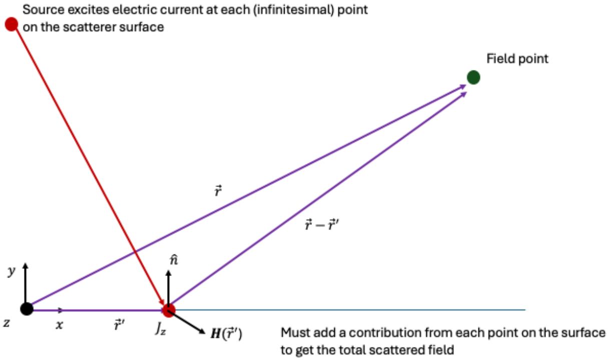
Physical Optics Code

April 16, 2025

Consider the situation in the image below.



An incident wave hits the metal plate of length w and scatters. Scattering is a general term that includes reflection, diffraction and multiple reflection/diffraction effects. To keep things simple we consider a 2D case. This means the electric field is in the z direction and we can treat it as a scalar (rather than worry about having to keep track of vector components).

The total electric fields vary throughout space and can be written at any point as

$$\begin{aligned}\mathbf{E}(\vec{r}) &= E_z(\vec{r})\,\hat{z} \\ &= \left(E_z^i(\vec{r}) + E_z^s(\vec{r})\right)\hat{z}\end{aligned}$$

where E_z^i is the (known) incident field (the field from the source) while E_z^s is the (unknown) scattered field (caused by interaction with the plate). Note that, in this formulation, the electric fields lie solely in the z direction. Similar expressions can be written for the magnetic field $\mathbf{H}$ which has an x and y component.

The scattered electric field at the point $\vec{r}$ can be written in terms of an integral as

$$E_z^s(\vec{r}) = \frac{k\eta_0}{4} \int_C J_z(\vec{r}') H_0^{(2)}\left(k\left|\vec{r} - \vec{r}'\right|\right) dl'$$

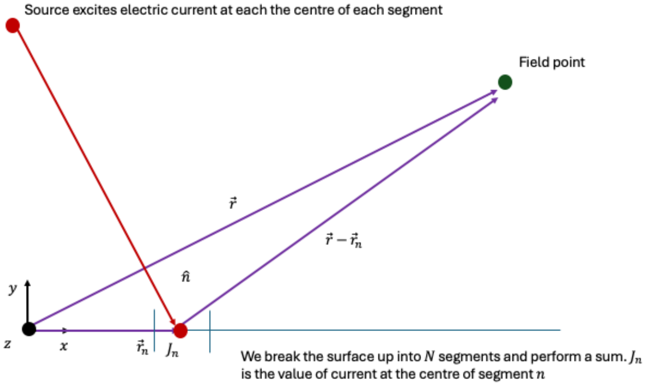
where k, η_0 are the wavenumber and free space impedance respectively.

$$\begin{aligned} k &= \frac{2\pi}{\lambda} \\ \eta &= \sqrt{\frac{\mu_0}{\epsilon_0}} \end{aligned}$$

J_z is the surface electric current defined along the plate and given by

$$J_z \hat{z} = 2\hat{n} \times \mathbf{H}$$

J_z is unknown as we do not know the magnetic field $\mathbf{H}$. Let's set aside the fact that we don't know J_z. As we shall see we can approximate it or numerically compute it (using the method of moments).



If we knew J_z we could discretise the plate into N little segments each with centre $\vec{r}_n$ and width Δ_n. We could then identify the value of J_z at the centre of each segment. Let's call that J_n (we drop the z at this stage to keep the notation simple). We can then approximate the integral by summing over these segments.

We get

$$\begin{aligned} E_z^s(\vec{r}) &= \frac{k\eta_0}{4}\int_C J_z(\vec{r}')\, H_0^{(2)}\left(k\left|\vec{r}-\vec{r}'\right|\right) dl' \\ &\simeq \frac{k\eta_0}{4}\sum_{n=1}^{N} J_n H_0^{(2)}\left(k\left|\vec{r}-\vec{r}_n\right|\right)\Delta_n \end{aligned}$$

J_n can be approximated at each point using Physical Optics. The physical optics current is the current that would exist if the scatterer were an infinitely long plane. Consequently it is a good approximation for points away from the edges. It is given by making the following approximation

$$\begin{aligned} J_n \hat{z} &= \hat{n} \times \mathbf{H}(\vec{r}_n) \\ &= \hat{n} \times \left(\mathbf{H}^i(\vec{r}_n) + \mathbf{H}^s(\vec{r}_n)\right) \\ &\simeq 2\hat{n} \times \mathbf{H}^i(\vec{r}_n) \end{aligned}$$

which essentially just makes the assumption that the scattered field is just a perfect reflection of the incident field.

