# Smoothing Structured Decomposable Circuits

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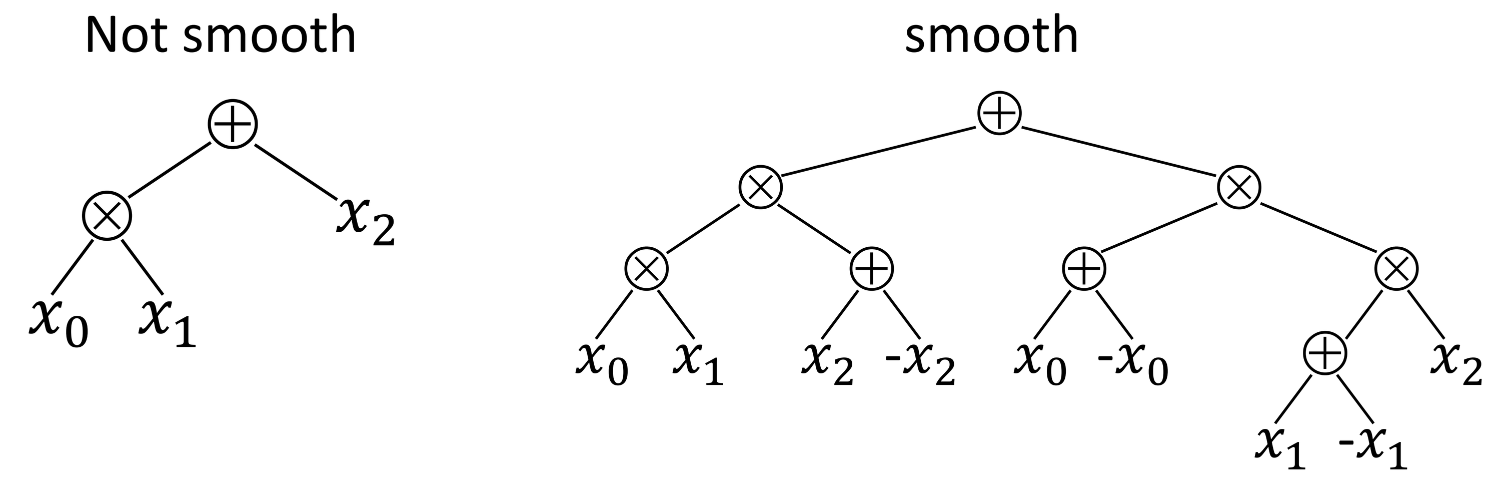
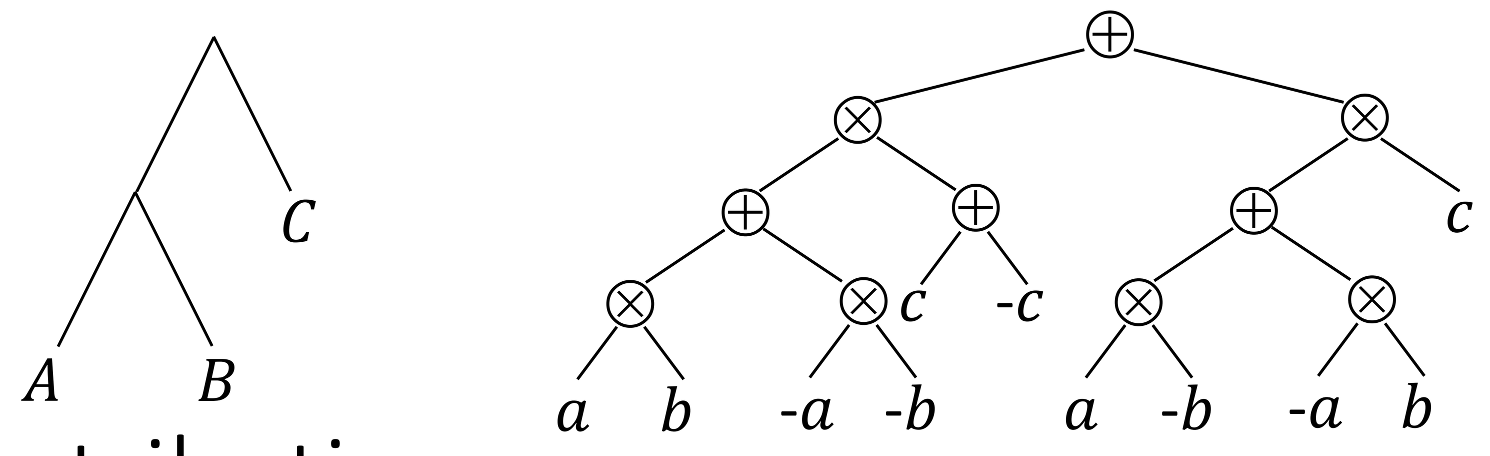
Smoothness is necessary for linear-time Weighted Model Counting and All-Marginals on circuits. But, the current smoothing algorithm is quadratic.




