Hong Kong Property Prices Time Series Analysis

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Contents

1 Introduction

Despite the recent hits to Hong Kong's economic growth, the city has continued to be the most expensive housing market in the world [1]. Applying various techniques from time series analysis to its real residential property prices helps us understand the changes over years and potentially forecast future prices. This project employs methods including Autoregressive integrated moving average (ARIMA), exponential smoothing and Facebook Prophet to analyze the data across time.

2 Data Visualization

Data are obtained from the BIS Residential Property Price database [2]. The inflation-adjusted price series ranges from 1979-10-1 to 2020-07-01 on a quarterly basis (around 164 data points). The price in 2010 is used as the base price and standardized to be 100, while other prices are converted accordingly and not seasonally adjusted (see figure 1).

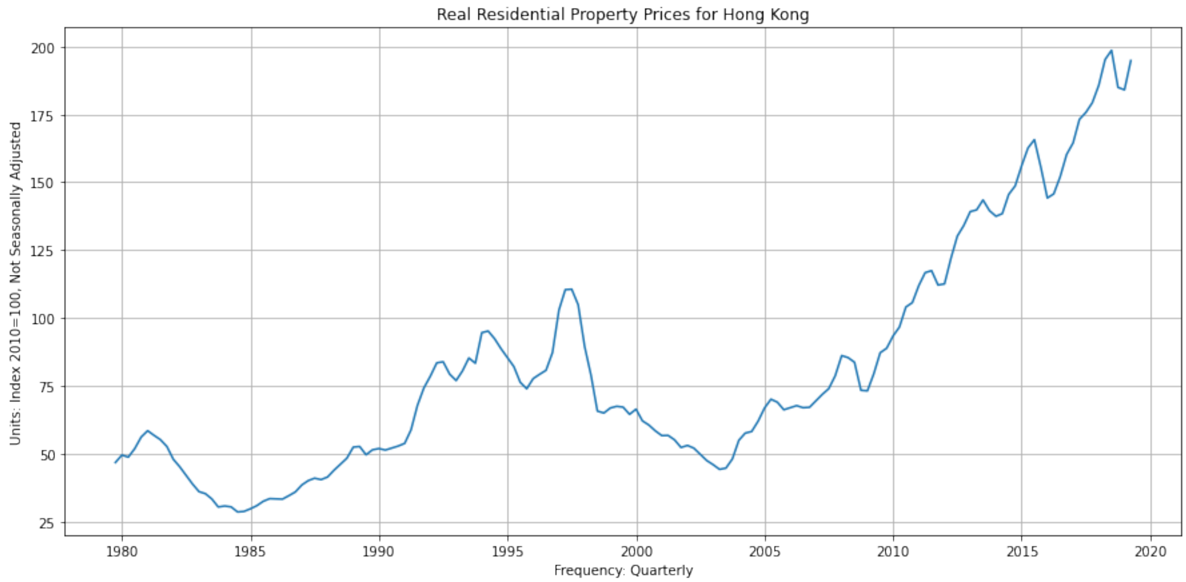

Figure 1: Original Price Series

The series demonstrates clear non-stationarity in both average level and variance. We perform Box-Cox transformation, which contains the frequently used Log transformation as a special case. Since λ is not pre-specified, the package `scipy.stats.boxcox` searches for the lambda that maximizes the log-likelihood function which returns $\lambda \approx -0.2093$ in this case. The resulting series is plotted in figure 2.

Figure 2 also contains the sample Autocorrelation Function (ACF), sample Partial Autocorrelation Function (PACF) and the p-value of the Augmented Dickey-Fuller (ADF) test of the series. The ACF decays slowly and the p-value is much higher than 0.05, which confirm that the current series is not stationary.

3 Box-Jenkins Method

Our model-building strategy would follow the Box-Jenkins method by cycling through model specification, fitting and diagnostics until an adequate model is found. Since the data collected are of quarterly nature, whereas its time plot does not demonstrate clear seasonality, we try out both seasonal and non-seasonal modeling in the this section.

3.1 Non-seasonal Models

Following the direction given by figure 2, we take first common difference and obtain figure 3. Now the series appears to fluctuate around zero and the p-value of the ADF test is almost zero.

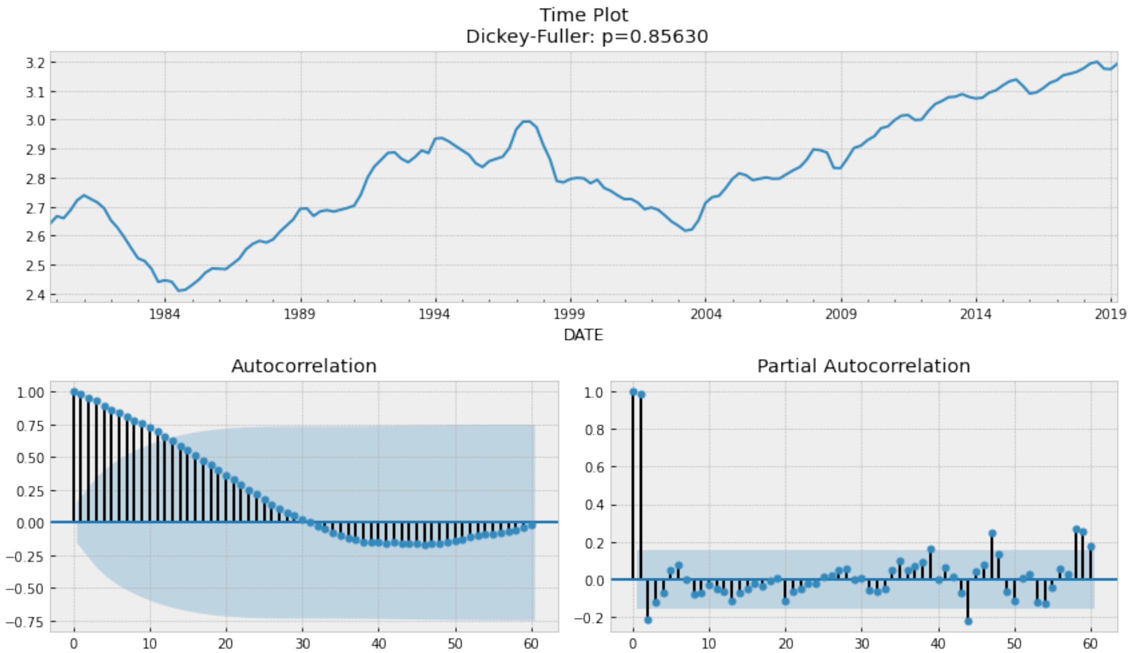

Figure 2: Time Plot after Box-Cox transformation

Therefore, we reject the null hypothesis and conclude that the series is stationary.

