MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) By issuing bonds, a government raises money to finance its spending.

   (b) On the one hand, the yield curve signals economic conditions: an upward sloping curve suggests economic expansion while an inverted curve indicates economic recession. On the other hand, the Treasury yield curve is often used as a proxy for risk-free interest rates, and thus a benchmark for other interest rates (e.g. mortgage rates).

   (c) Central banks can manipulate short-term interest rates (left end of the yield curve): lowering interest rates encourages borrowing and increases total money supply. Alternatively, central banks engage in open market operations: they buy Treasury bonds in the open market to directly inject cash into the economy, consequently pushing prices up and interest rates down.

2. Bonds to be used:
   "CAN 1.5 Mar 20", "CAN 0.75 Sep 20", "CAN 0.75 Mar 21", "CAN 0.75 Sep 21", "CAN 0.5 Mar 22", "CAN 2.75 Jun 22", "CAN 1.75 Mar 23", "CAN 1.5 Jun 23", "CAN 2.25 Mar 24", "CAN 1.5 Sep 24", "CAN 1.25 Mar 25".
   Explanation:
   Given the semi-annual coupon payment, we would like to ideally have 10 bonds with maturity dates evenly distributed over the time span of 5 years. Hence we narrow down from the available data set to bonds maturing in March and September, excluding those with a significantly older issue date. Due to incomplete bond data for two September bonds, we attempt to introduce two similar June bonds for later approximation.

3. According to a report by Moody's Analytics Research[1], Principal Component Analysis (PCA) aims to approximate a high dimensional system via fewer linearly uncorrelated vectors (principal components) without losing too much information. This could be achieved by the eigen-decomposition of the covariance matrix of the given stochastic processes. Since adjacent points along the stochastic curve tend to affect each other's movements, PCA attempts to capture the variability of the whole curve in a simple representation. Intuitively, the eigenvector corresponding to the largest eigenvalue (the first principal component) explains the most variability of the data. In the stochastic curve context, therefore, the first few principal components identify the primary ways in which different stochastic points on the curve move together.

Empirical Questions - 75 points

4.

(a) The yield to maturity of the chosen bonds is calculated using the Newton-Raphson method. Moreover, instantaneous compounding is used. As for interpolation, we adopt the B-spline of degree 3 for relatively smooth plotting.

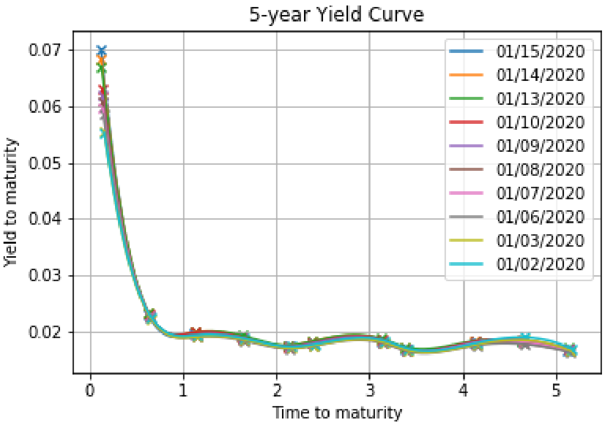

Figure 1: Q4(a) 5-year yield curve

(b)

Algorithm 1: Bootstrapping Spot Curve

Create empty lists storing time to maturity and calculated spot rates;
for *data of each day* **do**
  for *each bond* **do**
    Get timeToMaturity from today, semi-annual coupon and dirtyPrice;
    $couponSum \leftarrow 0$
    if *it is a non-zero-coupon bond* **then**
      for $j \leftarrow TTM - 0.5, TTM - 1, ..., TTM \bmod 0.5$ **do**
        $couponSum \leftarrow couponSum +$ discounted coupons by the spot rate on day j
      end
    end
    $spot \leftarrow -log(\frac{dirtyPrice - couponSum}{coupon + par}) \div timeToMaturity$
    Store current timeToMaturity and spot rate into lists;
  end
  Plot the (superimposed) spot rate curve with interpolation.
end

Please note that the following spot curve is zoomed in to terms ranging from 1-5 years.