## Structured decomposability

Example from Shen et al. "Tractable operations for arithmetic circuits of probabilistic models."



## Contributions

| Task | Operations | Complexity |
|---|---|---|
| Smoothing | $\oplus, \otimes$ | $O(m \cdot \alpha(m, n))$ |
| Smoothing* | $\oplus, \otimes$ | $\Omega(m \cdot \alpha(m, n))^*$ |
| All-Marginal | $\oplus, \ominus, \otimes, \oslash$ | $\Theta(m)$ |

* For *smoothing-gate algorithms* on decomposable circuits.

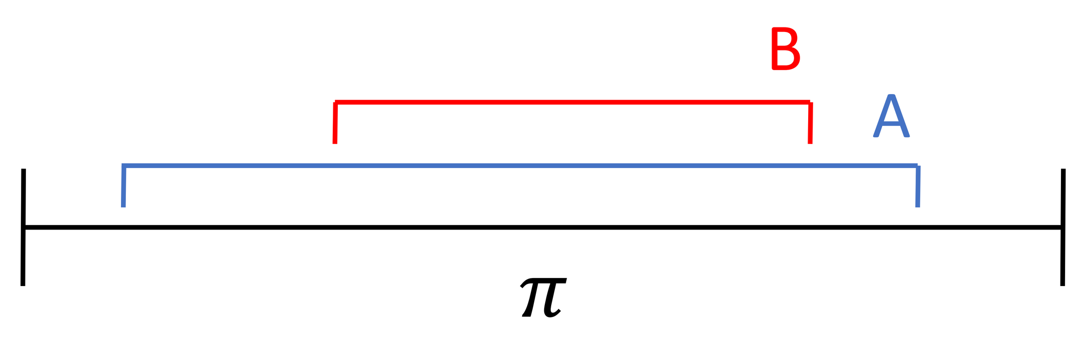
## Smoothing gate algorithms

Think "Filling in missing variables":
*The output circuit has a subcircuit that is isomorphic to input circuit (after edge contraction).*

## Missing variables form 2 intervals

Traverse vtree in-order to get ordering of vars $\pi$.
The vars of a subtree is a continuous interval in $\pi$.