3.1.1 Model Specification

Observing that the lag 1s of both ACF and PACF in figure 3 are significant while the lag 2 of ACF stands out a little, we tentatively add $ARIMA(1,1,1)$ and $ARIMA(1,1,2)$ as candidates.

3.1.2 Parameter Estimation

From figure 4, we see that the MA coefficient for $ARIMA(1,1,1)$ is not significant (p-value $0.191 > 0.05$) and all coefficients for $ARIMA(1,1,2)$ are insignificant. This prompts us to consider reducing the value of MA parameter, i.e. $ARIMA(1,1,0)$.

Figure 5 shows that the coefficient for AR is highly significant and the AIC has been the lowest up to now. Hence, we proceed to diagnose the $ARIMA(1,1,0)$ model.

3.1.3 Model Diagnostics

The residual analysis for $ARIMA(1,1,0)$ is visualized in figure 6. The standardized residuals behave similar to a white noise. Its histogram and Q-Q plot reinforce this similarity except that a few points in the tail might deviate from normality, which reveals potential heavy-tailedness. The p-value for **Shapiro-Wilk Normality Test** is 0.9538, which indicates that we cannot reject the normal hypothesis.

The correlogram looks fine apart from the imperfection that residuals might be correlated to the extent of lag 1 and 2, when we test the sample individually. However, the **Ljung-Box Test**

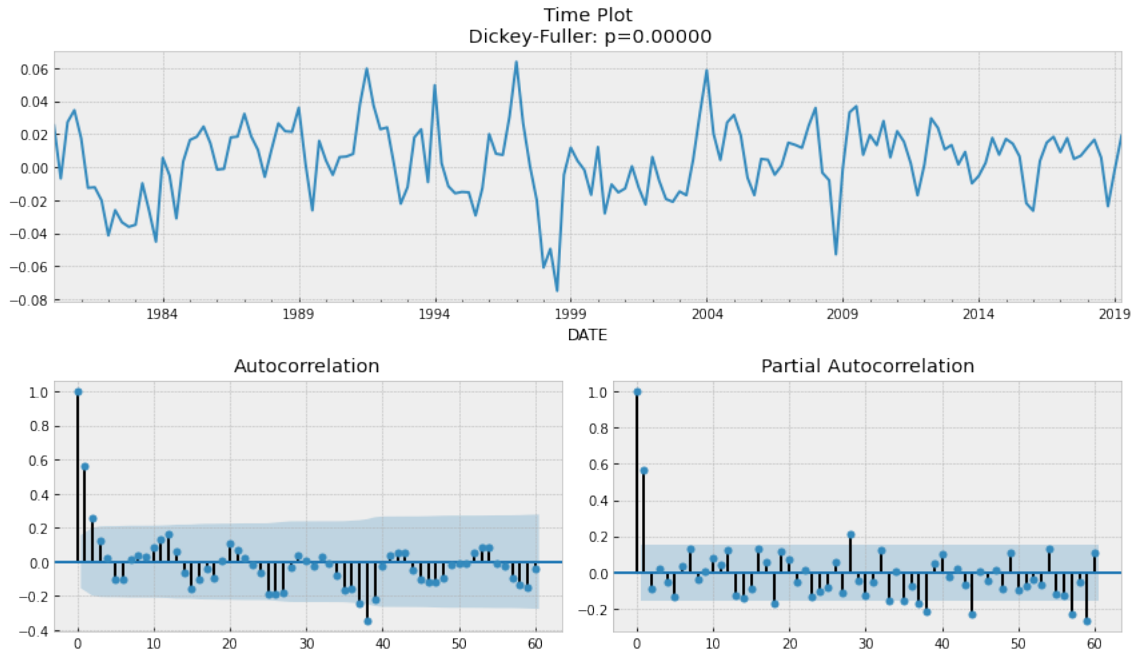

Figure 3: Time Plot after first common differencing

with results tabulated in Table 1 shows that when residuals are tested jointly, none of them are significant. So we shall not worry about the potential correlation.

Lag	1	2	3	4	5	6	7	8	9	10
p-value	0.51	0.55	0.76	0.86	0.54	0.35	0.34	0.42	0.49	0.56
Sig.	False	False	False	False	False	False	False	False	False	False

Table 1: Ljung-Box Test ARIMA(1,1,0)

3.1.4 Analysis of Over-parameterized Models

To check the adequacy of our candidate $ARIMA(1,1,0)$, we need to test it against over-parameterized ones. Incrementing MA parameters has been shown to be inferior in previous steps, here we increment AR parameters instead. According to figure 7, the incremented parameter of AR turns out to be insignificant and close to zero. This confirms that $ARIMA(1,1,0)$ is adequate, so we settle down on it as the final non-seasonal candidate.

3.2 Seasonal Models

Building on figure 3 where the sample ACF shows potential seasonality, we take the seasonal difference with $s = 4$ after taking the first common difference since the data are collected quarterly and obtain figure 8. By the same token, the p-value of the ADF test confirms the stationarity of this series.

