PSTAT 122 Final project - Paper Airplane experiment

Andy Chen

March 2025

1 Introduction

Paper airplanes are an important educational tool in our student life, in grades K-12 or at college. For a long time, it has been used for exploring fundamental principles of aerodynamics and experimental design. Researchers have investigated how different factors, such as paper weight, wing design, and folding techniques, influence flight performance (Puspita et al. 2019). Among these factors, suspending items on the airplane can also significantly affect flight performance, such as the placement of stabilizing clips at different locations on the airplane—nose, middle, and rear. Understanding these effects is essential for optimizing paper airplane designs for educational purposes and competitions.

In this study, we applied a full factorial design to systematically assess the effects of nose, middle, and rear clips on flight distance. It allows for us to examine both the main effects and interaction effects between factors. With the result, we can have a comprehensive understanding of how different clip placements will affect the flight distance result(Puspita et al. 2019). In the previous study, the investigator analyzed and found that the modification of the plane's structure, like the change in paper weight and shape, can affect the flying distance significantly(Puspita et al. 2019). Thus, it is crucial to analyze how different clip placements contribute to these aerodynamic variations. Additionally, we conducted statistical power analysis to calculate the optimal sample size to detect any significant effect. Thus, the reasonableness of the experiment will be guaranteed(Das, Mitra, and Mandal 2016).

Since this experiment is an initial experimental study, it follows the procedure of a pilot study, which requires us to check the feasibility of our experiment and the validation of the setup before the large-scale investigation(van Teijlingen and Hundley 2002). Pilot study is a vital step in our experiment because it helps us to identify the potential methodological issues and improve them. Furthermore, we also imported the model checking techniques including residual analysis to ensure the validity of the linear regression model used to analyze the results (Lin, Wei, and Ying 2002). This technique helps us again confirm the reliability of statistical conclusions and ensure that assumptions such as normality and homoscedasticity are met (Lin, Wei, and Ying 2002).

The aim of this paper airplane study is to determine whether the placement of clips significantly affects flight distance and to investigate whether interactions among clip placements further influence the final performance. The result that is derived from the experiment will give us an insight into how structural modifications impact the stability of the plane and provide a stepping-stone for future studies on paper airplane aerodynamics.

2 Methods

2.1 Experimental Design

For this study, we use a full factorial design. We adjust the suspending position of paper clips to test the flight distance of the airplane. There are three independent variables in our experiment as nose clip(no/yes), middle clip(no/yes), and rear clip(no/yes). Since there are 2 selections in these variables, it allows us to compose a 2^3 treatment combination. To ensure statistical reliability, we do each treatment condition multiple times to avoid outliers, etc.

2.2 Materials and Procedure

I folded the paper plane according to Suzanne's procedure (Make:2020). An airplane is folded with the A4 paper with the regular size and regular weight to maintain consistency across trials.[1] Then, three clip placements were tested: nose, middle, and rear, where each could either be present or absent. Other external factors are strictly controlled(will discuss further) in order to minimize variability, making the final launching of the airplane more reliable. The primary response variable measured was flight distance in inches.

In the experiment, each experimental condition (clip placement combination) was tested across multiple replicates. The number of replicates was determined based on statistical power analysis. I placed a sample throw data, to ensure a sufficient sample size to capture significant effects(Das, Mitra, and Mandal 2016).

2.3 Constants to Be Maintained

To ensure that only the designated independent variables influenced flight distance, several key experimental constants were strictly controlled throughout our study:

- Environmental Conditions: In my experiment, I chose an underground parking lot as a testing space. To alleviate the influences of the external airflow from the running car, I started my experiment at night to have a better performance.

[1] The picture of the airplane is in Figure 10, section B.

- Throwing Consistency: Each airplane I threw was keeping a constant posture, and launch angle to minimize variation caused by human factors.[2]
- Paper plane construction: In my experiment, the airplane was folded using A4 size paper, which ensures the regular paper weight, length, and thickness across trials. Also, I completely followed the instructions of Suzanne's airplane, which ensures I had a consistent wing shape to have the exact same performance for each trial.
- Clip Placement: I marked the exact nose, middle, and rear position of each clip to make sure all the trials with them are aligned.