A\B forms
2 intervals




## Smoothing in one pass

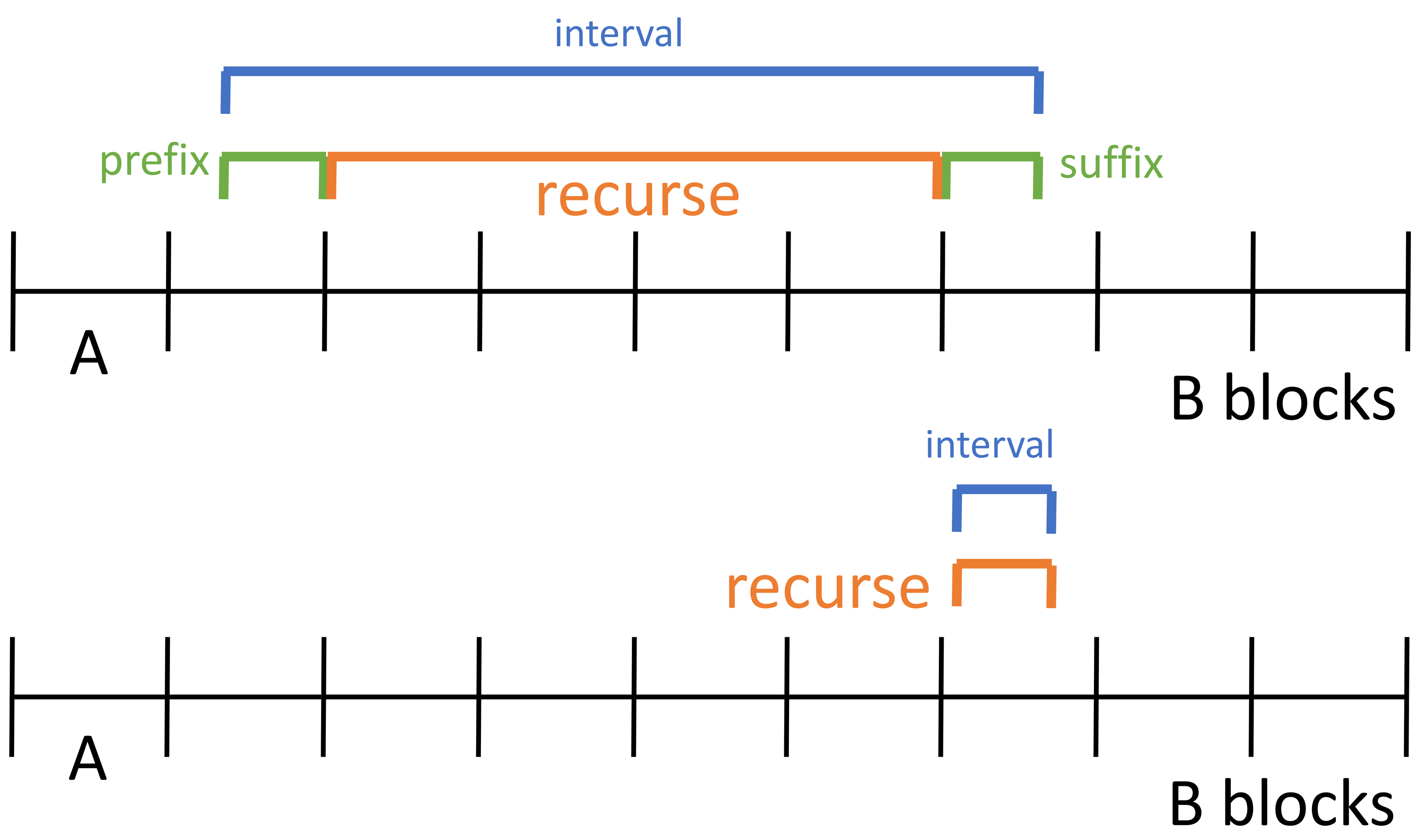
Gate p with interval A. Gate c with interval B.
$\oplus$-gate: replace child c with c $\otimes$ SG(A\B)

Gate $p_l$ with interval $A_l$. Gate $p_r$ with interval $A_r$.
Gate $c_l$ with interval $B_l$. Gate $c_r$ with interval $B_r$.
$\otimes$-gate: replace child $c_l$ with $c_l \otimes SG(A_l \backslash B_l)$
replace child $c_r$ with $c_r \otimes SG(A_r \backslash B_r)$

## Semigroup Range-Sum

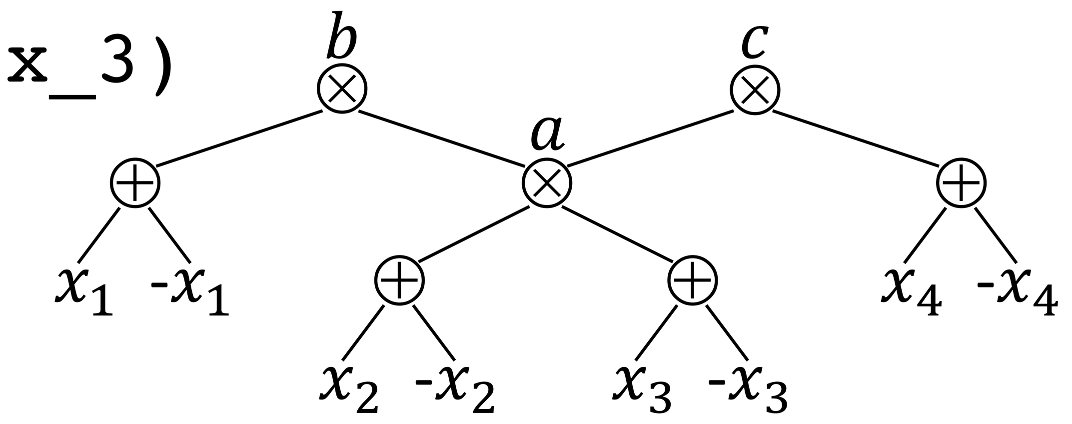
Given n variables defined over a semigroup and m intervals, the sum of each interval can be computed in time $O(m \cdot \alpha(m, n))$ [Chazelle and Rosenberg, 1989].