ARIMA Model Results

Dep. Variable:	D.y	**No. Observations:**	158
Model:	ARIMA(1, 1, 1)	**Log Likelihood**	407.108
Method:	css-mle	**S.D. of innovations**	0.018
Date:	Mon, 03 May 2021	**AIC**	-806.217
Time:	12:41:37	**BIC**	-793.966
Sample:	01-01-1980	**HQIC**	-801.242
	- 04-01-2019		

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.0038	0.003	1.223	0.223	-0.002	0.010
ar.L1.D.y	0.4418	0.125	3.540	0.001	0.197	0.686
ma.L1.D.y	0.1821	0.139	1.314	0.191	-0.090	0.454

Roots

	Real	Imaginary	Modulus	Frequency
AR.1	2.2633	+0.0000j	2.2633	0.0000
MA.1	-5.4911	+0.0000j	5.4911	0.5000

ARIMA Model Results

Dep. Variable:	D.y	**No. Observations:**	158
Model:	ARIMA(1, 1, 2)	**Log Likelihood**	407.117
Method:	css-mle	**S.D. of innovations**	0.018
Date:	Mon, 03 May 2021	**AIC**	-804.234
Time:	12:41:37	**BIC**	-788.921
Sample:	01-01-1980	**HQIC**	-798.015
	- 04-01-2019		

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.0038	0.003	1.212	0.227	-0.002	0.010
ar.L1.D.y	0.4700	0.241	1.947	0.053	-0.003	0.943
ma.L1.D.y	0.1530	0.255	0.600	0.549	-0.347	0.653
ma.L2.D.y	-0.0199	0.152	-0.131	0.896	-0.317	0.277

Roots

	Real	Imaginary	Modulus	Frequency
AR.1	2.1274	+0.0000j	2.1274	0.0000
MA.1	-4.2193	+0.0000j	4.2193	0.5000
MA.2	11.9017	+0.0000j	11.9017	0.0000

Figure 4: ARIMA(1,1,1) & ARIMA(1,1,2)

ARIMA Model Results

Dep. Variable:	D.y	**No. Observations:**	158
Model:	ARIMA(1, 1, 0)	**Log Likelihood**	406.307
Method:	css-mle	**S.D. of innovations**	0.018
Date:	Mon, 03 May 2021	**AIC**	-806.614
Time:	12:41:37	**BIC**	-797.426
Sample:	01-01-1980	**HQIC**	-802.883
	- 04-01-2019		

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.0038	0.003	1.133	0.259	-0.003	0.010
ar.L1.D.y	0.5639	0.066	8.602	0.000	0.435	0.692

Roots

	Real	Imaginary	Modulus	Frequency
AR.1	1.7735	+0.0000j	1.7735	0.0000

Figure 5: ARIMA(1,1,0)

3.2.1 Model Specification

By the principle of parsimony, we proceed with a seasonal $ARIMA(0,1,1) \times (0,1,1)_4$ model because

- d equals 1 because we had first common differencing
- D equals 1 because we had first seasonal differencing
- q equals 1 as seen on the ACF
- Q equals 1 since the 4-th lag on ACF is significant
- p & P are kept to be 0 to control the model complexity. They could be incremented as needed if the model turns out to be inadequate.

3.2.2 Parameter Estimation

The model is fitted and all coefficients are significant, as shown in figure 9.

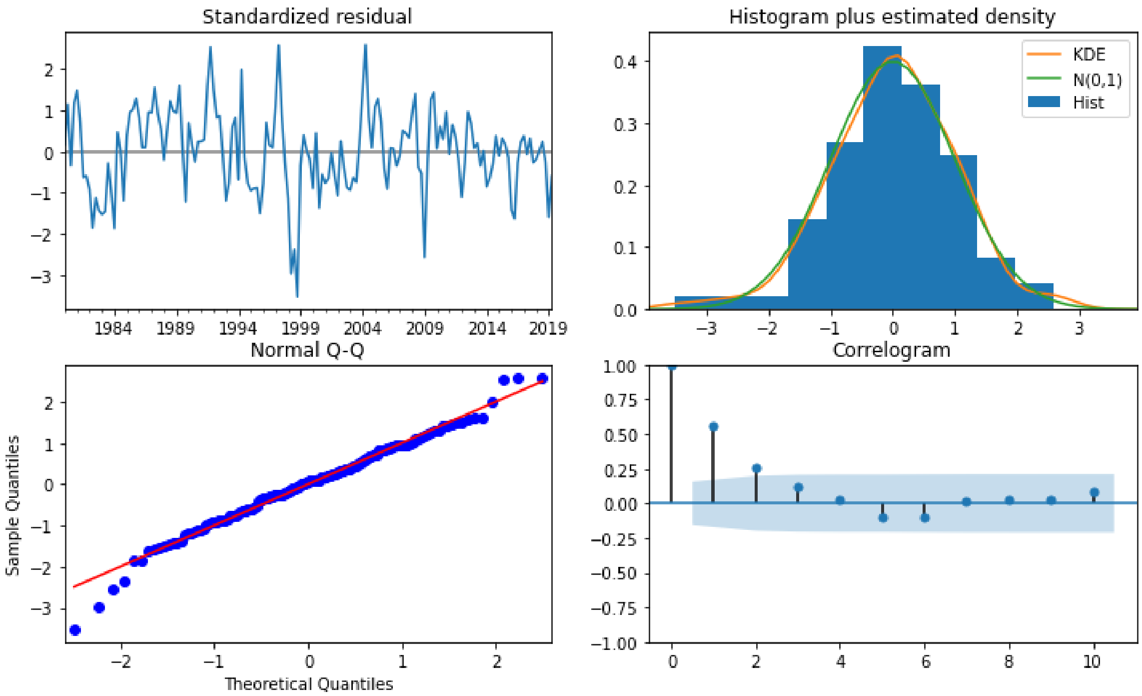

Figure 6: Residual Analysis ARIMA(1,1,0)

ARIMA Model Results

Dep. Variable:	D.y	**No. Observations:**	158
Model:	ARIMA(2, 1, 0)	**Log Likelihood**	407.048
Method:	css-mle	**S.D. of innovations**	0.018
Date:	Mon, 03 May 2021	**AIC**	-806.096
Time:	12:41:38	**BIC**	-793.846
Sample:	01-01-1980 - 04-01-2019	**HQIC**	-801.121

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
const	0.0038	0.003	1.234	0.219	-0.002	0.010
ar.L1.D.y	0.6197	0.080	7.776	0.000	0.463	0.776
ar.L2.D.y	-0.0971	0.080	-1.220	0.224	-0.253	0.059

Roots

	Real	**Imaginary**	**Modulus**	**Frequency**
AR.1	3.1916	-0.3388j	3.2095	-0.0168
AR.2	3.1916	+0.3388j	3.2095	0.0168

