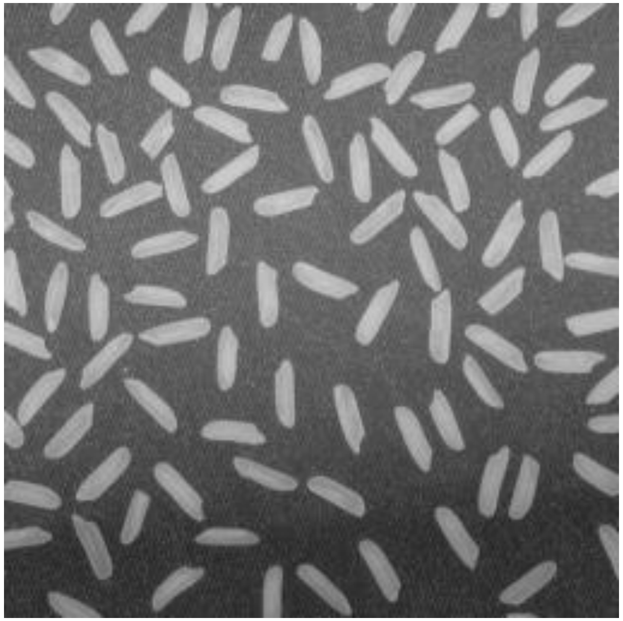
图像的阈值分割

许 向 阳
xuxy@hust.edu.cn

任务：算出图中有多少颗米粒？



参见：**MATLAB, Help/ Demos/Image Processing / Enhancement**

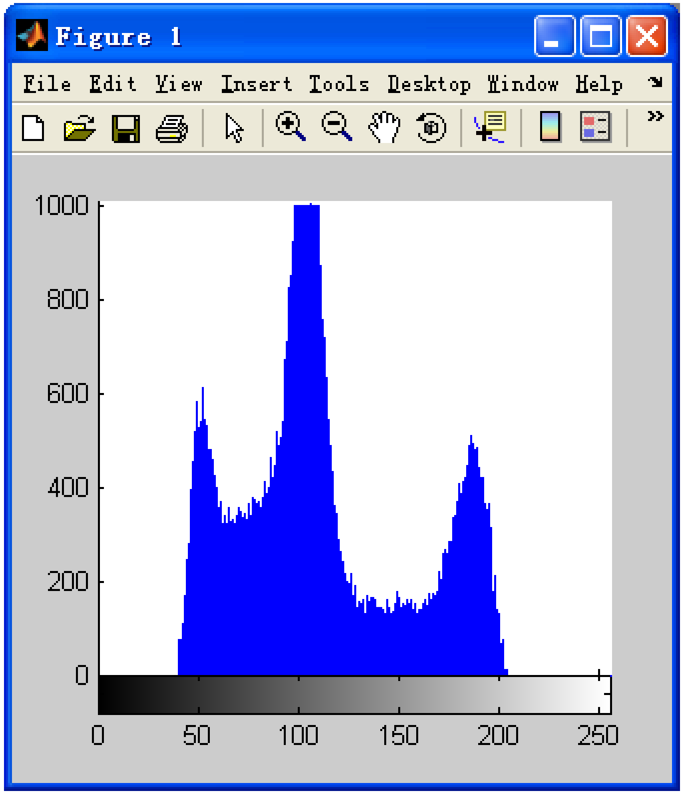


Figure 1
File Edit View Insert Tools Desktop Window Help
1000
800
600
400
200
0
0
50
100
150
200
250

T=131

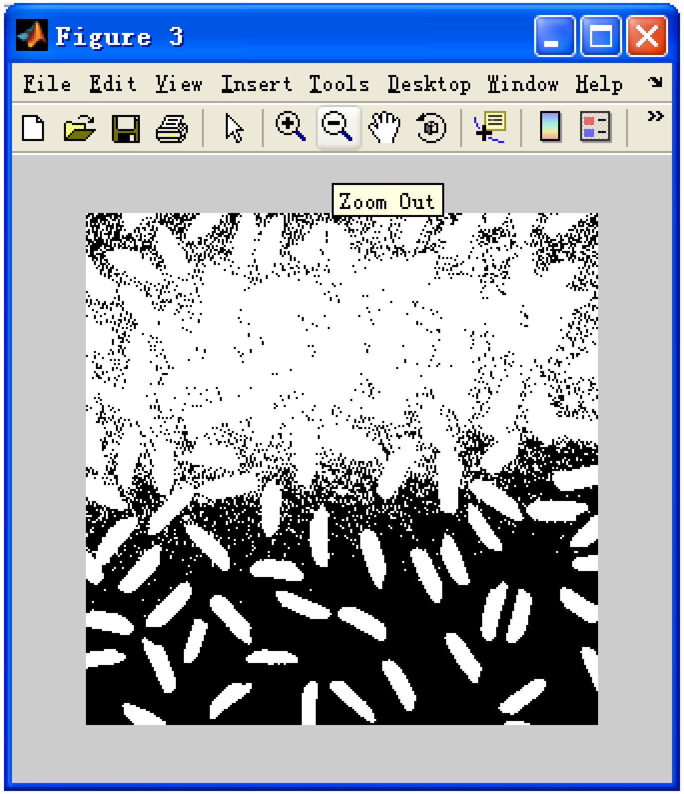


Figure 3
File Edit View Insert Tools Desktop Window Help
Zoom Out

T=100


Figure 4
File Edit View Insert Tools Desktop Window Help

T=150


Figure 1
File Edit View Insert Tools Desktop Window Help

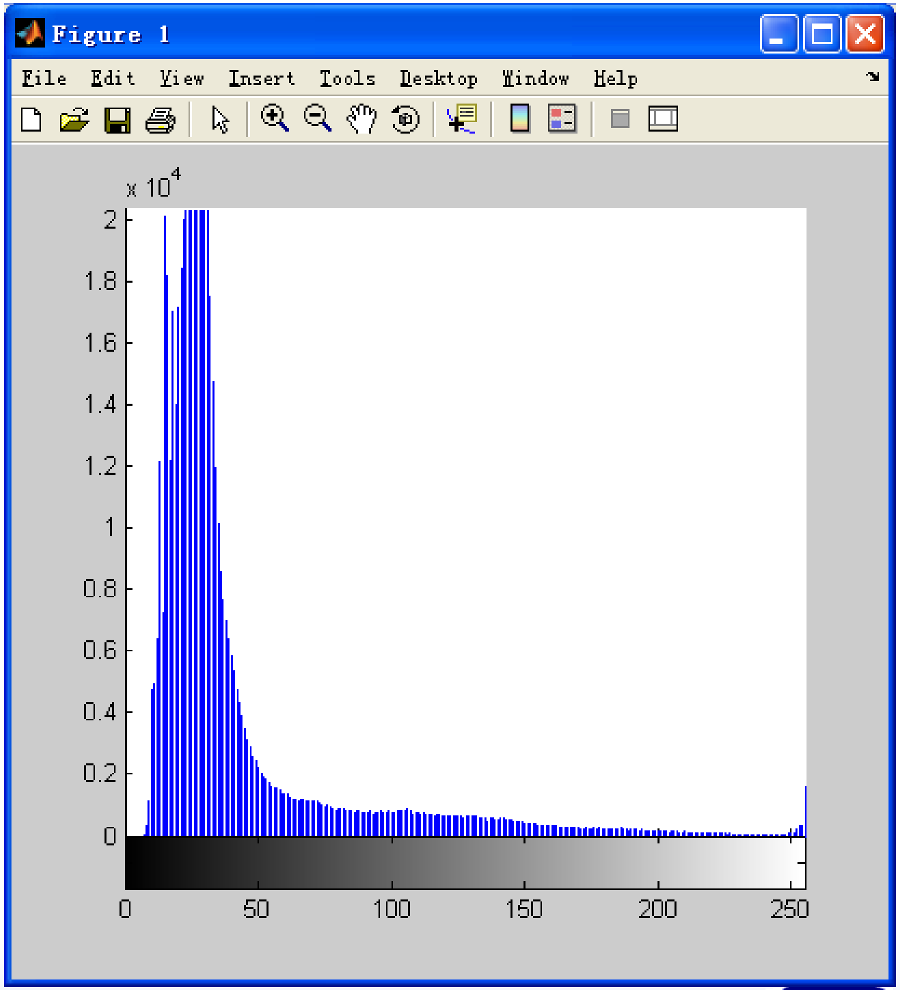


Figure 1
File Edit View Insert Tools Desktop Window Help
x 10^4
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250

