Assignment 3 12109502

1. For each node, there is $0.693 \times 23 = 15.939 \approx 16$ pointers.
Moreover, pointers are always one bigger than keys.

(i) level 1 :	16 nodes	$16 \times 15 = 240$ key entries	$16 \times 16 = 256$ children pointers
(ii) level 3 :	$16^3 = 4096$	$16^3 \times 15 = 61440$ key entries	$16^4 = 65536$ children ptrs
(iii) level 4 :	$16^4 = 65536$	$16^4 \times 15 = 983040$ key entries	$16^5 = 1048576$ children ptrs

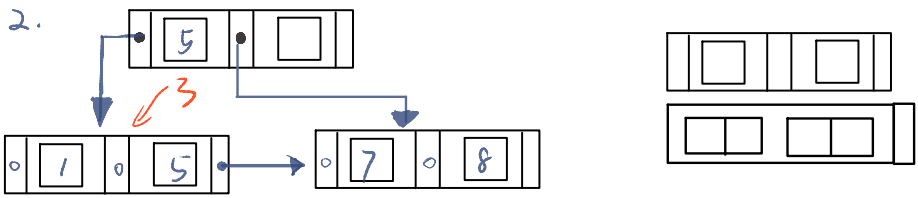
(iv) height 2 : $16^0 \times 15 + 16^1 \times 15 + 16^2 \times 15 = 4095$

(v) height 3 : $16^0 \times 15 + 16^1 \times 15 + 16^2 \times 15 + 16^3 \times 15 = 65535$

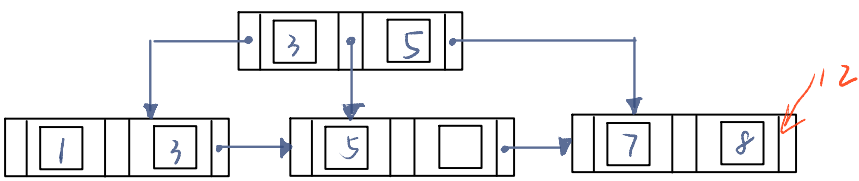
(vi) height 4 : $65535 + 16^4 \times 15 = 1048575$

$\Rightarrow$ average total number of entries $E = (16)^{h+1} - 1$

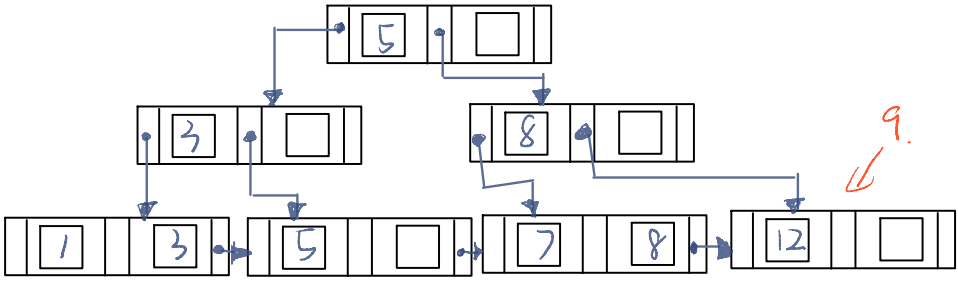
2.



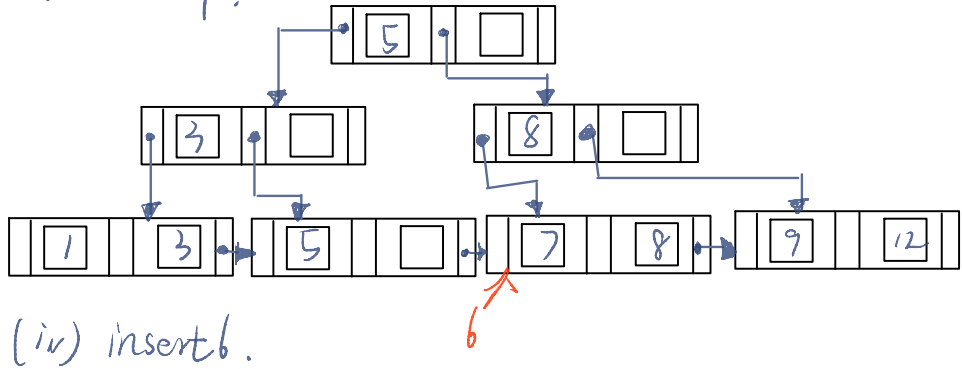
(i) insert 3, overflow, split



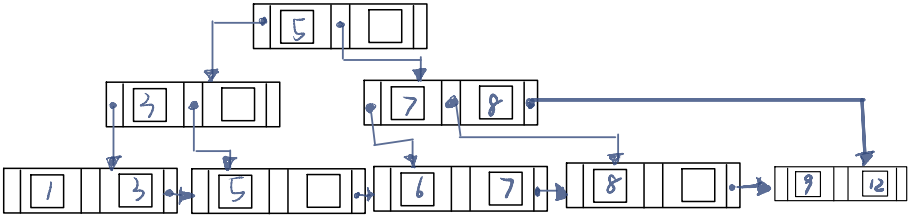
(ii) Insert 12, overflow, newlevel.