Figure 7: ARIMA(2,1,0)

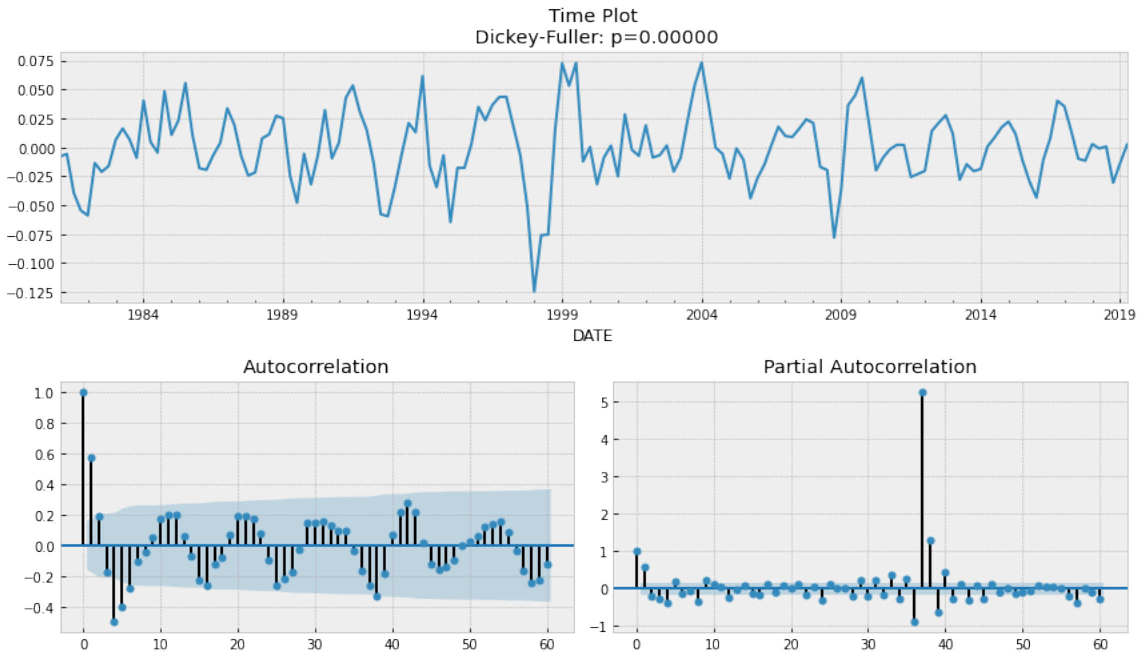

Figure 8: Time Plot after first common and first seasonal differencing

Statespace Model Results

Dep. Variable:	y	**No. Observations:**	159
Model:	SARIMAX(0, 1, 1)x(0, 1, 1, 4)	**Log Likelihood**	387.909
Date:	Mon, 03 May 2021	**AIC**	-769.819
Time:	12:41:40	**BIC**	-760.708
Sample:	10-01-1979	**HQIC**	-766.118
	- 04-01-2019		
Covariance Type:	opg		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
ma.L1	0.5563	0.066	8.366	0.000	0.426	0.687
ma.S.L4	-0.9788	0.161	-6.097	0.000	-1.293	-0.664
sigma2	0.0003	6.3e-05	5.547	0.000	0.000	0.000

Ljung-Box (Q):	75.62	**Jarque-Bera (JB):**	11.94
Prob(Q):	0.00	**Prob(JB):**	0.00
Heteroskedasticity (H):	0.63	**Skew:**	-0.49
Prob(H) (two-sided):	0.11	**Kurtosis:**	3.95

Figure 9: Seasonal $ARIMA(0,1,1) \times (0,1,1)_4$

3.2.3 Model Diagnostics

The diagnostics in figure 10 indicates that the residual errors do not seem to have a constant variance: more volatile around 1997 and 2008 when the two financial crises respectively took a toll on the housing market [3][4]. Its distribution is slightly left skewed compared to the standard normal. Further, the p-value for **Shapiro-Wilk Normality Test** is 0.0000, which rejects the normal hypothesis. This non-normality questions the assumption and validity of the model.

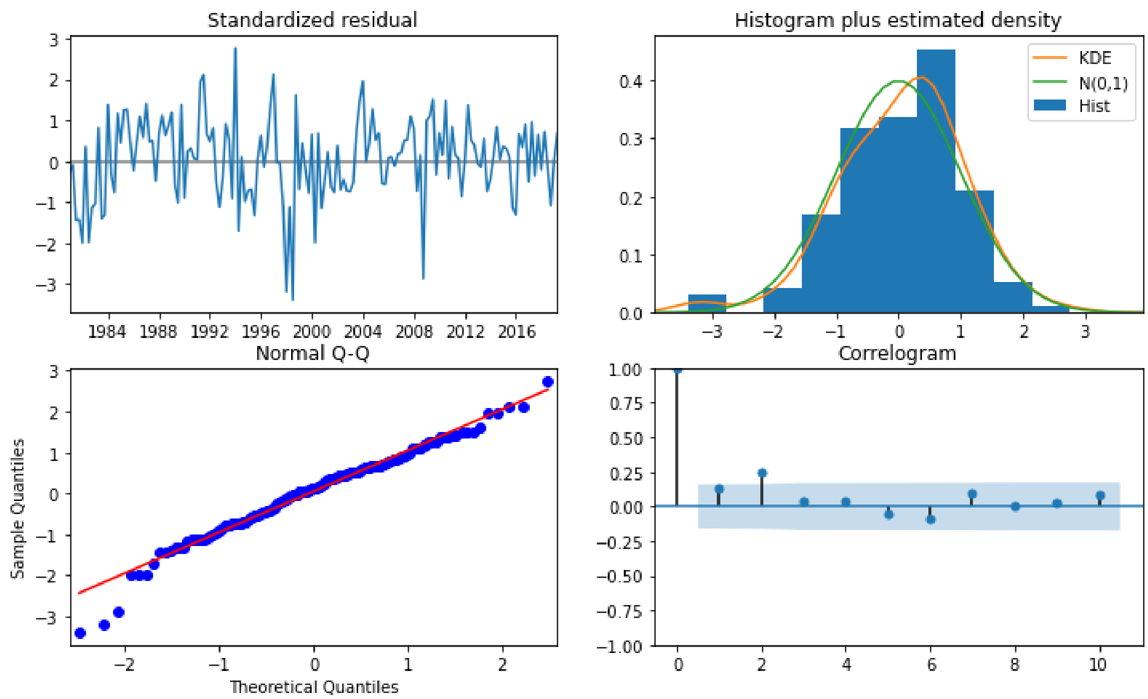

Figure 10: Residual Analysis Seasonal $ARIMA(0, 1, 1) \times (0, 1, 1)_4$

Although individually, the correlogram shows that almost all lags stay within the Bartlett's bounds, the **Ljung-Box Test** in Table 2 manifests the existence of correlation: jointly for all $4 \leq K \leq 10$ the results are significant.