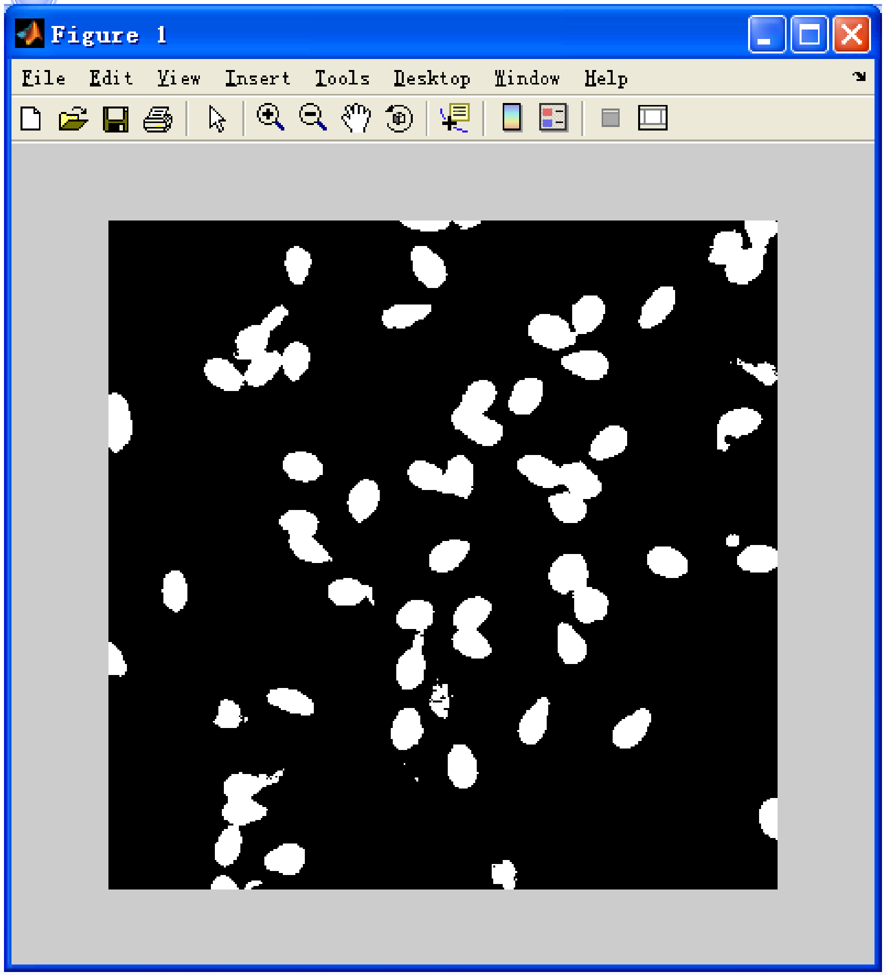


Figure 1
File Edit View Insert Tools Desktop Window Help

T=70

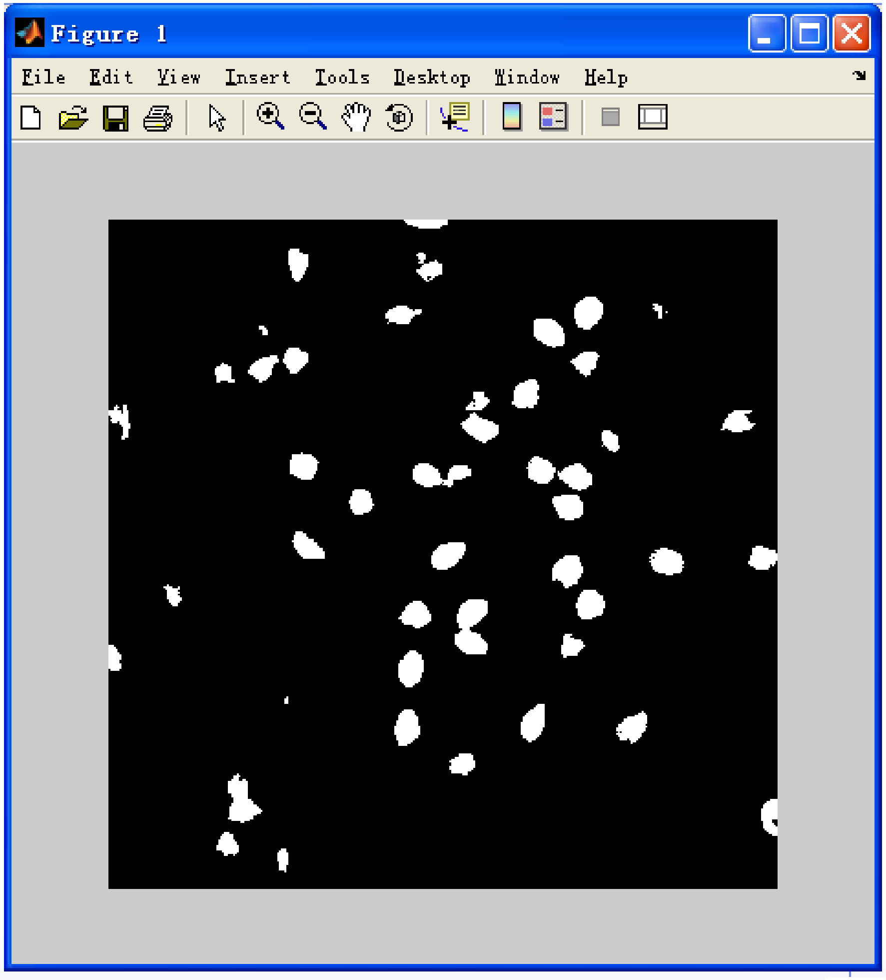


Figure 1
File Edit View Insert Tools Desktop Window Help

T=86

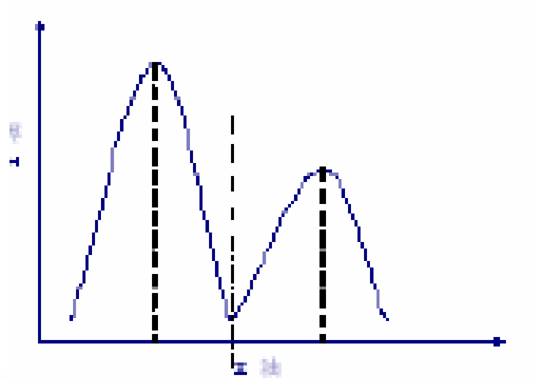
参考文献

Mehmet Sezgin. Survey over image thresholding techniques and quantitative performance evaluation，Journal of Electronic Imaging 13(1), 146–165 (January 2004).

Xiangyang Xu. Characteristic analysis of Otsu threshold and its applications .Pattern Recognition Letters, 2011, 32(7), 956-961

出发点

- 目标或背景内的相邻像素间的灰度值是相似的
- 不同目标或背景的像素在灰度上有差异
- 在直方图上，不同目标和背景对应不同的峰
- 选取阈值将各个峰分开



优缺点

华中科技大学

- 阈值分割的优点

  简单，常作为预处理方法

- 阈值分割的缺点

  - 不适用于多通道图像
  - 不适用于特征值相差不大的图像
  - 不适用于各物体灰度值有较大重叠的图像
  - 对噪声和灰度不均匀敏感

关键：选取阈值

阈值分割的三种技术方案

- 直接阈值法
- 间接阈值法

  对图像进行预处理后再运用阈值法。

  拉氏或梯度运算，邻域平均
- 多阈值法

阈值的确定方法

华中科技大学

- 根据直方图确定阈值
- 最小误判概率准则下的最佳阈值
- 最大类间距准则下的最佳阈值
- 最大类间类内距离比准则下的最佳阈值
- 最大熵准则下的最佳阈值
- 根据二维直方图确定图像分割阈值
- 边缘灰度作为分割阈值
- 分水岭方法

The Otsu Algorithm

For each potential threshold T,

1. Separate the pixels into two clusters according to the threshold.
2. Find the mean of each cluster.
3. Square the difference between the means.
4. Calculate the object function of $\sigma^2_{BETWEEN}(t)$
5. Find the optimal threshold T* that maximizes the value of $\sigma^2_{BETWEEN}(t)$.

