

## G. Jolly Jumpers

Given an array  $a$  containing  $0 < n \leq 3000$  integers. Check if  $|a_i - a_{i+1}|$  takes on all values between 1 and  $n - 1$ .

### Solution

- Store (count) all values  $|a_i - a_{i+1}|$  ( $1 \leq i \leq n - 1$ ) into a *map* structure. Then, you check for all keys between 1 and  $n - 1$  if they are non-empty. If there is an empty key, then it means that the sequence is not a jolly jumper.
- This can be achieved by either using `std::vector` or `std::map` since  $n \leq 3000$ .

```
if n <= 1 then
    print JOLLY and return
for i = 1 to n-1 do
    diff = abs(a[i]-a[i+1]) // Note: diff can be greater than n-1!!
    cnt[diff] += 1
for d = 1 to n-1 do
    if cnt[d] <= 0 then
        print NOT JOLLY and return
print JOLLY
```

- Time Complexity:  $O(n)$ .

## H. Good Sequence

A sequence  $b$  is a good sequence if: *For each element  $x$  in  $b$ ,  $x$  has to occur exactly  $x$  times in  $b$ .* Given an array  $a$  containing  $N$  integers, find the minimum number of elements that need to be removed in order to make  $a$  a *good* sequence.

### Solution

- If an element  $x$  occurs less than  $x$  times in  $a$ , we cannot add more  $x$ 's in there. Then, the best move is to remove all occurrences of  $x$ .
- If an element  $x$  occurs  $y$  times such that  $y \geq x$ . Then, we should remove  $y - x$  occurrences.
- Let  $cnt_x$  be the number of  $x$ 's in  $a$ . If  $cnt_x < x$  then  $ans := ans + cnt_x$ . Otherwise,  $ans := ans + (cnt_x - x)$ .

```
for x = 1 to 1e9 do // Note: TLE => How can you optimize this?
    if cnt[x] < x then ans += cnt[x]
    else then ans += (cnt[x]-x)
print ans and return
```

- Time Complexity:  $O(n)$

## I. Two Sets

Divide all the numbers 1, 2, 3, ..., n into two sets of equal sum or state that it is impossible.

### Solution

- If the sum of all elements is odd, then it is obviously IMPOSSIBLE. Recall that the sum can be computed using the formula  $S_n = \frac{n(n+1)}{2}$
- Suppose we divide the numbers into two sets  $set_1$  and  $set_2$  respectively. The sum of all the elements in  $set_1$  is equal to  $set_2$ , which is also equal to  $\frac{S_n}{2}$ . Then, if we could construct the first set  $set_1$  such that the sum of its elements is equal to  $\frac{S_n}{2}$ , then the remaining elements in the original sequence can be used to construct the  $set_2$ .
- We can start by adding elements one-by-one into  $set_1$ , *starting with the largest elements first*, while keeping track of the sum  $sum_1$ . Consider an element  $x$  in the original set, if  $sum_1 + x > \frac{S_n}{2}$ , then we add  $x$  to  $set_2$ . Otherwise, we add  $x$  to  $set_1$ . If at any point  $sum_1 = \frac{S_n}{2}$ , we stop. The rest of the elements are added then to  $set_2$ .

```
total_sum = n*(n+1)/2
if total_sum is odd then
    print NO and return
half_sum = total_sum / 2
for x = N to 1 do
    if current_sum + x <= half_sum then
        add x to set_1
    else then
        add x to set_2
```

- Time Complexity:  $O(n)$

**Extra.** If you start by adding elements from 1 to  $n$  instead of the other way around, you would notice that the solution no longer works. Can you prove why this is the case?