Lag	1	2	3	4	5	6	7	8	9	10
p-value	0.928	0.995	0.998	0.000	0.000	0.000	0.001	0.002	0.004	0.006
Sig.	False	False	False	True	True	True	True	True	True	True

Table 2: Ljung-Box Test Seasonal $ARIMA(0, 1, 1) \times (0, 1, 1)_4$

3.2.4 Exploration of Adjacent Models

In the Jupyter Notebook (see link in the "Link to Python Code" section), several adjacent models are explored, in hope of finding more adequate models for which the residual assumptions would not be violated. To avoid repetition, the details of model fitting and diagnostics are not presented here. The problems above unfortunately persist in all these trials. Then, a grid search of an optimal parameter combination is carried out. Please find table 3 for the resulting top 5 combinations in terms of AICs. The most promising seasonal candidate is $ARIMA(1,1,0) \times (0,1,1)_4$, and yet its diagnostics again fails to justify the assumptions of normality and independence. Its AIC is lower than the non-seasonal candidate as well. Therefore, the **final model** selected through the Box-Jenkins method is $ARIMA(1,1,0)$.

Parameters (p, q, P, Q)	(1, 0, 0, 1)	(2, 0, 0, 1)	(1, 0, 1, 1)	(2, 3, 0, 1)	(2, 1, 0, 1)
AIC	-779.308	-778.549	-777.464	-776.871	-776.792

Table 3: Grid Search Result

3.3 Forecasting

Since the predicted values and intervals are based on the transformed series. We need to transform them back through inverse Box-Cox transformation: this preserves the coverage of the prediction intervals, and the back-transformed point forecast can be considered the median of the forecast densities, assuming the forecast densities on the transformed scale are symmetric. The forecasting results are summarized in table 4 and figure 11. We can observe that, despite the actual values lying between the 95% confidence interval, the predicted values consistently overestimate the price growth. This is understandable considering the special situations during this particular period: Hong Kong's average home prices plummeted as protests and trade war weakened demand [5]. It would be hard for the ARIMA model to predict accurately during unforeseen political and economic turmoils like this. Lastly, two metrics are calculated to help evaluate the model performance: the Mean Squared Error (MSE) and Mean Absolute Percentage Error (MAPE) of the out-of-sample forecasts are 589.37 and 11.09%, respectively.

Steps	Period	Actual	Predicted	Difference	95% L.B.	95% U.B.
1	2019-07-01	189.63	202.22	-12.59	181.38	226.02
2	2019-10-01	185.75	207.59	-21.85	169.79	256.06
3	2020-01-01	184.53	211.77	-27.24	159.28	286.71
4	2020-04-01	186.87	215.26	-28.39	150.24	317.59
5	2020-07-01	190.75	218.37	-27.62	142.59	348.70

Table 4: Summary of Forecasts

4 Exponential Smoothing

Exponential smoothing methods utilize weighted averages of past observations, with the weights decaying exponentially as the observations get older. That is, a higher weight is associated with a more recent observation. This framework is widely adopted in industry because of its quick, reliable and easy-to-interpret forecasts.

4.1 Moving Average

A basic case is moving average: the future value of a variable depends on the average of its k previous values. Here, if we set $k = 4$, then the property price next quarter is calculated as the

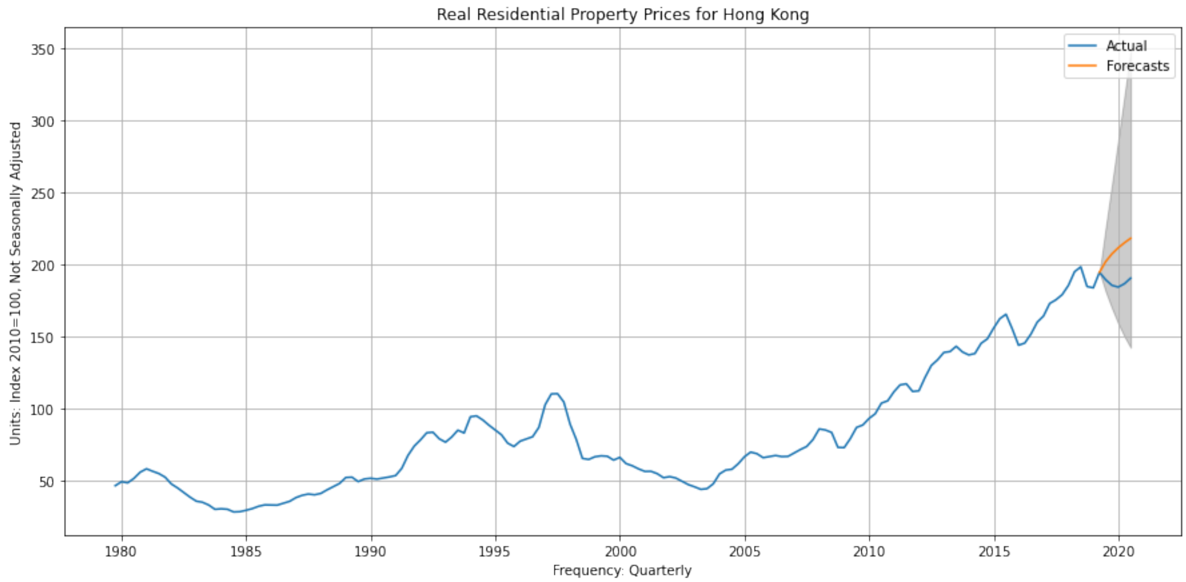

Figure 11: Forecasts by ARIMA(1,1,0)

average of the 4 quarterly prices in the past year. Figure 12 shows the rolling window of the moving average that is constructed by the upper and lower bounds of the confidence interval. Three anomaly points are detected, which correspond to the 1997 Asian financial crisis as mentioned before. A general model is Weighted Average: rather than assigning weights to the past k values equally, other combinations that sum up to 1 could be used. Domain knowledge might help, such as assigning larger weights to more recent observations.