The Otsu Algorithm

最大类间距准则下的最佳阈值

$$\sigma^2_{BETWEEN}(t) = w_0 \times [\mu_0 - \mu_{TOTAL}]^2 + w_1 \times [\mu_1 - \mu_{TOTAL}]^2$$

W0：灰度小于等于t 的像素在全体像素中所占的比例；
W1：灰度大于t 的像素在全体像素中所占的比例；
W0+W1=1;
Utotal：图像的灰度平均值
u0: 灰度小于等于t 的像素的灰度平均值
u1: 灰度大于t 的像素的灰度平均值

The Otsu Algorithm

最大类间距准则下的最佳阈值

$$\sigma^2_{BETWEEN}(t) = w_0 \times [\mu_0 - \mu_{TOTAL}]^2 + w_1 \times [\mu_1 - \mu_{TOTAL}]^2$$

$$w_0(t) \times \mu_0(t) + w_1(t) \times \mu(t)_1 = \mu_{TOTAL}$$

$$\sigma^2_{BETWEEN}(t) = w_0(t) w_w(t) [\mu_0(t) - \mu_1(t)]^2$$

$$T^* = \arg(t) \max \sigma^2_{BETWEEN}(t)$$

The Otsu Algorithm

华中科技大学

最小类内距准则下的最佳阈值

$$\sigma_w^2(T_0) = \sum_{i=1}^{T_0}(i-\mu_0(T_0))^2 p_i + \sum_{i=T_0+1}^{L}(i-\mu_1(T_0))^2 p_i$$

两个类的类内方差

$$\sigma_0^2(T) = \sum_{i=1}^{T}(i-\mu_0(T))^2 \frac{p_i}{P_0(T)} \qquad P_0(T) = \sum_{i=1}^{T} p_i$$

$$\sigma_1^2(T) = \sum_{i=T+1}^{L}(i-\mu_1(T))^2 \frac{p_i}{P_1(T)} \qquad P_1(T) = \sum_{i=T+1}^{L} p_i = 1 - P_0(T)$$

两个类的类内方差的加权和

$$\sigma_w^2(T) = P_0(T)\sigma_0^2(T) + P_1(T)\sigma_1^2(T)$$

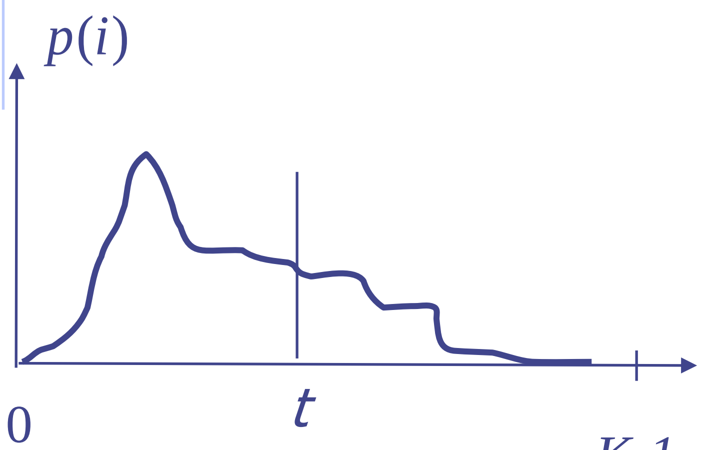
The Otsu Algorithm

最小类内距准则下的最佳阈值 $T^* = \arg\min_{1 \leqslant T < L} \{\sigma_w^2(T)\}$

= 最大类间距准则下的最佳阈值 $T^* = \arg\max_{1 \leqslant T < L} \{\sigma_b^2(T)\}$

The Otsu Algorithm

- If t is chosen as a threshold, and $p(i)$ is the normalized histogram



N bits means $K = 2^N$

$$P(i \mid H_0, t) = \frac{p(i)}{\sum_{i=0}^{t} p(i)} = \frac{p(i)}{w_0(t)}$$

$$P(i \mid H_1, t) = \frac{p(i)}{\sum_{i=t+1}^{K-1} p(i)} = \frac{p(i)}{w_1(t)}$$

$$w_0(t) + w_1(t) = 1, \text{ since } \sum_{i=0}^{K\text{-}1} p(i) = 1$$

The Otsu Algorithm

华中科技大学

- Means and variance for each class

$$\left.\begin{aligned} \mu_0(t) &= \sum_{i=0}^{t} i \times \frac{p(i)}{w_0(t)} \\ \mu_1(t) &= \sum_{i=t+1}^{K-1} i \times \frac{p(i)}{w_1(t)} \end{aligned}\right\} w_0(t)\times\mu_0(t) + w_1(t)\times\mu(t)_1 = \mu_{TOTAL}$$

means

$$\left.\begin{aligned} \sigma_0^2(t) &= \sum_{i=0}^{t} [i-\mu_0(t)]^2 \times \frac{p(i)}{w_0(t)} \\ \sigma_1^2(t) &= \sum_{i=t+1}^{K-1} [i-\mu_1(t)]^2 \times \frac{p(i)}{w_1(t)} \end{aligned}\right\} \text{Variances}$$

Variances 类内方差

The Otsu Algorithm

Statistical discrimination measure based on variance between classes:

$$\sigma^2_{BETWEEN}(t) = w_0 \times [\mu_0 - \mu_{TOTAL}]^2 + w_1 \times [\mu_1 - \mu_{TOTAL}]^2$$

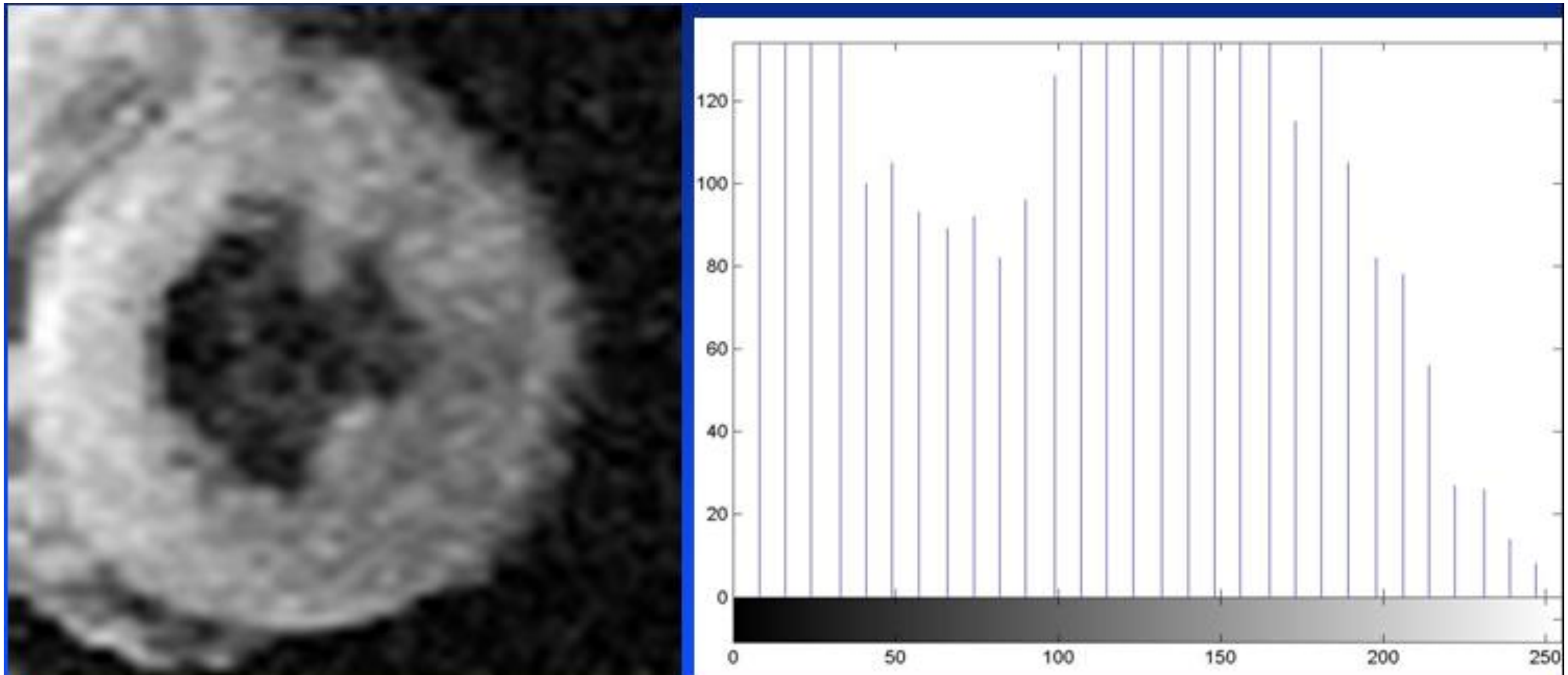
Run through all possible values of *t*, and pick the one that maximizes the discrimination measure:

$$T^* = \underset{t=0,1,\ldots K-1}{\arg\max} \left[\sigma^2_{BETWEEN}(t)\right]$$

$\uparrow$

Chosen Threshold

Determination of Otsu's threshold



(a)心臟超音波影像　　(b)Otsu's threshold=94

K-means clustering（目标函数）

This algorithm aims at minimizing an *objective function*, in this case a squared error function. The objective function

$$J(c_1, \dots, c_k) = \arg\ \min \sum_{j=1}^{k} \sum_{i=1}^{n} \left\| x_i^j - c_j \right\|_2^2$$

where $\left\| x_i^j - c_j \right\|$ is a chosen distance measure between a data point x_i^j and the cluster centre c_j, is an indicator of the distance of the *n* data points from their respective cluster centroids.