By maintaining these controlled variables, the study ensures that differences in flight distance can be attributed solely to the placement of stabilizing clips.

2.4 Sample Size calculation

To determine an appropriate sample size, I conducted an initial pilot experiment. The first step is to do a sample throw with each replicate of 2. Then with the collected data, I fitted a linear model to get the set standard error of coefficients 'beta_se', which was then used in power calculations. Finally, I chose appropriate standard errors to estimate the power between 2:10 replicates and formed a graph after that. Hence I can find the least replicates of my experiment.

Using these preliminary estimates, my final dataset was collected with an adjusted number of replicates. The final sample size was optimized to ensure that the statistical power exceeded 0.8, which is the least acceptable power for capturing the significance of effects.(Das, Mitra, and Mandal 2016).

2.5 Hypothesis test Setup

Before the statistical analysis, we need to set up the null and alternative hypotheses first.

The null hypothesis(H_0) is that:

$$H_0 : \mu_{\text{nose}} = \mu_{\text{middle}} = \mu_{\text{rear}} = \mu_{\text{interaction}} = 0 \tag{1}$$

The equation means that all the main effect(nose, middle, rear) and their interaction effect are not significant. Thus, the different flight distance is not affected by the position of the clips.

The alternative hypothesis is that at least one factor of the position of the clips will affect the flight distance. Based on the hypothesis, we will use the linear model to check the result with the p value on the summary.

[2] The test photo in Figure 11, section B

2.6 Statistical Analysis

For this experiment, I introduced a linear regression model to assess the impact of clip placements on flight distance:

$$\text{distance} = \beta_0 + \beta_1\text{nose} + \beta_2\text{middle} + \beta_3\text{rear} \tag{2}$$

The β's are the coefficients to be determined. Based on this model, the interaction terms are allowed for the evaluation of combined effects between different clip placements. We will also use the ANOVA model to check the full model and the model with only the main effect to check the significance of the interaction term. With the interaction term, the model is:

$$\begin{aligned}\text{distance} = \beta_0 &+ \beta_1\text{nose} + \beta_2\text{middle} + \beta_3\text{rear} + \beta_1\beta_2\text{nose\&middle} \\ &+ \beta_1\beta_3\text{nose\&rear} + \beta_2\beta_3\text{middle\&rear} + \beta_1\beta_2\beta_3\text{nose\&middle\&rear}\end{aligned}$$

We will compare this model with the main effect model(Equation 2).

To check the reasonableness of the model, a model-checking technique was performed using residual analysis and permutation testing to validate the assumptions of the linear model (Lin, Wei, and Ying 2002). Also, to determine the necessary sample size for achieving sufficient power, a power analysis was conducted using estimates from the fitted model, and I will not reiterate it here[3](Das, Mitra, and Mandal 2016).

2.7 Model Validation

To verify the assumptions of the linear model, I plotted residual plots and Q-Q plots to assess normality and homoscedasticity. The model check helps to guarantee the factorial design effectively captured meaningful relationships between clip placement and flight distance (Lin, Wei, and Ying 2002).

3 Results

Previous note:
The full code of any graph or knitted kable is shown in Appendix Section B to make the result analysis more concise. All of the code is successfully run and the graph is directly extracted from the code.

3.1 Sample Size calculation

As discussed in Das et al. (2016), ensuring adequate sample size is critical for maintaining statistical power and minimizing errors in experimental design. Therefore, to ensure the experiment had sufficient statistical power to detect significant effects, a power analysis was conducted based on the sample throws

[3]refer to section 2.4

with the estimates of standard errors (beta_se) as shown in Figure 1.[4] The Figure 2 below explains the relationship between the number of replicates per condition and the resulting power for three different values of beta_se.

distance	nose	middle	rear
252	no	no	no
216	no	no	no
92	no	no	yes
95	no	no	yes
209	no	yes	no
200	no	yes	no
181	no	yes	yes
279	no	yes	yes
278	yes	no	no
265	yes	no	no
192	yes	no	yes
178	yes	no	yes
248	yes	yes	no
290	yes	yes	no
199	yes	yes	yes
217	yes	yes	yes