Physical Optics Current

$$J_n^{PO} \hat{z} = 2\hat{n} \times \mathbf{H}^i(\vec{r}_n)$$

With this we get the PO scattered field

Physical Optics Scattered Field

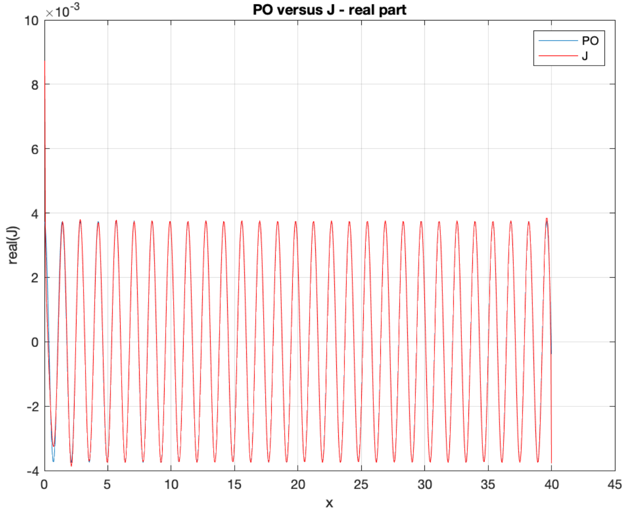
$$E_z^s(\vec{r}) = \frac{k\eta_0}{4} \sum_{n=1}^{N} J_n^{PO} H_0^{(2)}(k|\vec{r} - \vec{r}_n|) \Delta_n$$

In the code we specify the incident field as being a plane wave with amplitude E_0. The strip runs along the x axis with normal vector $\hat{n} = \hat{z}$. The wave vector $\vec{k}$ is a vector of amplitude k_0 in the direction of wave propagation.

Plane wave incident field

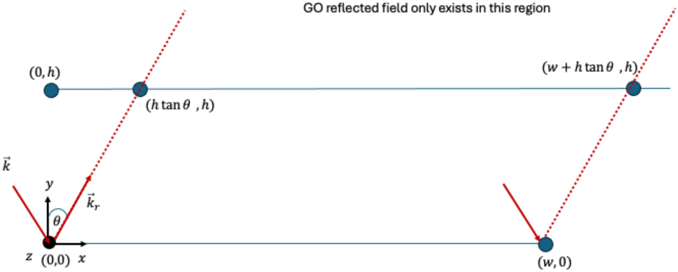
$$
\begin{aligned}
\mathbf{E}^i(\vec{r}) &= E_0 e^{-\jmath \vec{k}\cdot\vec{r}} \hat{z} \\
\mathbf{H}^i(\vec{r}) &= \frac{\jmath}{\omega\mu_0} \nabla \times \mathbf{E}^i(\vec{r}) \\
&= \frac{E_0}{\omega\mu_0} \left(k_y \hat{\imath} - k_x \hat{\jmath}\right) e^{-\jmath \vec{k}\cdot\vec{r}} \\
J^{PO} \hat{z} &= 2\hat{n} \times \mathbf{H}^i \\
&= -2 \frac{E_0 k_y}{\omega\mu_0} e^{-\jmath \vec{k}\cdot\vec{r}} \hat{z}
\end{aligned}
$$

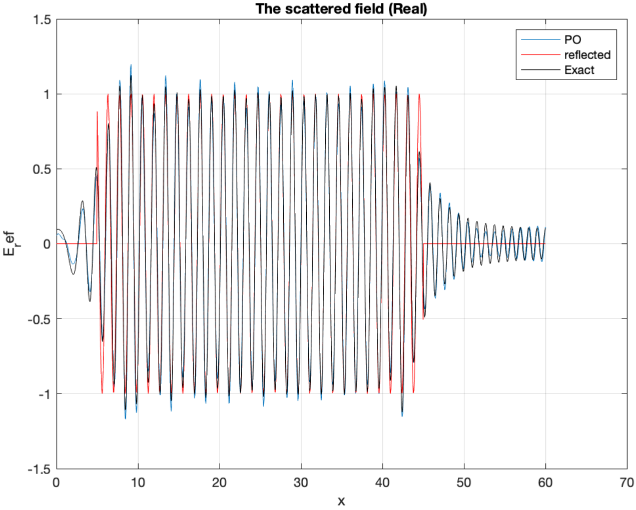
As a validation the code also computes J_z exactly using the method of moments. Below is an output showing the real value of J_z versus J_z^{PO}. Note the excellent agreement (except near the leading edge where the truncation effect is evident).



The code computes the scattered field along a line of receivers. This is compared to fields obtained using the method of moments and geometric optics. The reflected field exists only between $(h\tan\theta, h)$ and $(w + h\tan\theta, h)$ and is given by

$$\mathbf{E}^r(\vec{r}) = E_0 e^{-\jmath\vec{k}_r\cdot\vec{r}}\hat{z}$$




The scattered field (Real)
PO
reflected
Exact
1.5
1
0.5
0
-0.5
-1
-1.5
0
10
20
30
40
50
60
70
x
E_r^{ef}