K-means clustering Algorithm

目标函数优化的一种求解方法

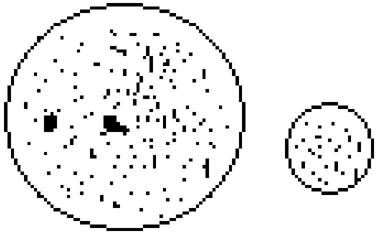
1. Place K points into the space represented by the objects that are being clustered. These points represent initial group centroids.
2. Assign each object to the group that has the closest centroid.
3. When all objects have been assigned, recalculate the positions of the K centroids.
4. Repeat Steps 2 and 3 until the centroids no longer move. This produces a separation of the objects into groups from which the metric to be minimized can be calculated.

K-means clustering Algorithm

华中科技大学

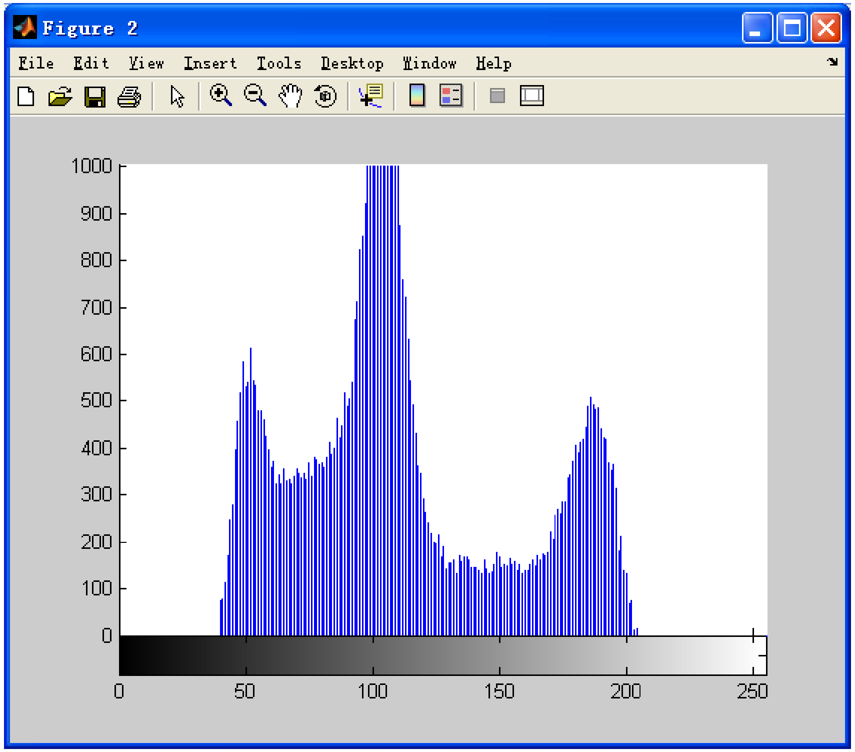
算法过程演示：

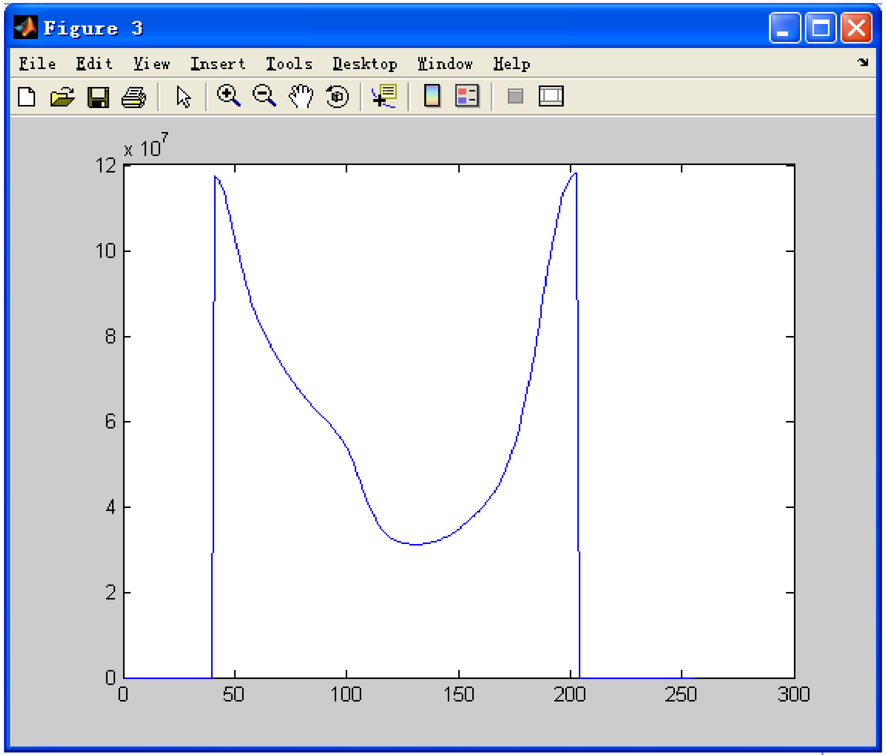
编写一个程序，可动态观察类中心的移动，直至收敛。



K-means clustering

成功例子：rice.png





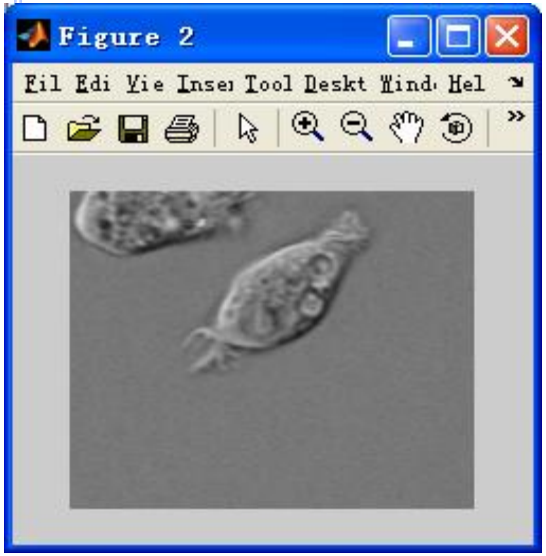
目标函数曲线

T=131

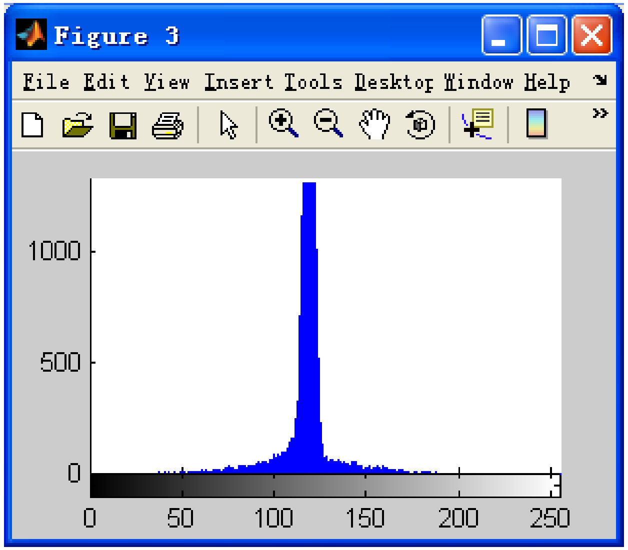
K-means clustering

目标函数的目标极值问题

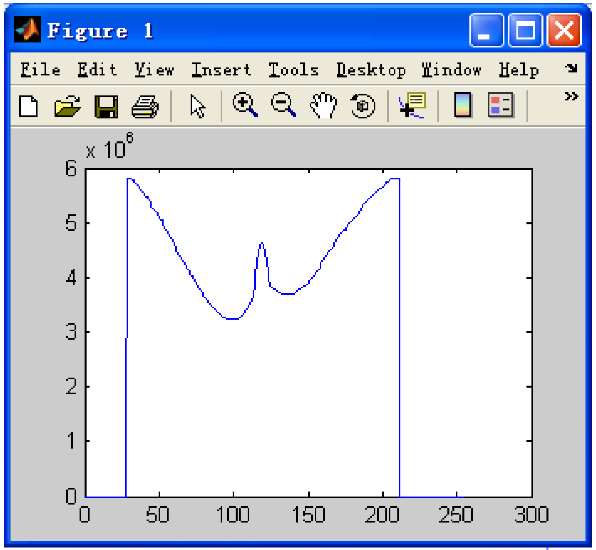
Cell.tif (matlab例子图像)



