Balanis Quick Reference

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Contents

1 Time-Varying and Time-Harmonic Electromagnetic Fields

1. Circuit theory is only valid when the physical dimensions of the circuit are small compared to the wavelength

2. Boundary conditions are derived by considering

   (a) the integral curl equations on a tiny rectangle along the boundary and taking the limit as the height goes to zero, and

   (b) the integral divergence equations on a cylinder along the boundary and taking the limit as the height goes to zero.

3. To get conservation of energy relations, consider

   (a) a finite volume of space V,

   (b) scalar multiplying each curl equation by the other field,

   (c) subtracting the equations, and

   (d) use vector identity for divergence of cross product

4. Poynting vector $\mathscr{S}$ and complex Poynting vector $\mathbf{S}$ are given by

$$\mathscr{S} \stackrel{\text{def}}{=} \mathscr{E} \times \mathscr{H} \tag{1}$$

$$\begin{aligned} \mathbf{S} &\stackrel{\text{def}}{=} \mathcal{F}(\mathscr{S}) \stackrel{(1)}{=} \mathcal{F}(\mathscr{E} \times \mathscr{H}) \\ &= \frac{1}{2\pi} \int_{\mathbb{R}} \mathbf{E}(\omega') \times \mathbf{H}(\omega - \omega') d\omega' \end{aligned} \tag{2}$$

$$\begin{aligned} \langle \mathscr{S} \rangle &\stackrel{\text{def}}{=} \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathscr{S}(t) dt = \mathbf{S}(\omega = 0) \\ &\stackrel{(2)}{=} \frac{1}{2\pi} \int_{\mathbb{R}} \mathbf{E}(\omega') \times \mathbf{H}^*(\omega') d\omega' \end{aligned} \tag{3}$$

2 Electrical Properties of Matter

1. Dielectric $\implies$ no free charges $\implies$ less resistance to electrical forces, and is quantified by ϵ. Think of bound charges storing elastic energy

2. Magnetic $\implies$ less resistance to magnetic forces, and is quantified by μ.

3. Conductor $\implies$ free charges and is quantified by σ. Think of free charges and dissipative energy

4. Media Properties:

   (a) *Linear* : parameters are not functions of applied field (or are linearly mapped to them)

(b) *Homogeneous* : parameters not function of position

(c) *Isotropic* : parameters not function of field direction

(d) *Dispersive* : parameters are function of frequency

3 Wave Equation and its Solutions

1. Assume a source-free, homogeneous medium, then we can derive the Helmholtz equations in the frequency domain:

$$(\nabla^2 - \gamma^2)E = 0 \tag{4a}$$

$$(\nabla^2 - \gamma^2)H = 0 \tag{4b}$$