point on the object

$R_f X$ be the point on the reflecⁿ.

$R_f \rightarrow$ 3D reflection

for camera, the points are,

PX, PR_fX

$$R_f = H \begin{pmatrix} \Delta & 0 \\ 0^T & 1 \end{pmatrix} H^{-1}$$

$H \rightarrow$ euclidean transformation

$\Delta = \text{diag}(-1, 1, 1)$

$$H = \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix}$$

taking, $P = K[I|0]$

$PR_f = K[I|0] H \Lambda H^{-1}$

$\Rightarrow$ now, $PR_f = K[R\Lambda R^T | -R\Lambda R^T t + t]$
$= K[R\Lambda R^T | R\Gamma R^T t]$

here $\Gamma = I - \Lambda = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\Rightarrow$ the corresponding canonical cameras will be,
$[I|0]$
$K[R\Lambda R^T K^{-1} | R\Gamma R^T t]$

$\Rightarrow$ the fundamental matrix relating these two,

$$F = [KR\Gamma R^T t]_\times KR\Lambda R^T K^{-1}$$

$\Rightarrow$ to check if it is skew-symmetric

$x^T F x = 0$ should hold $\forall x$

$$\Rightarrow x^T F x = x^T [KR\Gamma R^T t]_\times KR\Lambda R^T K^{-1} x$$
$$= (K^{-1}x)^T [R\Gamma R^T t]_\times R\Lambda R^T (K^{-1}x)$$

⇒ substituting $K^{-1}x = x'$

$$x'^T [R\Gamma R^T t]_\times R\Lambda R^T x'$$

⇒ now, checking if $[R[R^T t]_\times R\Lambda R^T$ is skew symmetric

$$R[\Gamma R^T t]_\times \Lambda R^T \rightarrow \text{skew-symmetric}$$

as $$[\Gamma R^T t]_\times = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -x \\ 0 & x & 0 \end{pmatrix}$$

∴ we can conclude that
F is autoepipolar

reference: Immensely Happy