原始图像



灰度直方图



目标函数曲线

有四个极值点：**99，100，119，136**

Fuzzy K-means clustering

Fuzzy K-means clustering algorithm aims at minimizing the following objective function with respect to the membership function μ_{ij} and the centroids c_j:

$$J = \sum_{j=1}^{K} \sum_{i=1}^{n} \mu_{ij} \left\| x_i^j - c_j \right\| \qquad (1)$$

where K is a number of clusters or classes, n is the total number of feature points or vectors and $m \in (1, \infty)$ is a weighting exponent. And we have:

$$0 \le \mu_{ij} \le 1, \ \sum_{j=1}^{K} \mu_{ij} = 1, \ 0 < \sum_{i=1}^{n} \mu_{ij} < n \qquad (2)$$

Fuzzy K-means clustering

- For each input vector x_i^j , using Lagrangian multiplier method, for m>1, local minimum solutions of equation (1) was demonstrated if and only if:

$$c_j = \frac{\sum_{i=1}^{n} (\mu_{ij})^m x_i^j}{\sum_{i=1}^{n} (\mu_{ij})^m}, \quad \mu_{ij} = \frac{1}{\sum_{k=1}^{K} (d_{ij} / d_{kj})^{\frac{2}{m-1}}}$$

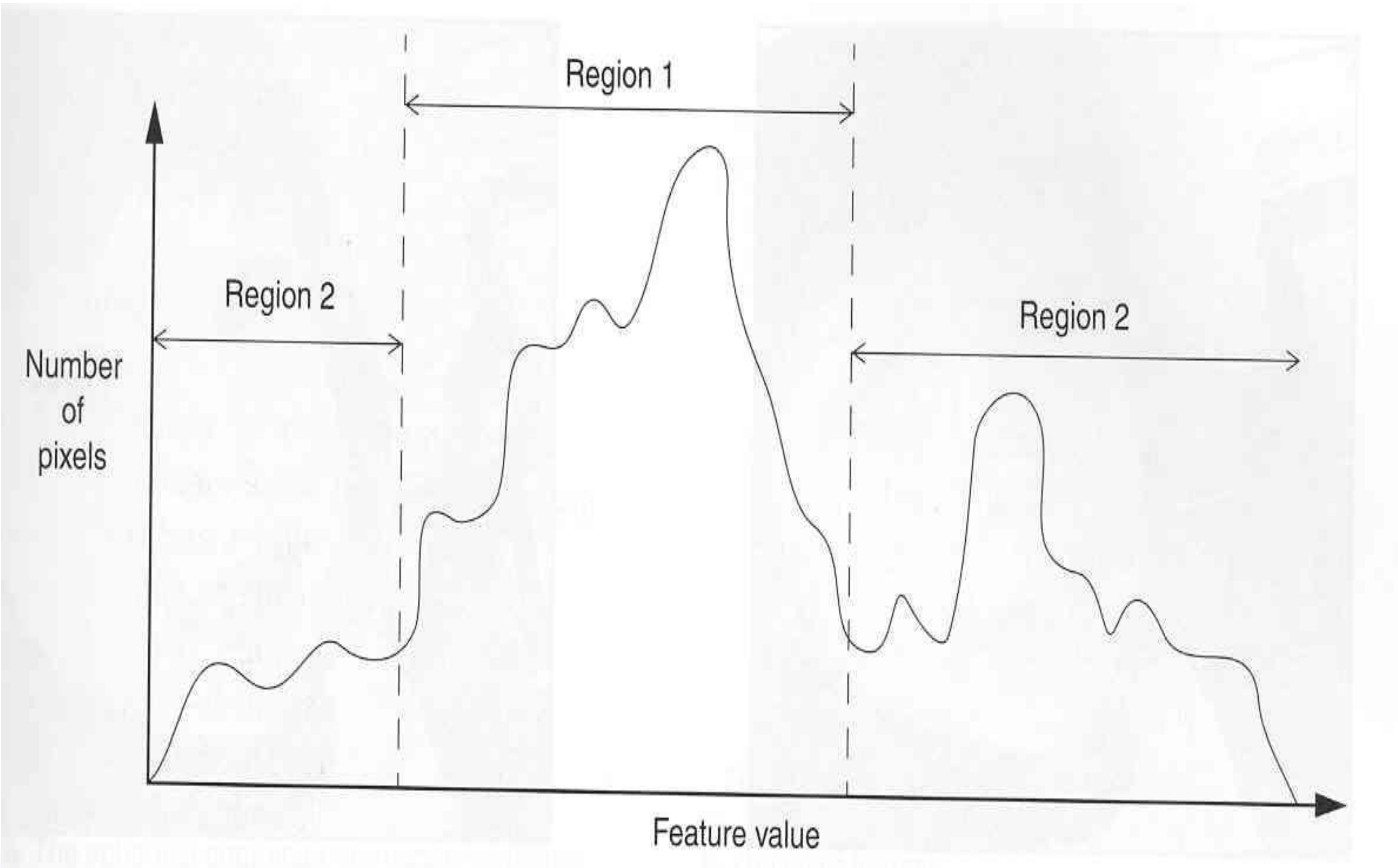
Using the histogram to select a threshold

华中科技大学

- Using the histogram to select a threshold is a very common theme in thresholding. If the histogram has two peaks, then a threshold occurs at the low point between these two peaks.

- Finding the first peak in the histogram is simple: it is the bin having the largest value. However, the second largest value is probably in the bin right next to the largest, rather than being the second peak. Locating the second peak is harder.

Using the histogram to select a threshold



Using the histogram to select a threshold

- A simple trick that frequently works well enough is to look for the second peak by multiplying the histogram values by the square of the distance from the first peak. So, if the largest peak is at level j in the histogram, select the second peak as:

$$\mathrm{Max}\{((k - j)^2 h[k])|(0 \le k \le 255)\}$$

- where h is the histogram, and there are 256 grey levels.

Using the histogram to select a threshold

華中科技大學

- Weszka (1974) developed a thresholding method based on the digital Laplacian (a nondirectional edge-detection operator).
- Now a histogram of the original image is found considering only those pixels having large Laplacians; those in the 85th percentile and above will do nicely.

- Using edge pixels: an edge pixel must be near to the boundary between an object and the background, or between two objects.

Iterative Selection

- Iterative selection is a process in which an initial guess at a threshold is refined by consecutive passes through the image.
- Algorithm:
- Step 1: take the mean gray level (T_i) as the initial guess.
- Step 2: use T_i to collect statistics on the black and white regions obtained: the mean gray level for all pixels below the threshold (T_b), and the mean gray level of the pixels greater than or equal to the initial threshold (T_w).
- Step 3: A new estimate of the threshold is computed as $(T_b + T_w)/2$.
- Step 4: if there is no change in threshold in two consecutive passes through the image, the process stops. Otherwise, repeat steps 2 and 3.

Iterative Selection …

- The method of iterative threshold selection presented in the above can be mathematically described by the iteration

$$T_{k+1} = \frac{\sum_{b=0}^{T_k} bn(b)}{2\sum_{b=0}^{T_k} n(b)} + \frac{\sum_{b=T_k+1}^{N} bn(b)}{2\sum_{b=T_k+1}^{N} n(b)}$$

T_k : threshold at the K^{th} iteration,

b : brightness level, and

N(b): number of occurrences of level b, $0 \le b \le N$ in the image (i.e. the histogram).

Automatic Threshold based on mean and standard deviation

華中科技大學

Automatic threshold based on mean and standard deviation:

$$T(i,j) = \overline{f(i,j)} + k\sigma(i,j)$$

where $T(i,j)$, $\overline{f(i,j)}$, $\sigma(i,j)$ are the automatic threshold at the point *(i,j)*, the mean and standard deviation of the neighbors of *(i,j)*, i.e., a local window, *k* is the weight and can be a real number.