Figure 1: Sample throw data

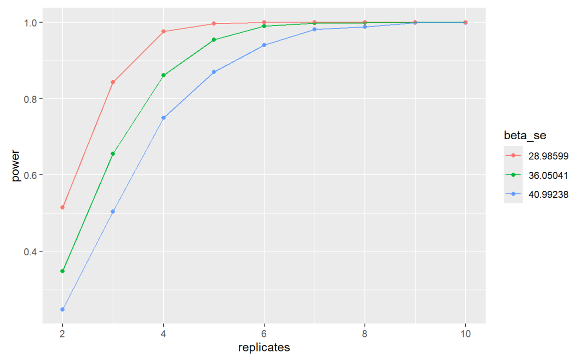


Figure 2: Sample size calculation of the paper airplane

The three stand error values are all extracted from the linear regression model function. Among them, the red line(28.98599) is the lower bar value, the green(36.05041) is the mean bar value, and the blue(40.99238) is the higher bar value. Since I deem the lower and higher bar value may be extreme and cannot represent the whole data, I finally select the mean standard error. For the estimated standard error of 36.05041, we can see the power surpasses 0.8 at a sample size equal to 4. This result gives us the final complete dataset 'final_project_adjusted_r.csv', which ensures reliable detection of factorial effects.

With the expected sample size, the experiment holds the reasonableness and feasibility balance, and also reduces unnecessary trials while ensuring accurate estimation of the effects of clip placements.

3.2 Descriptive Statistics

3.2.1 Visual representation of flight distance

To illustrate the effect of clip position, Figure 3 and Figure 4 present the distribution and interaction patterns of flight distance.

From the Interaction plot of Clip placements on Flight Distance(Figure 3). Two vital patterns are noticeable. One is that, when the middle clip and the rear clip are all present, the negative impact from the rear clip is significantly mitigated. This conveys a strong interaction effect between the middle and rear clips, as the middle clip can compensate for the loss by the rear clips. The other one is that the nose clip's effect remains relatively consistent across conditions, which suggests it has an independent contribution to the flight distance.

[4]The procedure refers to section 2.4

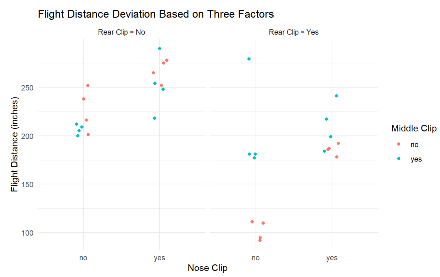


Figure 3: Interaction Plot of Clip Placements on Flight Distance

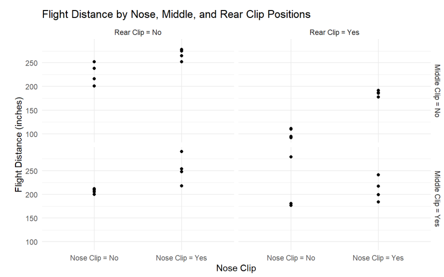


Figure 4: Flight Distance by Nose, Middle, and Rear Clip Positions

From Figure 4, we can clearly observe that when the rear clip is absent(Rear clip = No), the airplane could have longer flight distances. However, when the rear clip is added(Rear clip = Yes), the fight distance is significantly reduced, which supports the regression findings. The nose clip does have a positive contribution to the flight distance, as the flight distance becomes higher in the data points when Nose clip = Yes.

3.2.2 Regression model result

The regression analysis provides a quantitative understanding of the trends observed in the visual representations, we use the ‘lm()‘ function to explain the full model of my paper airplane data. Figure 5 below shows the summary of each group:

Table 1: Result of linear regression model of paper airplane

Variable	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	226.75	12.1913	18.5993	9.24e-16
noseyes	40.75	17.2411	2.3635	0.0265
middleyes	-20.25	17.2411	-1.1745	0.2517
rearyes	-124.75	17.2411	-7.2356	1.78e-07
noseyes:middleyes	5.25	24.3826	0.2153	0.8313
noseyes:rearyes	43.00	24.3826	1.7636	0.0905
middleyes:rearyes	122.75	24.3826	5.0343	3.81e-05
noseyes:middleyes:rearyes	-83.25	34.4822	-2.4143	0.0238

Figure 5: Linear Regression Model for Paper Airplane Flight Distance

From the table, we can see some trends of each clip that are significant:

- Nose clip effect: When the nose clip is attached, the final flight distance has a significant increase by 40.75 inches (p = 0.0265). This result matched our findings in Figure 4, where the distribution of flight distances shifts upward when a nose clip is applied, especially when the rear clip is not present.