Split N into B blocks of size A



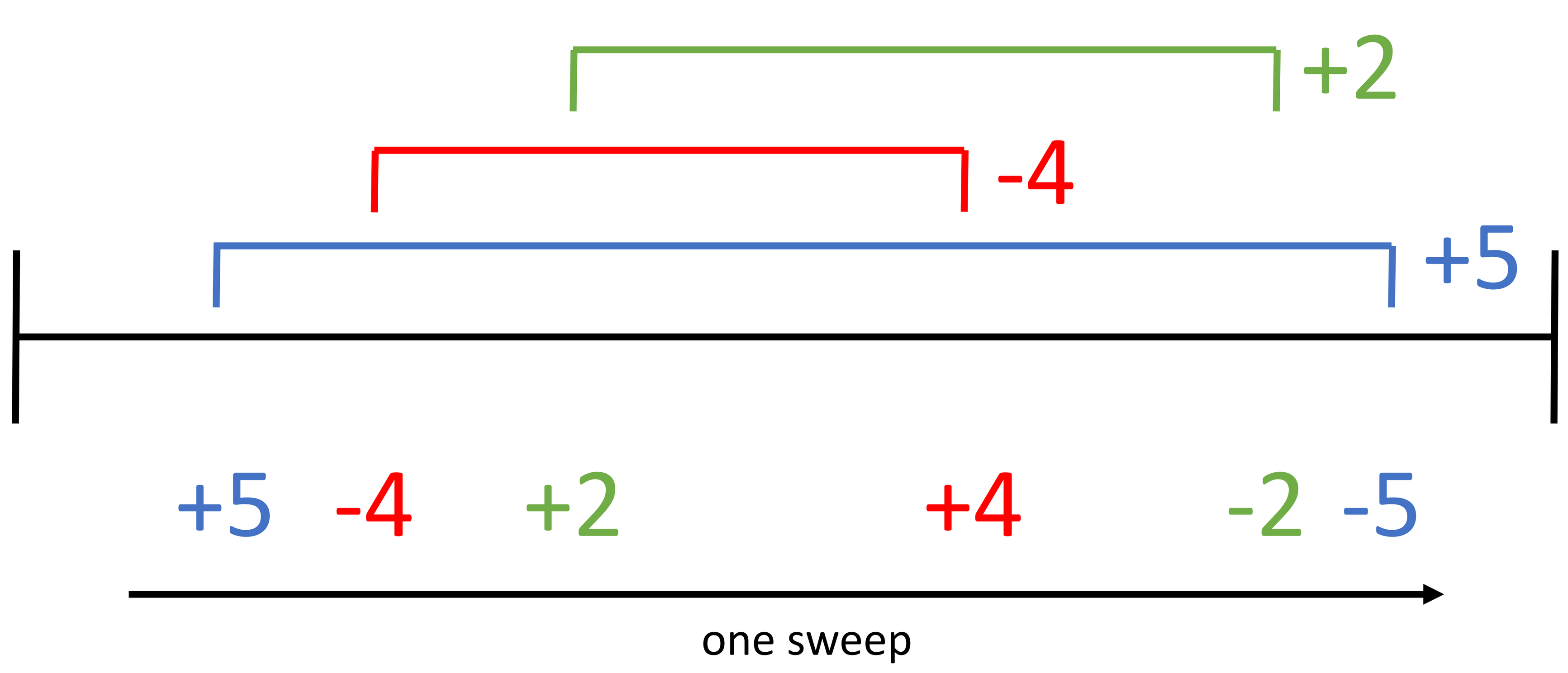

## Trace additions using circuits

```
a = z(x_2) + z(x_3)
b = a + z(x_1)
c = a + z(x_4)
output b, c
```



## All-Marginals by range increments

1. Backpropagate to get marginals for node
2. Update a range of missing variables