Determination of threshold by maximum entropy

华中科技大学

- What is an entropy?
- Entropy is the measurement of the information content in a probability distribution

$$H = -\sum_{i=0}^{N-1} p_i \lg p_i$$

- Maximum entropy segmentation is to select such a threshold that the entropies in both object and background areas have maximum distributions.

$$H = H_o + H_b$$

根据二维直方图确定图像分割阈值

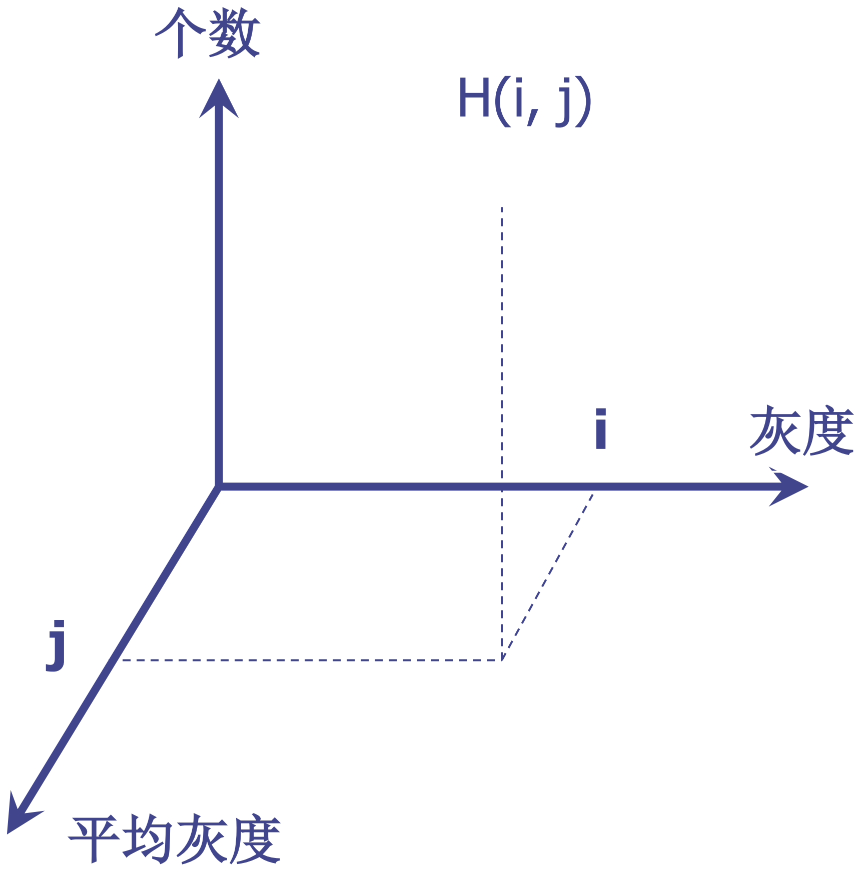
- 灰度－平均灰度直方图
- 平均灰度－局部方差直方图

  最大熵
- 灰度－梯度直方图

  采用聚类的方法，分三类
- 平均灰度－局部方差直方图

  最大熵

灰度－平均灰度直方图



一个像素点的平均灰度，是指其邻域内像素灰度的平均值。

对每个像素可用其灰度和平均灰度来描述

灰度－平均灰度直方图

Determination of threshold by 2-D Histogram

- The average gray level at the point (x,y) of its *nxn* neighbors is:

$$g(x,y)=\frac{1}{n^2}\sum_{i=-\frac{n}{2}}^{\frac{n}{2}}\sum_{j=-\frac{n}{2}}^{\frac{n}{2}} f(x+i,y+j)$$

  where $n < N$

- For the 2D thresholding method, it considers the average gray level of the point *(x,y)* simultaneously, i.e., use *(f(x,y),g(x,y))* to represent an image and to segment the image with 2D vector threshold *(S,T):*

Determination of threshold by 2-D Histogram

$$f_{S,T}(x,y)=\begin{cases} b_0 & f(x,y)\le S \,\&\, g(x,y)\le T \\ b_1 & f(x,y)> S \,\&\, g(x,y)> T \end{cases}$$

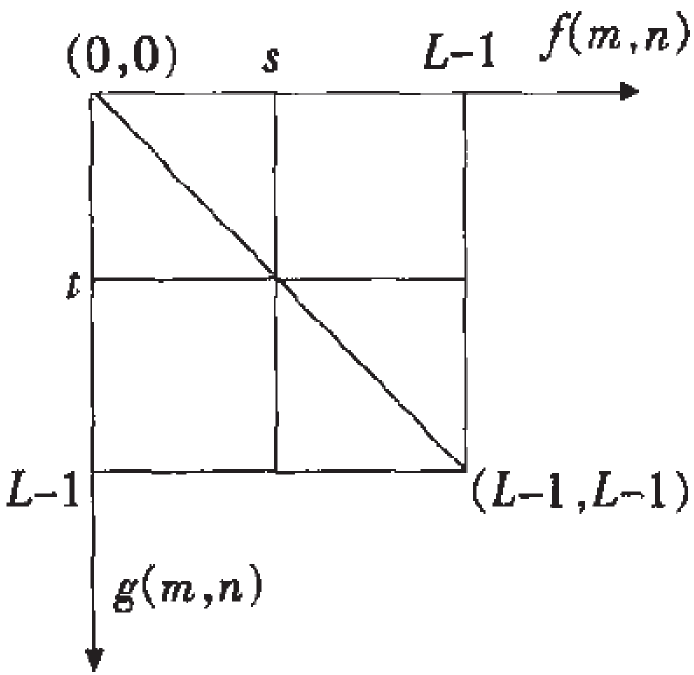
where $0\le b_0, S, T, b_1 \le L-1$

- For one image, let r_{ij} to be the occurrence number of gray level *i* and the average gray level *j*, we can define the joint probability as:

$$P_{i,j}=\frac{r_{ij}}{N^2} \qquad\qquad 0\le i,j\le L-1$$

- *P* is called the 2D histogram of the image *f(x,y)*

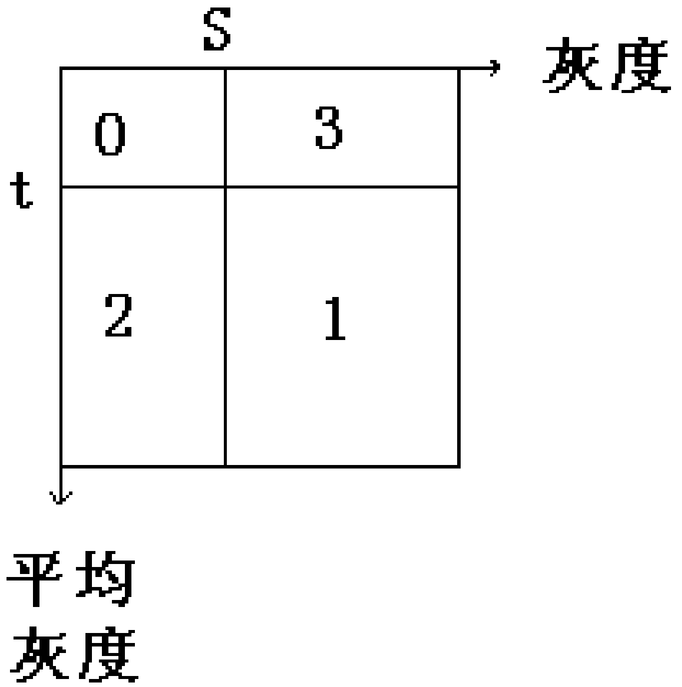
Determination of threshold by 2-D Histogram



2D histogram of image

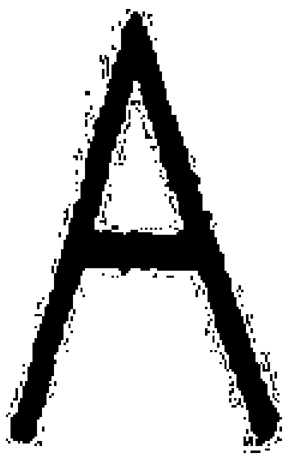
If the threshold vector is (*S*, *T*), the 2D histogram will be divided into 4 parts: In Part 0 and Part 1, i.e., the object or background, the gray level and the average is close, while in Part 2 and part 3, the difference between the gray level and the average is big, which is corresponding to the boundary points.

