Homework 5

1. The exponential moving average predicition equations are given by the following:

If the values of a function X(t) are known for t = 0, 1, 2, ..., t an estimate of the future value at time t + T is given by

$$\hat{X}(t+T) = \bar{X}(t) + \bar{b}(t)\left[\frac{1}{\alpha} + T - 1\right]$$

where $$\bar{X}(t) = \alpha X(t) + (1-\alpha)\bar{X}(t-1) \qquad 0<\alpha<1$$

and $$\bar{b}(t) = \alpha\,[\bar{X}(t) - \bar{X}(t-1)] + (1-\alpha)\bar{b}(t-1)$$

Use the IBM data you downloaded in homework 4, and perform one day look ahead prediction of data. Compare your predicted values to the true values.

2. (Hagan, ch. 10)

Now consider the convergence of the system of Problems P10.3 and P10.4. What is the maximum stable learning rate for the LMS algorithm?

3. (Hagan, ch. 10)

P10.6 Consider the adaptive filter ADALINE shown in Figure P10.5. The purpose of this filter is to predict the next value of the input signal from the two previous values. Suppose that the input signal is a stationary random process, with autocorrelation function given by:

$$C_y(n) = E[y(k)\,y(k+n)]$$

$$C_y(0) = 3,\ C_y(1) = -1,\ C_y(2) = -1.$$

i. **Sketch the contour plot of the performance index (mean square error).**

ii. **What is the maximum stable value of the learning rate (α) for the LMS algorithm?**

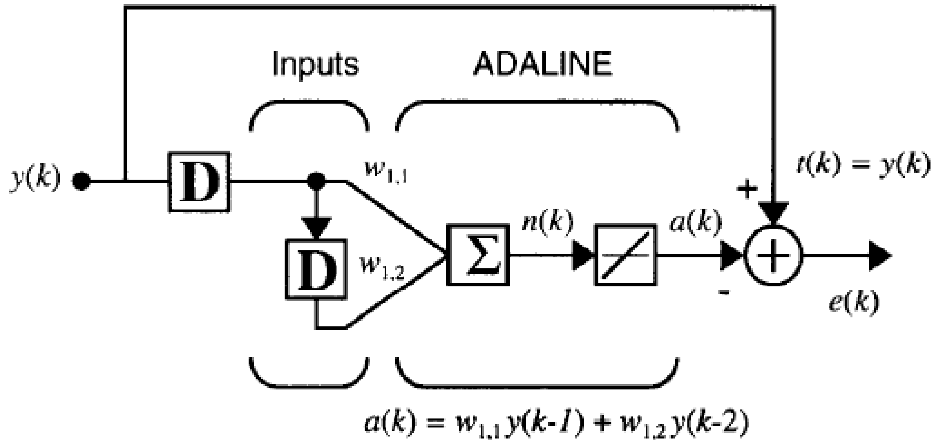

Figure P10.5 Adaptive Predictor

4. (Gonzalez, ch. 4)

4.1 Show that $F(u)$ and $f(x)$ in Eqs. (4.2-5) and (4.2-6) are a Fourier transform pair. You can do this by substituting Eq. (4.2-6) for $f(x)$ into Eq. (4.2-5) and showing that the left and right sides are equal. Repeat the process by substituting Eq. (4.2-5) for $F(u)$ into Eq. (4.2-6). You will need the following orthogonality property of exponentials:

$$\sum_{x=0}^{M-1} e^{j2\pi rx/M} e^{-j2\pi ux/M} = \begin{cases} M & \text{if } r = u \\ 0 & \text{otherwise.} \end{cases}$$

5. (Gonzalez, ch. 4)

4.4 A Gaussian lowpass filter in the frequency domain has the transfer function

$$H(u, v) = Ae^{-(u^2+v^2)/2\sigma^2}.$$

Show that the corresponding filter in the *spatial* domain has the form

$$h(x, y) = A2\pi\sigma^2 e^{-2\pi^2\sigma^2(x^2+y^2)}.$$

(*Hint:* Treat the variables as continuous to simplify manipulation.)