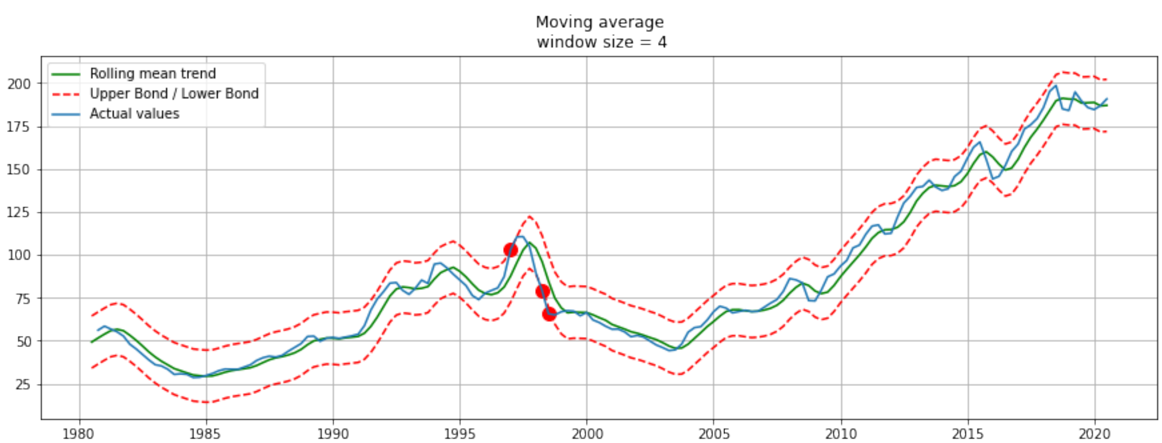

Figure 12: Moving Average

4.2 Simple Exponential Smoothing

As introduced, we start weighting all available observations while exponentially decreasing the weights as we move further back in time. Refer to equation 1 and the α indicates how quickly the last available true observation will be "forgotten". The smaller α is, the more influence the previous observations have and the smoother the series is. Figure 13 confirms that the smaller

$\alpha = 0.05$ leads to a smoother curve.

$$\hat{y}_t = \alpha \cdot y_t + (1 - \alpha) \cdot \hat{y}_{t-1} \tag{1}$$

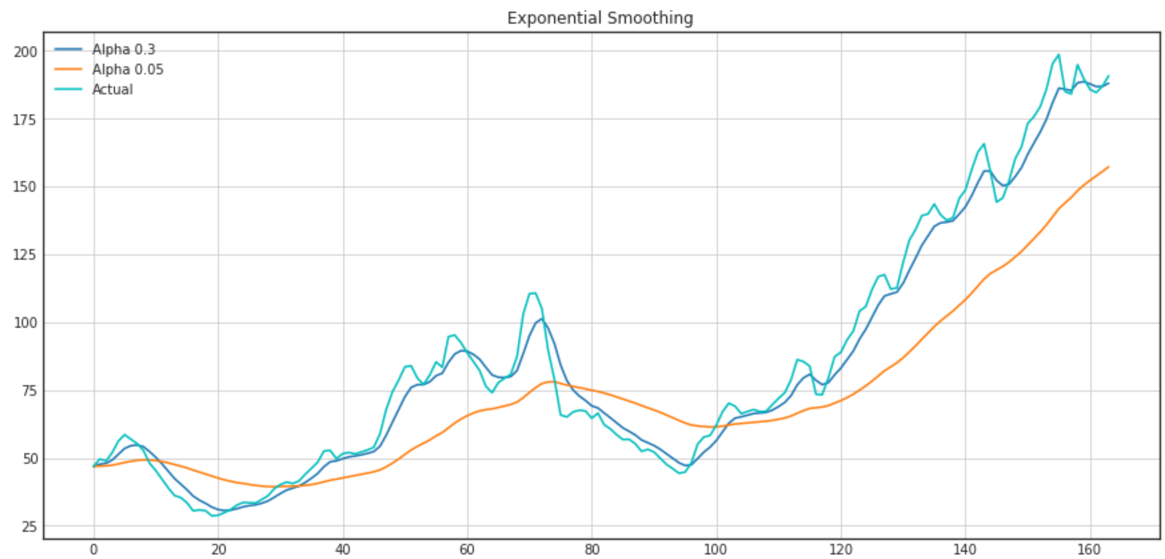

Figure 13: Simple Exponential Smoothing with Different α

4.3 Double Exponential Smoothing

In equations (2)-(4), we further decompose the series into intercept (i.e. level) l and slope (i.e. trend) b. We have predicted intercept (or expected series value) above. Then we apply the same exponential smoothing to the trend by assuming that the future direction of the time series changes depends on the previous weighted changes. The β here functions similarly as a weight for exponential smoothing. The final prediction is obtained by summing up the model values of the intercept and trend. Figure 14 shows how α and β control the smoothness of the curve.

$$l_x = \alpha y_x + (1 - \alpha)(l_{x-1} + b_{x-1}) \tag{2}$$

$$b_x = \beta(l_x - l_{x-1}) + (1 - \beta) b_{x-1} \tag{3}$$

$$\hat{y}_{x+1} = l_x + b_x \tag{4}$$

4.4 Triple Exponential Smoothing (Holt-Winters)

As its name would suggest, the Triple Exponential Smoothing goes beyond level and trend smoothing to incorporate another dimension: seasonality smoothing. Please refer to the paper by Winters (1960) [6] for details of the equations. When the number of parameters increases, the problem of choosing the optimal combination becomes tricky. A natural idea from machine learning is to perform cross validation. However, time series or other intrinsically ordered data require a modified version of cross validation. Among others, one canonical way studied in this