- Rear clip effect: The rear clip has the most significant negative impact on the flight performance when it is applied on the plane(coefficient = -124.75), by reducing flight distance by 124.75 inches (p = 1.78e-07). This finding also matches the conclusion in Figure 4, where the right facet (Rear Clip = Yes) shows a clear reduction in flight distances across all conditions. It is also verified by the experiment with Puspita et al. (2019), who observed weight distribution and shape modifications affect flight performance, finding that heavier configurations can significantly alter aerodynamic behavior. Our results extend this analysis by demonstrating that a rear clip placement, which shifts weight backward, can significantly reduce flight distance.

- Middle clip effect: The middle clip alone is not statistically significant by our result (p = 0.2517), but its interaction with the rear clip (p = 3.81e-05) is highly significant. As we concluded in the interaction plot(Figure 3), the negative impact of rear clip is alleviated when a middle clip is also present, which suggests a compensatory effect.

The next table shows the overall model fit statistics, as shown in Figure 6:

Table 2: Overall result for the paper airplane data

Statistic	Value
Multiple R-squared	0.8316
Adjusted R-squared	0.7825
Overall p-value	7.18e-08

Figure 6: Overall Model Fit for Paper Airplane Flight Distance

As the table shown, the model demonstrates a strong power of explanation, with an R-squared value of 0.8316, which means that 83.16% of the variance in flight distance is explained by the clip placements. According to Lin et al. (2002), model checking on the residuals are crucial for validating the assumptions of linear regression. Our model validation in the next section confirms that while minor deviations from normality exist, they do not significantly affect the conclusions.

Additionally, the overall p-value (p = 7.18e-08) confirms that at least one of the clip placements significantly influences flight performance. And the impact from the rear clip has proven the accuracy of the overall p-value.

I also used the ANOVA function to check whether the interaction term is significant as I applied a model with no interaction to test with interation, the result in shown in the table (Figure 7):

Table 3: ANOVA result for the interaction coefficients

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
28	33546.12	NA	NA	NA	NA
24	14268.25	4	19277.88	8.10662	0.00028

Figure 7: Interaction of paper plane coefficients

From the ANOVA table, the overall p-value is 0.00028 which is less than 0.05, so the interaction between the factors are highly significant. Thus, we cannot drop the interaction term to analysis the model.

To conclude these statistical results with the graphs, we confirm that clip placements strongly dictate the flight performance of paper airplanes, where the nose clip will contribute to the flight distance and the rear clip vice versa. The middle clip is not that significant, but it has an interaction effect on the rear clips. Also, by the strong statistical significance (p less than 0.001 for multiple factors) and the high R-squared value(0.8316), we reject the null hypothesis and conclude that clip placements significantly affect flight performance.

3.3 Model Validation

To ensure the reliability and validity of the regression model, I conducted two key diagnostics: residual analysis and normality testing. Also, the Shapiro-Wilk test is also applied in this model.

3.3.1 Checks on the models

A residual vs. fitted values plot (Figure 9) was generated to examine the homoscedasticity and potential model misspecifications.

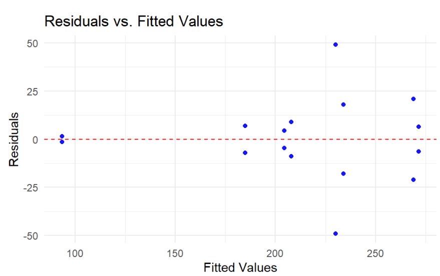


Figure 8: Residuals vs. Fitted Values Plot

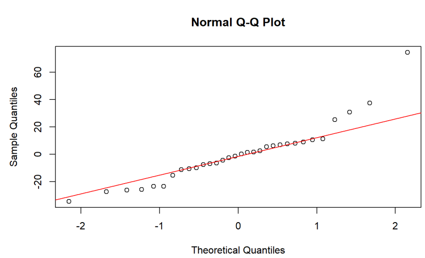


Figure 9: Q-Q Plot of Residuals

In the ideal situation, if the residuals are randomly scattered around zero without any systematic pattern, we can say the homoscedasticity is not violated. As we can see from the plot, the residuals appear to be randomly distributed, but we can see a clear funnel shape which indicates that the assumption of constant variance (homoscedasticity) is violated. Nevertheless, a few extreme residuals have deviated from the line, which suggests potential outliers. Hence, we need to do the further examination on the p value to check validity,