Determination of threshold by 2-D Histogram

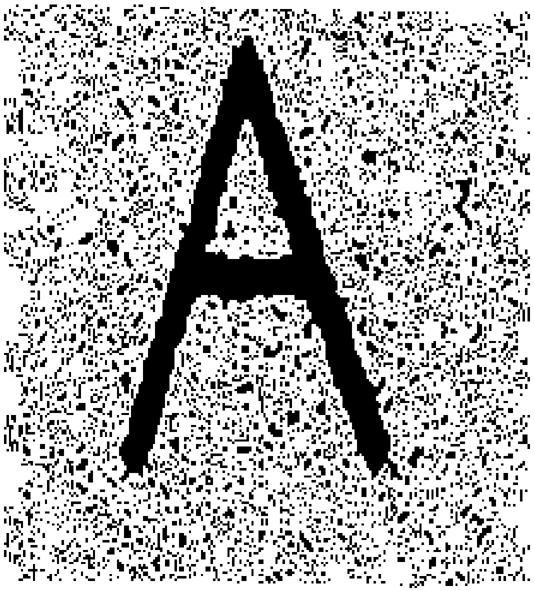


方案1：
0区：背景
1区：目标
2、3区：
都归为目标
或者都归为背景

Determination of threshold by 2-D Histogram



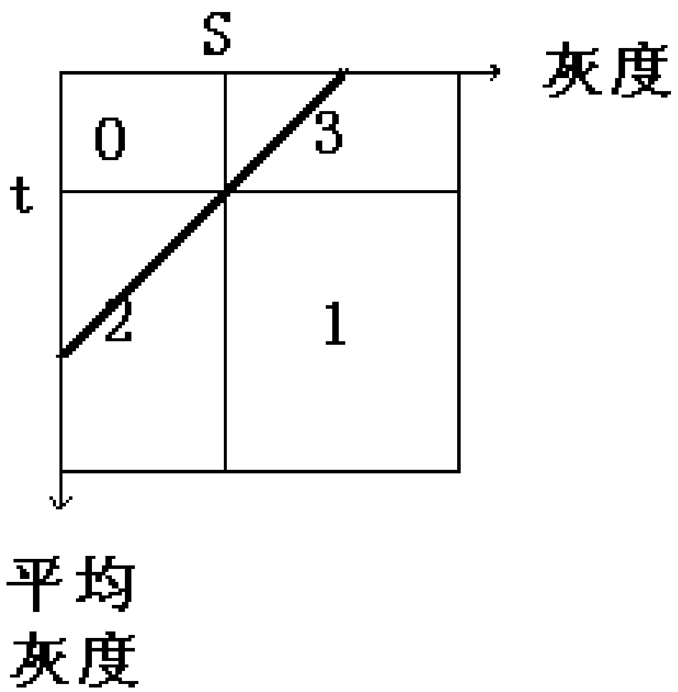
区域2、3归为背景



区域2、3归为目标

范九伦，灰度图像的二维**Otsu** 曲线阈值分割法，电子学报，**2007**

Determination of threshold by 2-D Histogram



方案2:
0区：背景
1区：目标

粗斜线之上:
　　归为背景
粗斜线之下:
　　都归为目标
粗斜线为过(s,t)的45度线

Determination of threshold by 2-D Histogram

- The maximum entropy for the 2D histogram is to determine a threshold vector (*S,T*) such that we can divide the image into object (A) and background (B) with the probability of

$$A: \quad \frac{P_{0,0}}{P_{st}}, \frac{P_{0,1}}{P_{st}}, \ldots, \frac{P_{s,t}}{P_{st}}$$

$$B: \quad \frac{P_{(s+1),0}}{1-P_{st}}, \frac{P_{(s+1),1}}{1-P_{st}}, \ldots, \frac{P_{(L-1),(L-1)}}{1-P_{st}}$$

where $P_{st} = \sum_{i=0}^{s} \sum_{j=0}^{t} P_{i,j}$

Determination of threshold by 2-D Histogram

华中科技大学

- The goal of segmentation is to let the entropies in the object and background areas as big as possible,

$$H_A(s,t) = \sum_{i=0}^{s}\sum_{j=0}^{t} \frac{P_{i,j}}{P_{st}} \ln \frac{P_{i,j}}{P_{st}}$$

$$H_B(s,t) = \sum_{i=s}^{L-1}\sum_{j=t}^{L-1} \frac{P_{i,j}}{P_{st}} \ln \frac{P_{i,j}}{P_{st}}$$

$$H(s,t) = H_A(s,t) + H_B(s,t)$$

- The maximum entropies of the object and background will correspond to the optimal threshold vector *(S,T)*.

Determination of threshold by Fuzzy Entropy

- The BlockB and BlockW are defined in Fig. 1(a) and (b). Four fuzzy sets, BrightX, DarkX, BrightY, DarkY, are defined based on the S-function and the corresponding Z-functions as follows: (Z()=1-s())

$$BrightX = \sum_{x \in X} \frac{\mu_{BrightX}(x)}{x} = \sum_{x \in X} \frac{S(x,a,b,c)}{x}$$

$$DarkX = \sum_{x \in X} \frac{\mu_{DarkX}(x)}{x} = \sum_{x \in X} \frac{Z(x,a,b,c)}{x}$$

$$BrightY = \sum_{y \in Y} \frac{\mu_{BrightY}(y)}{y} = \sum_{y \in Y} \frac{S(y,a,b,c)}{y}$$

$$DarkY = \sum_{y \in Y} \frac{\mu_{DarkY}(y)}{y} = \sum_{y \in Y} \frac{Z(y,a,b,c)}{y}$$

Determination of Threshold by Fuzzy Entropy

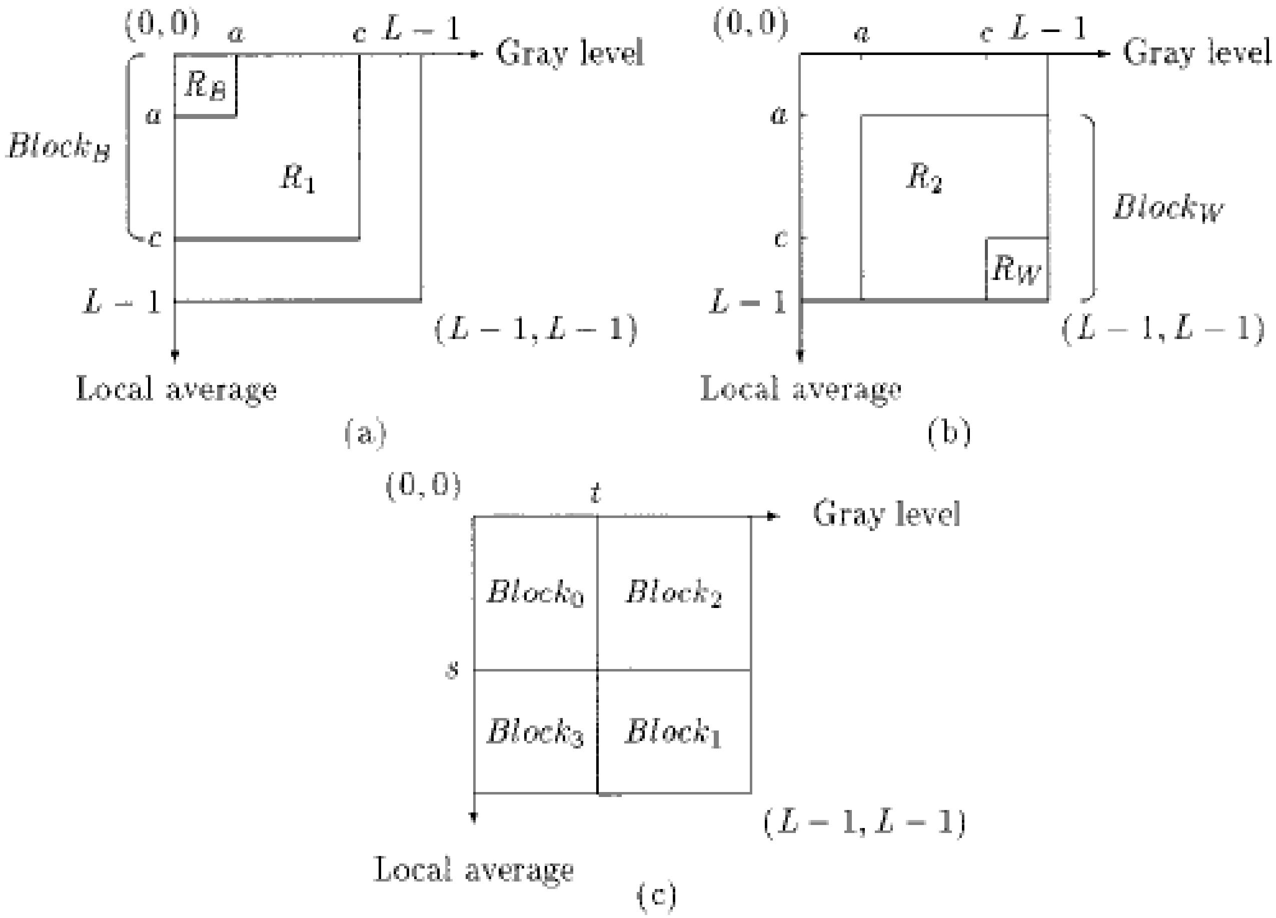


Fig. 1. (a) Block_B is divided into R_B and R_1, (b) Block_W is divided into R_W and R_2, and (c) 2-D histogram is divided into four blocks.