## Efficient smoothing / all-marginals

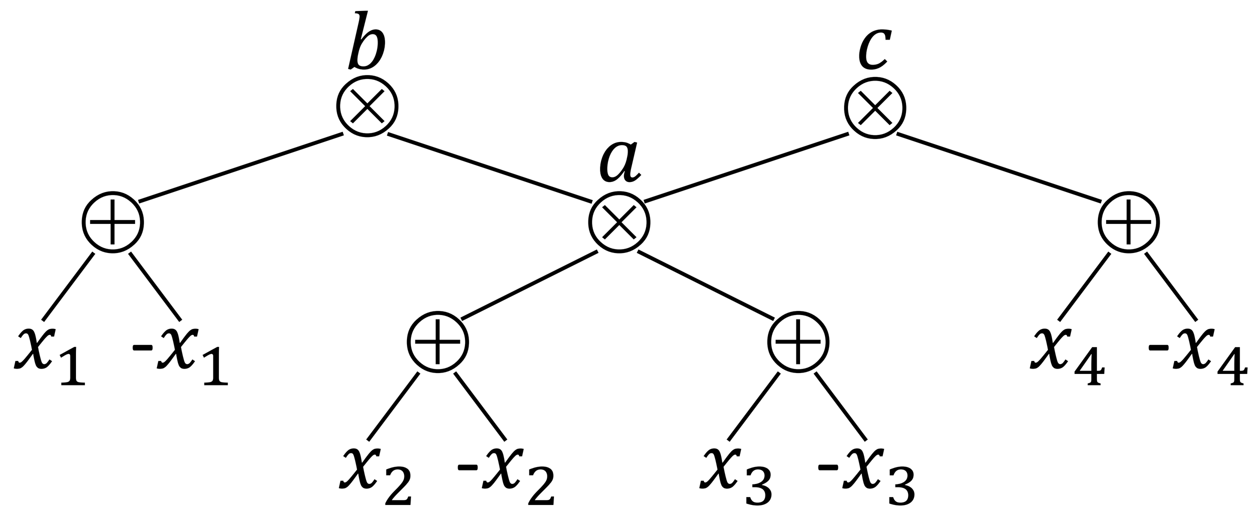
Table 2: Experiments on smoothing hand-crafted circuits and experiments on computing All-Marginals as part of the collapsed sampling algorithm. Sizes are reported in thousands (k).

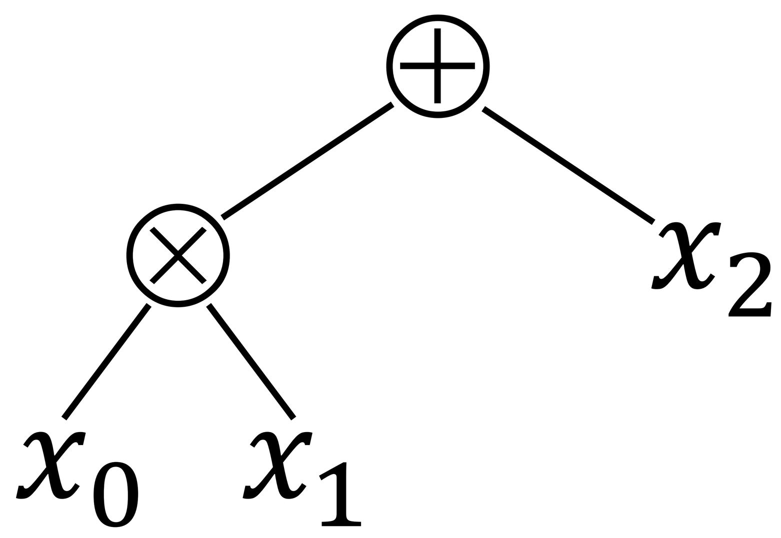
(a) Time (in seconds) taken to smooth circuits.

