

CG lab 2

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Exercise 2

(a)

$$u - v = (9, 8, 5) - (6, 6, 3) = (9 - 6, 8 - 6, 5 - 3) = (3, 2, 2)$$

(b)

$$u + 5v = (9, 8, 5) + 5(6, 6, 3) = (9 + 5 \times 6, 8 + 5 \times 6, 5 + 3 \times 5) = (39, 38, 20)$$

Exercise 5

$$|u| = \sqrt{(u \cdot u)} = \sqrt{(7, 3, 8) \cdot (7, 3, 8)} = \sqrt{(7 \times 7 + 3 \times 3 + 8 \times 8)} = \sqrt{122}$$

Exercise 11 (c)

$$G(f + g) - (G(f) + G(g)) = \int_0^1 (\sin x + e^x) dx - \left(\int_0^1 \sin x dx + \int_0^1 e^x dx \right) = 0$$

Exercise 12 (a)

$$a = \frac{u \cdot e}{|e|^2} e = (9/\sqrt{2} + 4/\sqrt{2})(1/\sqrt{2}, 1/\sqrt{2}) = (13/2, 13/2)$$

$$|a| = 13/\sqrt{2}$$

Exercise 16

$$\begin{aligned} \begin{pmatrix} 9 & 8 \\ -8 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 4 \\ 3 \end{pmatrix} \\ A = \begin{pmatrix} 9 & 8 \\ -8 & 9 \end{pmatrix} \text{ so } A^{-1} &= \frac{1}{9 \times 9 - (-8 \times 8)} \begin{pmatrix} 9 & -8 \\ 8 & 9 \end{pmatrix} = \begin{pmatrix} 9/145 & -8/145 \\ 8/145 & 9/145 \end{pmatrix} \\ \text{so } (x, y) &= \begin{pmatrix} 9/145 & -8/145 \\ 8/145 & 9/145 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = (12/145, 59/145) \end{aligned}$$

Exercise 20

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$$Q(f) = \int_0^1 \left(\frac{df}{dx} \right)^2 dx \quad \backslash \backslash$$

$$B(f, g) = \frac{1}{2} (Q(f+g) - Q(f) - Q(g)) = \quad \backslash \backslash$$

$$\frac{1}{2} \int_0^1 \left[\left(\frac{d(6x + e^{2x})}{dx} \right)^2 - \left(\frac{d(6x)}{dx} \right)^2 - \left(\frac{d(e^{2x})}{dx} \right)^2 \right] dx$$

$$= \int_0^1 \frac{d6x \cdot de^{4x}}{dx} dx$$

$$= \int_0^1 16 \cdot 2 \cdot e^{2x} = 6e^2 - 6$$

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$$Q(f) = \int_0^1 \left(\frac{df}{dx}\right)^2 dx$$

$$\begin{aligned} B(f, g) &= \frac{1}{2}(Q(f+g) - Q(f) - Q(g)) = \\ \frac{1}{2} \int_0^1 &\left[\left(\frac{d(6x + e^{2x})}{dx}\right)^2 - \left(\frac{d(6x)}{dx}\right)^2 - \left(\frac{d(e^{2x})}{dx}\right)^2\right] dx \\ &= \int_0^1 \frac{d6x \cdot de^x}{dx} dx \\ &= \int_0^1 6 \cdot 2 \cdot e^{2x} = 6e^2 - 6 \end{aligned}$$

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Exercise 22 (a)

$$A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5x_1 \\ 6x_2 \\ x_1 + x_2 \end{pmatrix}$$

$$\text{so } A = \begin{pmatrix} 5 & 0 \\ 0 & 6 \\ 1 & 1 \end{pmatrix}$$