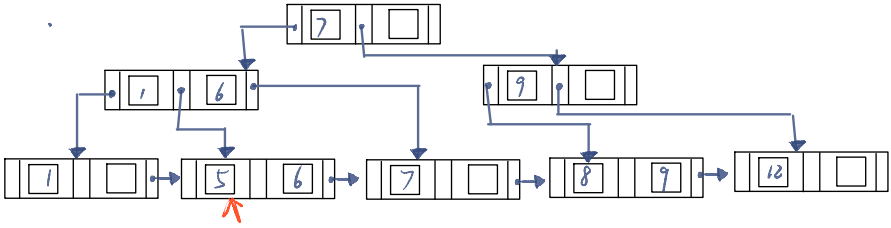
(iii) insert 9.



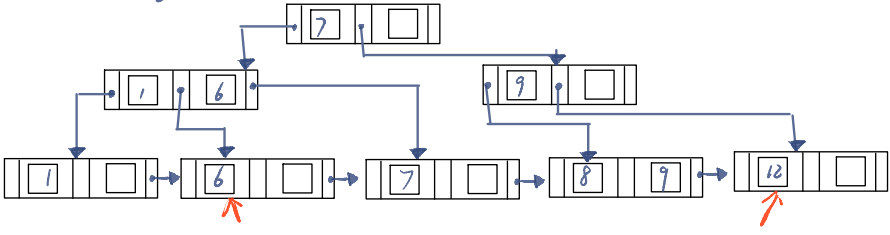
(iv) insert 6.



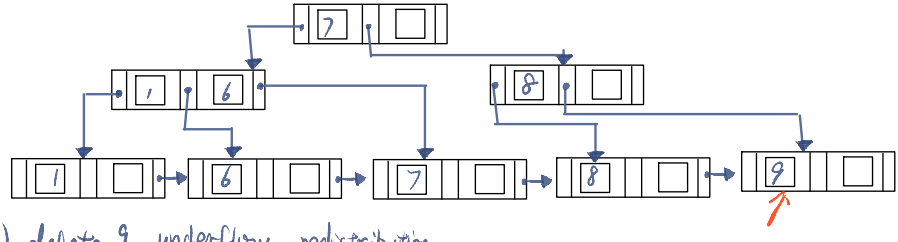
3. delete.



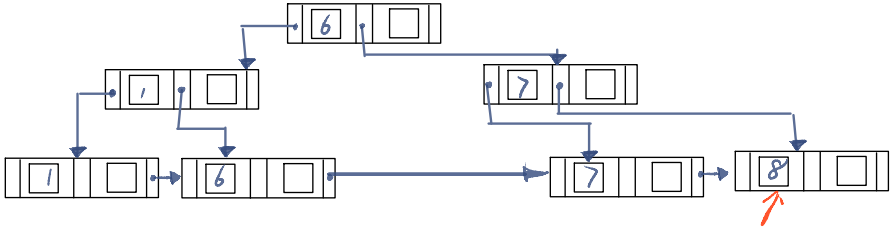
(i) delete 5



(ii) delete 12. underflow, redistribution.



(iii) delete 9 underflow, redistribution.



4. (i) mean storage utilization $E(\rho)$

$$E(\rho) = \frac{f}{f-1}\ln\frac{1}{f} = 3\cdot\ln\left(\frac{4}{3}\right) \approx 86.3\%$$

(ii) Standard deviation $sd(\rho)$

$$\sigma_f^2 = f - \left(\frac{f}{f-1}\right)^2\left[\ln\left(\frac{1}{f}\right)\right]^2 \qquad \sigma = sd(\rho) \approx 0.07$$

(iii) $P(0.8 \le \rho \le 0.9) = P(\rho \le 0.9) - P(\rho \le 0.8)$

$$= \frac{1}{1-f}\left(1-\frac{f}{0.9}\right) - \frac{1}{1-f}\left(1-\frac{f}{0.8}\right) \approx 0.42$$

(iv) median of storage utilization.

median $m \Rightarrow \frac{1}{1-f}\left(1-\frac{f}{m}\right) = \frac{1}{2} \Rightarrow m \approx 0.857 <$ mean $= 0.863$

(v) $$\frac{f}{m} = \frac{1-f}{2} - 1$$

$$\frac{f}{m} = 1 - \frac{1-f}{2} = \frac{1+f}{2} \Rightarrow m = \frac{2f}{1+f}$$

5. PARTS

1) Static Hashing

2305 mod 8 = 1
1168 mod 8 = 0
2580 mod 8 = 4
4871 mod 8 = 7
5659 mod 8 = 3
1821 mod 8 = 5
1074 mod 8 = 2
7115 mod 8 = 3

1626 mod 8 = 4
2428 mod 8 = 4 overflow
3963 mod 8 = 7
4750 mod 8 = 6
6975 mod 8 = 7 overflow
4981 mod 8 = 5
9208 mod 8 = 0

$\frac{13}{15}\times 1 + \frac{2}{15}\times 2 = \frac{17}{15} \approx 1.13$ (blocks)

ii) extendible hashing

2305 mod 128 = 1 => 00001

1168 mod 128 = 16 => 10000

2580 mod 128 = 20 => 10100

4871 mod 128 = 7 => 00111

5659 mod 128 = 27 => 11011

1821 mod 128 = 29 => 11101