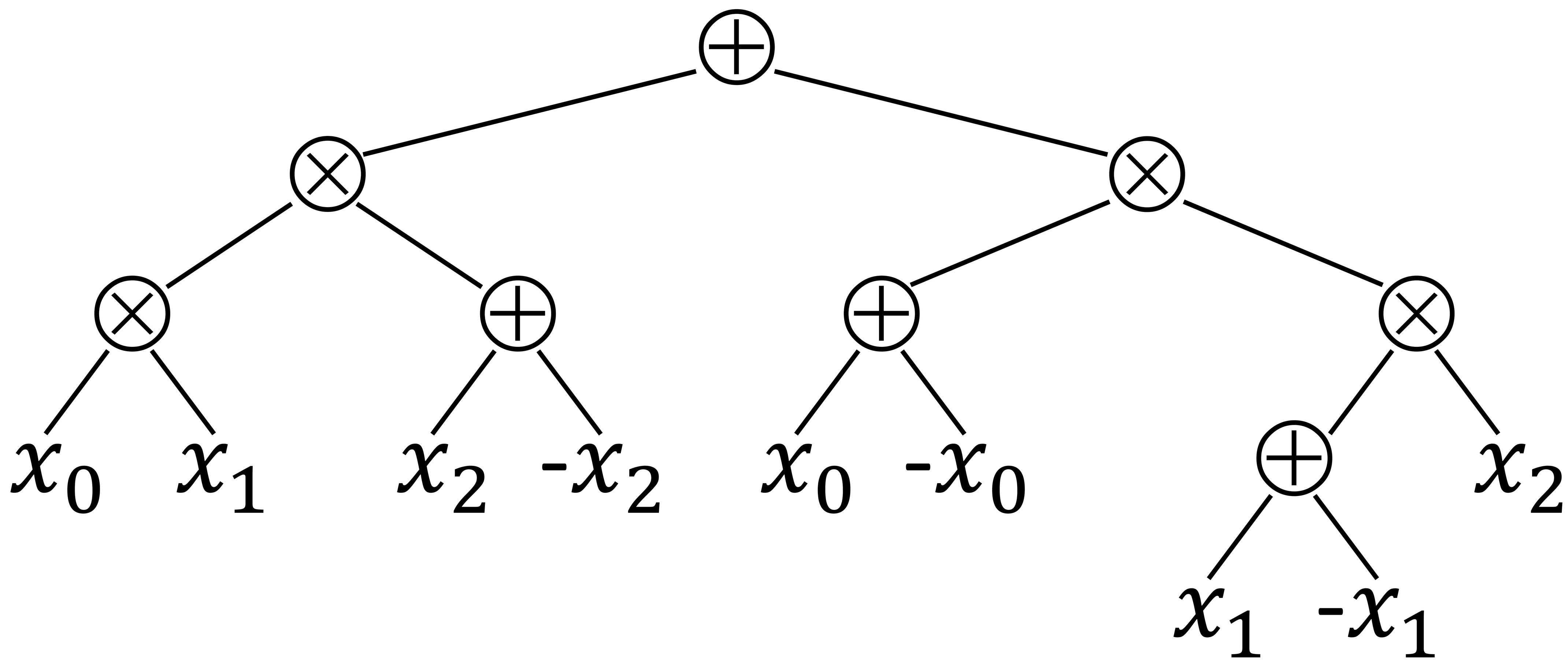
| Size | Naive | Ours | Speedup × |
|---|---|---|---|
| 40k | $0.82 \pm 0.01$ | $0.04 \pm 0.01$ | $21 \pm 1$ |
| 416k | $50 \pm 0.3$ | $0.31 \pm 0.01$ | $161 \pm 6$ |
| 1,620k | $293 \pm 2$ | $0.74 \pm 0.04$ | $390 \pm 30$ |
| 8,500k | $6050 \pm 20$ | $4.13 \pm 0.09$ | $1470 \pm 40$ |

(b) Number of $\oplus, \ominus, \otimes, \oslash$ operations to compute All-Marginals when sampling the Segmentation-11 network.

| Size | Naive | Ours | Impr % |
|---|---|---|---|
| 100k | $28{,}494 \pm 598$ | $20{,}207 \pm 411$ | $29 \pm 3$ |
| 200k | $55{,}875 \pm 1{,}198$ | $36{,}101 \pm 1{,}522$ | $35 \pm 5$ |
| 400k | $86{,}886 \pm 6{,}330$ | $56{,}094 \pm 817$ | $35 \pm 6$ |


b
c
a
$x_1$ $-x_1$
$x_2$ $-x_2$
$x_3$ $-x_3$
$x_4$ $-x_4$


$x_0$ $x_1$ $x_2$


$x_0$ $x_1$ $x_2$ $-x_2$ $x_0$ $-x_0$ $x_2$
$x_1$ $-x_1$

Shen, Yujia, Choi, Arthur, and Darwiche, Adnan. Tractable operations for arithmetic circuits of probabilistic models.