(c)

Algorithm 2: Derive Forward Curve from Spot Curve

for *data of each day* **do**
  Repeat Algorithm 1 to generate enough points $\{(t_1, r_1), (t_2, r_2), \dots\}$ from the spot curve;
  for *each point* **do**
    Derive forward rate by $f(0, t_1, t_2) = \frac{(1+r_2)^{t_2}}{(1+r_1)^{t_1}} - 1$
  end
  Plot the (superimposed) forward rate curve over the required interval.
end

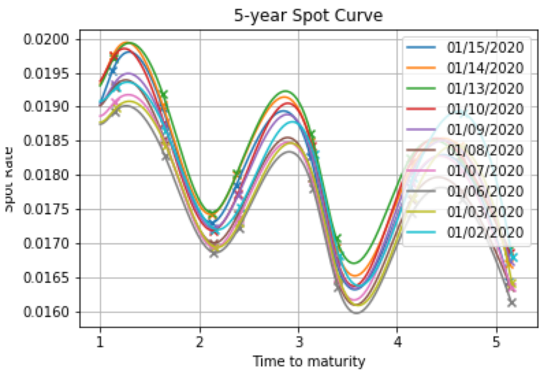

Figure 2: Q4(b) Spot curve in years 1-5

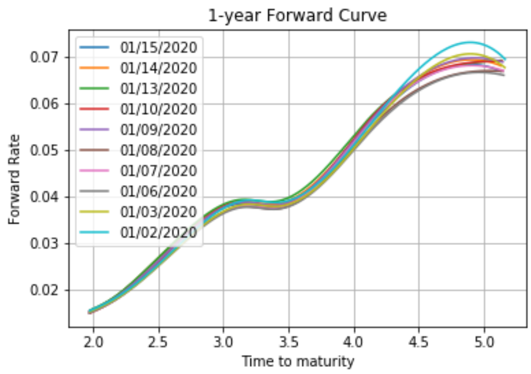

Figure 3: Q4(c) 1-year forward curve

5. Please find the two covariance matrices either in the code output or in the screenshots below.

	1-year yiel	2-year yiel	3-year yiel	4-year yiel	5-year yield log
1-year yiel	0.000117	2.93E-05	8.24E-05	7.76E-05	8.58E-05
2-year yiel	2.93E-05	0.0001	9.20E-05	0.000102	0.000149
3-year yiel	8.24E-05	9.20E-05	0.000127	0.000139	0.000189
4-year yiel	7.76E-05	0.000102	0.000139	0.000198	0.000224
5-year yiel	8.58E-05	0.000149	0.000189	0.000224	0.000465

Figure 4: Q5 Yield covariance matrix

	1yr-1-yr lc	1yr-2-yr lc	1yr-3-yr lc	1yr-4-yr log
1yr-1-yr lc	0.000407	0.000198	0.000221	0.000337
1yr-2-yr lc	0.000198	0.000206	0.000203	0.000314
1yr-3-yr lc	0.000221	0.000203	0.000312	0.000356
1yr-4-yr lc	0.000337	0.000314	0.000356	0.000784

Figure 5: Q5 Forward covariance matrix

6. (a) For yield, the greatest eigenvalue is $7.80935903 \times 10^{-4}$, and its associated eigenvector (first principal component) is $[0.20581782, 0.71944902, -0.54886962, 0.33607808, -0.16070406]$. This implies that the left end and the middle part of the 5-year yield curve would primarily move in the direction opposite to that of the rest, presenting fluctuations.

   (b) For forward rates, the greatest eigenvalue is $1.34009228 \times 10^{-3}$, and its associated eigenvector (first principal component) is $[-0.43243633, -0.81741008, 0.35924501, 0.12562886]$. Therefore, it indicates that the short-term spot rates one year later would move in the opposite direction to the then long-term rates and furthermore in greater magnitude.

GitHub Link to Code

`https://github.com/AndyYFTao/Yield-Curve-Empirical-Exercise/blob/master/A1_yield_curve.ipynb`

References

[1] Moody's Analytics Research. *Principal Component Analysis for Yield Curve Modelling.* URL: `https://www.moodysanalytics.com/-/media/whitepaper/2014/2014-29-08-PCA-for-Yield-Curve-Modelling.pdf`.