Determination of threshold by Fuzzy Entropy

华中科技大学

- The fuzzy relation Bright is a subset of the full Cartesian product space $X \times Y$

$$Bright = BrightX \times BrightY \subset X \times Y$$

$$\mu_{Bright}(x, y) = \mu_{BrightX \times BrightY}(x, y)$$

$$= \min(\mu_{BrightX}(\mathrm{x}), \mu_{BrightY}(\mathrm{y}))$$

- Similarly,

$$Dark = DarkX \times DarkY \subset X \times Y$$

$$\mu_{Dark}(x, y) = \mu_{DarkX \times DarkY}(x, y)$$

$$= \min(\mu_{DarkX}(\mathrm{x}), \mu_{DarkY}(\mathrm{y}))$$

Definition of Fuzzy Entropy

- Let A be a fuzzy set with membership function $\mu_A(x_i)$, where $x_i, i = 1, \ldots, N,$ are the possible outputs from source A with the probability . The fuzzy entropy set A is defined as:

$$H_{fuzzy}(A) = -\sum_{i=1}^{N} \mu_A(x_i) P(x_i) \ln P(x_i)$$

- The total image entropy is defined as:

$$H(image) = H(Block_B) + H(Block_W)$$

Determination of threshold by Fuzzy Entropy

華中科技大学

- As shown in Fig. 1(a), the dark block $Block_B$ can be divided into a nonfuzzy region R_B and a fuzzy region R_1

$$Block_B = \{(x, y) | \mu_{Dark}(x, y) = 1, (x, y) \in Block_B\}$$

$$R_1 = \{(x, y) | \mu_{Dark}(x, y) < 1, (x, y) \in Block_B\}$$

- Similarly, the bright block $Block_W$ is composed of a nonfuzzy region RW and a fuzzy region R_2, as shown in Fig. 1(b)

$$Block_W = \{(x, y) | \mu_{Bright}(x, y) = 1, (x, y) \in Block_W\}$$

$$R_2 = \{(x, y) | \mu_{Bright}(x, y) < 1, (x, y) \in Block_W\}$$

Determination of threshold by Fuzzy Entropy

- The following four entropies can be calculated:

$$H_{\text{nonfuzzy}}(R_B) = -\sum_{(x,y)\in R_B} \frac{n_{xy}}{\sum_{(x,y)\in R_B} n_{xy}} \cdot \log \frac{n_{xy}}{\sum_{(x,y)\in R_B} n_{xy}} \qquad (7)$$

$$H_{\text{fuzzy}}(R_1) = -\sum_{(x,y)\in R_1} \mu_{\text{Dark}}(x,y) \frac{n_{xy}}{\sum_{(x,y)\in R_1} n_{xy}} \cdot \log \frac{n_{xy}}{\sum_{(x,y)\in R_1} n_{xy}} \qquad (6)$$

$$H_{\text{fuzzy}}(R_2) = -\sum_{(x,y)\in R_2} \mu_{\text{Bright}}(x,y) \frac{n_{xy}}{\sum_{(x,y)\in R_2} n_{xy}} \cdot \log \frac{n_{xy}}{\sum_{(x,y)\in R_2} n_{xy}} \qquad (8)$$

$$H_{\text{nonfuzzy}}(R_W) = -\sum_{(x,y)\in R_W} \frac{n_{xy}}{\sum_{(x,y)\in R_W} n_{xy}} \cdot \log \frac{n_{xy}}{\sum_{(x,y)\in R_W} n_{xy}} \qquad (9)$$

where n_{xy} is the element in the 2-D histogram which represents the number of occurences of the pair *(x,y)*

Determination of threshold by Fuzzy Entropy

- To find the best set of a,b,and c is an optimization problem which can be solved by different optimization methods. For example, we can use genetic algorithm to search for the optimal solution. The proposed method consists of the following three major steps:
  1) find the 2-D histogram of the image;
  2) perform fuzzy partition on the 2-D histogram;
  3) compute the fuzzy entropy.
- Step 1) needs to be execute only once while Steps 2) and 3) are performed iteratively for each set of (a,b,c). The optimum (a,b,c) determines the fuzzy region (i.e., interval [a,c]). The threshold is selected as the crossover point of the membership function which has membership 0.5 implying the largest fuzziness.

Determination of threshold by Fuzzy Entropy

Once the threshold vector (s, t) is obtained, it divides the 2-D histogram into four blocks, i.e., a dark block Block_0, a bright block Block_1, and two noise (edge) blocks, Block_2 and Block_3, as shown in Fig. 1(c). The bright extraction method is expressed as [6]

$$f_{s,t}(x, y, \text{bright}) = \begin{cases} g_1, & f(x, y) \geq t \wedge g(x, y) \geq s \\ g_0, & \text{otherwise.} \end{cases} \tag{10}$$

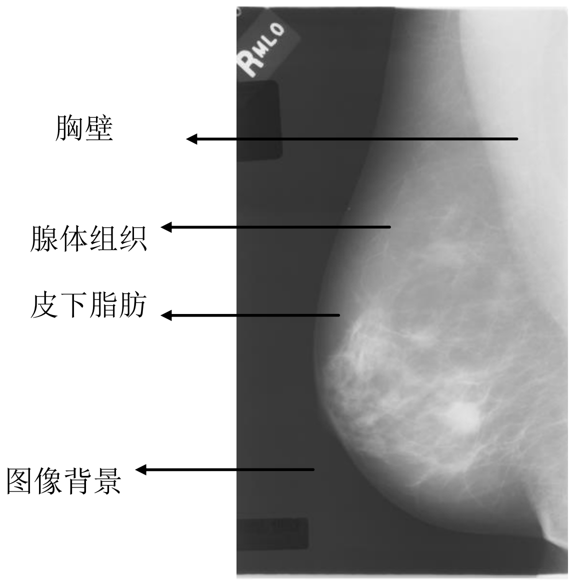
Conversely, the dark portion extraction is

$$f_{s,t}(x, y, \text{dark}) = \begin{cases} g_0, & f(x, y) < t \wedge g(x, y) < s \\ g_1, & \text{otherwise.} \end{cases} \tag{11}$$

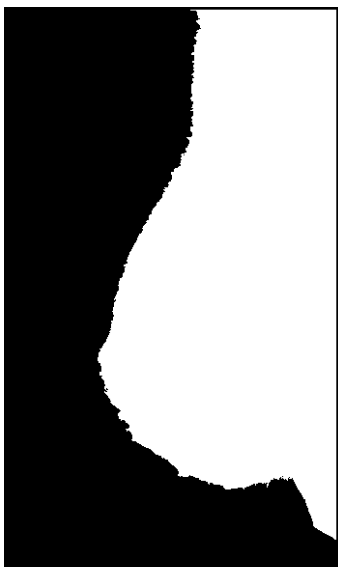
阈值分割的改进

- 利用像素邻域的局部信息：基于过渡区的方法
- 利用像素点空间位置：变化阈值法
- 结合局部灰度
- 结合连通信息
- 基于最大熵原则的阈值选择方法

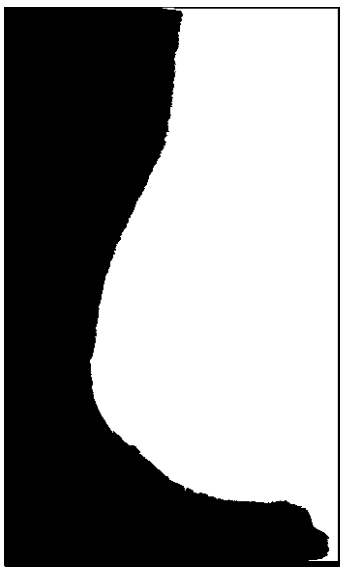
多阈值法



乳腺钼靶图像



单阈值分割



多阈值分割