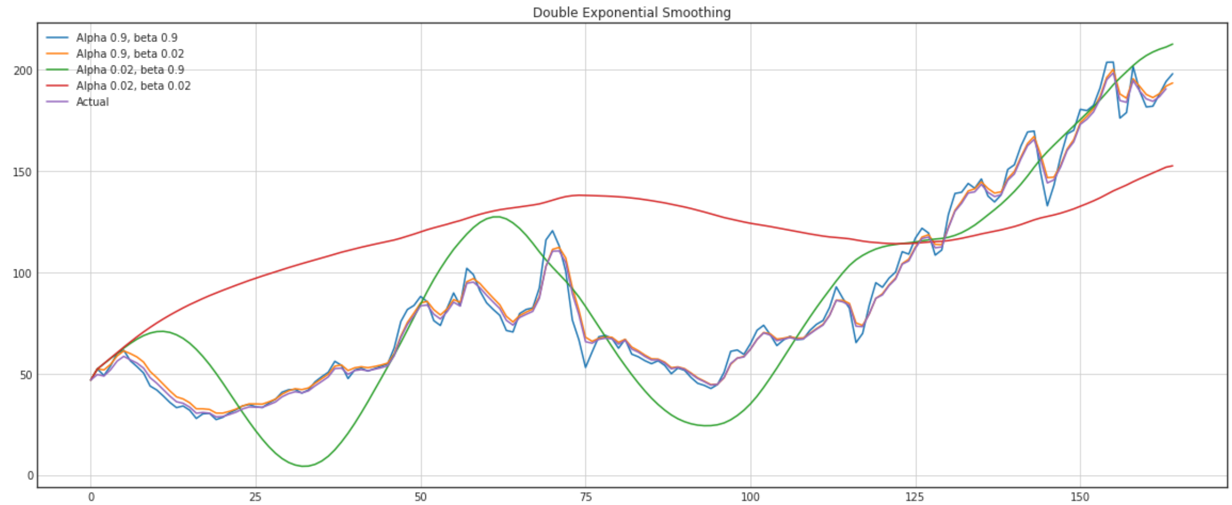

Figure 14: Double Exponential Smoothing with Different α and β

paper [7] is briefly exemplified here: suppose we have 5 time periods for training and 1 for testing, we should proceed as follows to preserve the time order.

- fold 1: training [1], test [2]
- fold 2: training [1 2], test [3]
- fold 3: training [1 2 3], test [4]
- fold 4: training [1 2 3 4], test [5]
- fold 5: training [1 2 3 4 5], test [6]

Figure 15 shows the forecasting results by this method. There is no anomaly detected within the training set but many in the test set. This on the one hand may be explained by the unprecedented political and economic situations in Hong Kong housing market. On the other hand, however, it implies potential overfitting: perhaps the seasonality is not strong enough for the Holt-Winters model to work well. The MAPE of the whole fitted series including the test set being 1.49%, in contrast to the mispredictions, adds to the possibility of overfitting.

5 Facebook Prophet

An API created by Facebook named Prophet [8] is employed on this time series. Essentially the Prophet procedure is an additive regression model that captures trend, seasonality and change points. Figure 16 shows the fitted model: many data points are treated as outliers including those around the two financial crises, while the overall trend of steady growth in the 2010s is clear. Figure 17 illustrates the different components (weekly component is not applicable to the quarterly data here): we learn from the yearly seasonality that property prices drop during Christmas but would rise after New Year. The MAPE of the out-of-sample forecasts is 3.43%, the lowest of all that have been discussed.

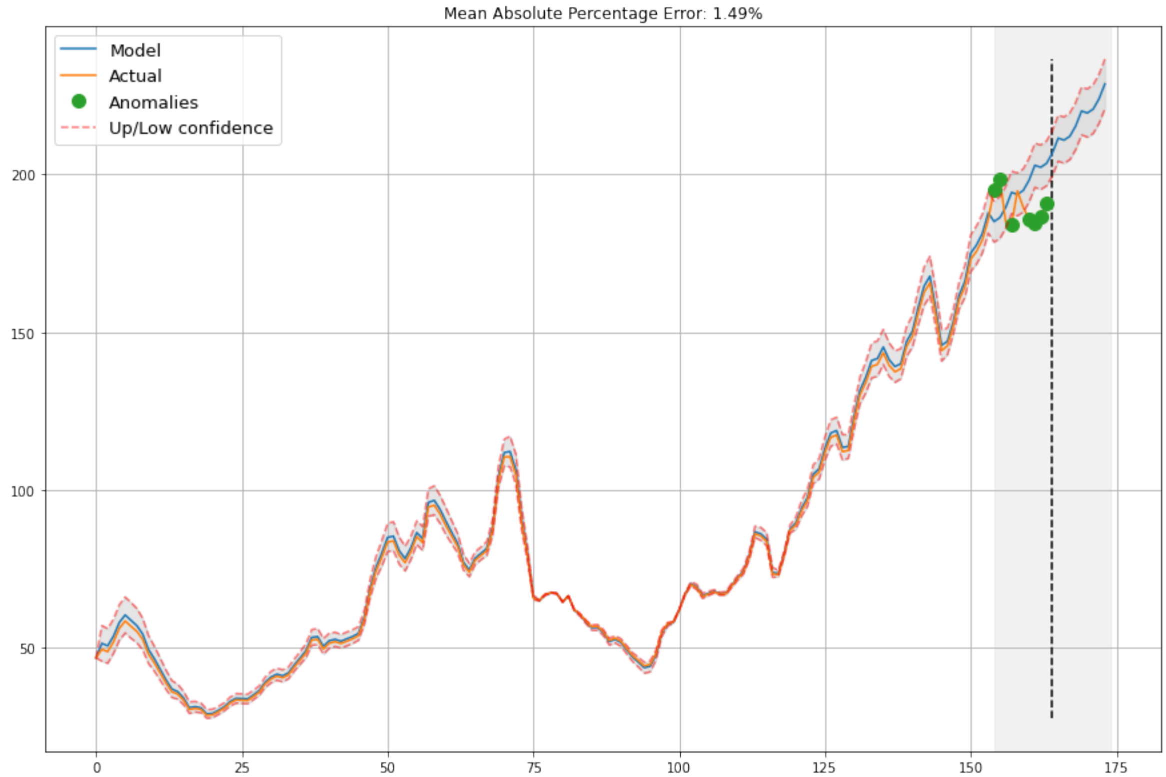

Figure 15: Triple Exponential Smoothing (Holt-Winters) Forecasts

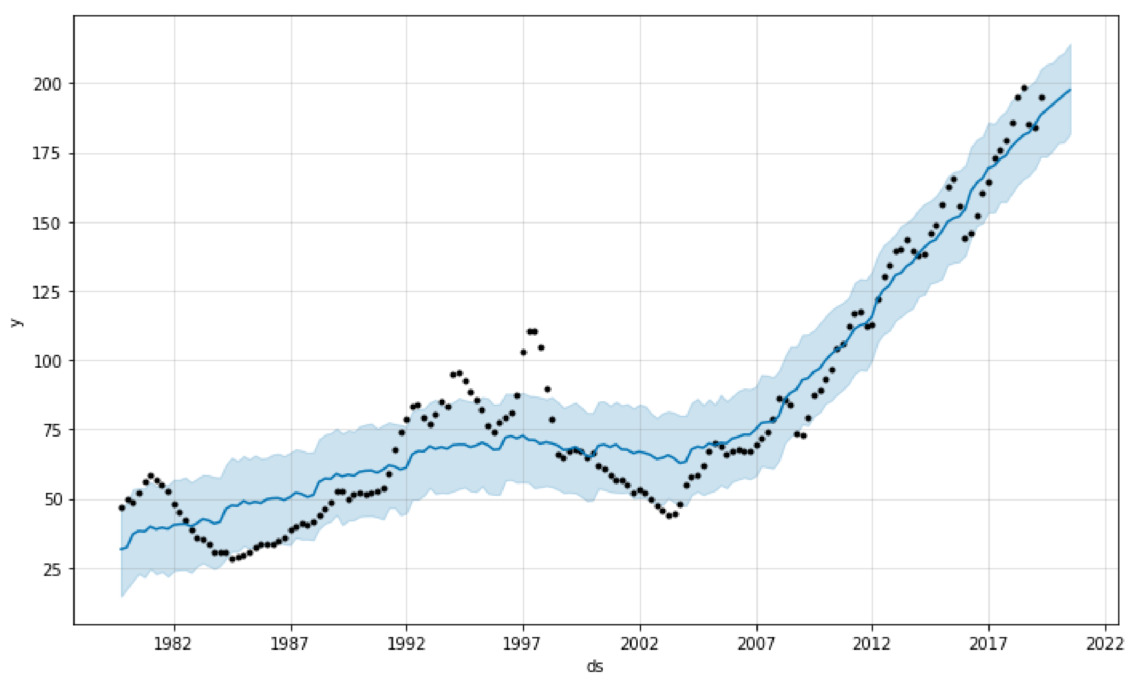

Figure 16: Facebook Prophet Model Fitting

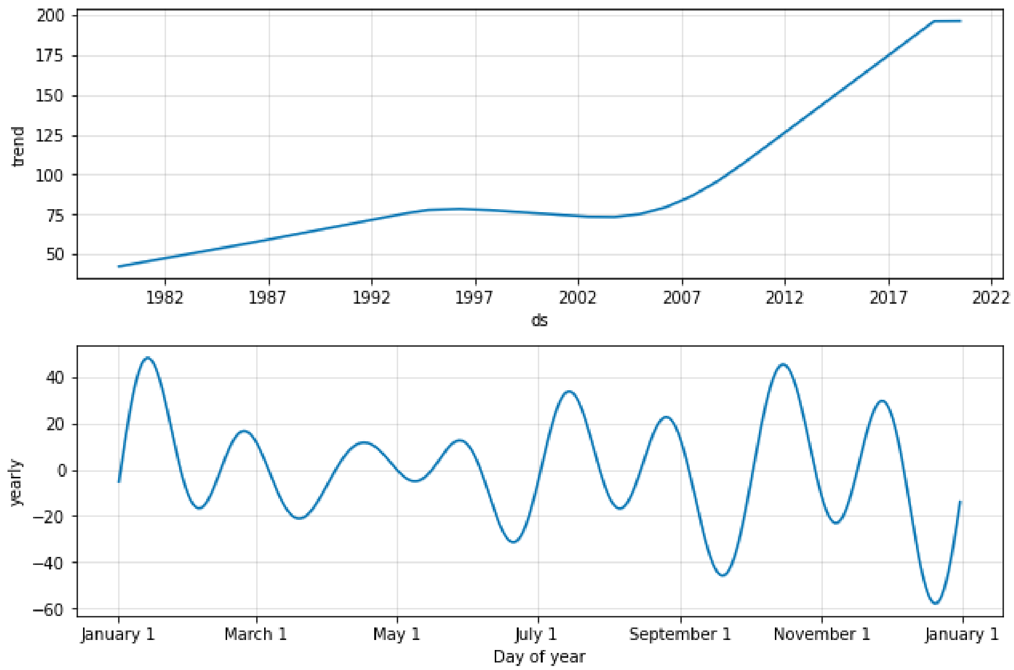

Figure 17: Facebook Prophet Series Decomposition

6 Conclusion

Through various modeling processes, a better understanding of the real residential property prices in Hong Kong is obtained: its overall trend, seasonality and outliers that are affected by historical events and the law of supply & demand. The comparative advantages of different models also become clear: the Box-Jenkins method being rigorous but lengthy, the exponential smoothing being straightforward but prone to overfitting, the Facebook Prophet being flexible but sophisticated. The analysis could be improved by better tuning model parameters, considering alternative data processing techniques, or including more data points. More sophisticated models such as LSTMs and RNNs in deep learning would also be valuable to explore.

7 Link to Python Code

Please find the accompanying Jupyter Notebook **here**. It also includes a section exploring Machine Learning algorithms including linear regression and XGBoost. But due to time constraint, the models are preliminary and require further tuning to have a satisfactory performance.

References

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