Then, I checked the normality of the residuals by the QQ plot, as shown in Figure 8. On the graph, we observe that most residuals align well with the theoretical quantiles, but there is a heavy tail that deviated from the line. This implies that the assumption of normality may not be met. So I applied the Shapiro-Wilk test to check the p-value and got a p-value of 0.04216, which is slightly below the conventional significance threshold of 0.05. That suggests again the normality of the residuals may be violated. So to ensure the validity of the model, we applied a permutation test to validate the significance.

3.3.2 Permutation test

The basic principle of Permutation Test is to randomly disrupt the dependent variable (flight distance), then we recalculate the F statistic to see whether the randomly arranged data can produce a larger F value than the original data. The p-value is calculated by:

$$p = \frac{\text{times of the F value in the permutation test exceed the original value}}{\text{Total repetition times}} \tag{3}$$

Based on this principle, I wrote a code to meet the requirement of the permutation test:

```
# Permutation test
set.seed(123)
perm_f <- NA
reps <- 100000

for(i in 1:reps){
  perm_data <- df
  perm_data$distance <- sample(perm_data$distance)

  perm_model <- lm(distance ~ nose * middle * rear,
        data = perm_data)
  perm_f[i] <- summary(perm_model)$fstatistic[1]
}

# Permutation p-value
model <- lm(distance ~ nose * middle * rear, data = df)
F <- summary(model)$fstatistic[1]
perm_p_value <- sum(perm_f >= F) / reps
```

I set the repetition time to 100000 times to find any different p values in the test. However, the result p-value is shown as 0, meaning that the p-value of the 100,000 randomized datasets produced an F-statistic is less than 1e-06, providing strong evidence that the observed effects are not due to random variation and the overall p-value is truly significant. This confirms that the observed relationships in our model are highly unlikely to have occurred due to random chance.

Hence, by the result of residual diagnostics, normality assessment, and permutation testing. I can conclude that the linear regression model is valid for drawing meaningful inferences. Though there are some outliers that affect normality(which will be discussed further in the discussion part), they will not affect the significance of this experiment.

4 Discussion

4.1 Summary of Findings

In this study, we used a 2^3 factorial design to investigate the effect of clip placements on the flight distance of the paper airplane. The findings reveal distinct influences of different clip positions on flight performance, as we rejected the null hypothesis of the experiment since significant main effects and interaction effects were observed.

To be more specific, we found that the rear clip had a significant negative impact on the flight distance as it reduced the distance by 124.75 inches with an extremely small p value (1.78e-07). The result is similar with the experiment by Puspita et al. (2019), which indicated that weight distribution can influence aerodynamic stability and reduce overall flight performance.

Conversely, the nose clip significantly increased flight distance by 40.75 inches (p=0.0265), which also had a significant effect. The middle clip did not show a statistically significant impact on flight distance, which is only placed on the airplane(p=0.2517). However, its interaction with the rear clip was highly significant(p = 3.81e-05), which moderated the negative effect by the clip on the rear.

In terms of overall model performance, we used the linear regression model to explain the data. From the model, it explained a strong R-squared value(0.8316), and with a significant overall p-value(7.18e-08). For the ANOVA test on the interaction term, the p-value is also highly significant(0.00028). They all confirm that at least one clip placement configuration has a substantial impact on flight performance. These findings provide strong evidence that clip placement is a crucial factor in determining paper airplane flight distance.

4.2 Model Assumption and Validation

To ensure the reliability of our regression model, we conducted model diagnostics to assess whether the assumptions of linear regression were met. In the residual vs. fitted values plot, the distribution reveals heteroscedasticity as the funnel shape shown, which suggests the assumption of constant variance is not satisfied. Also, the residuals appeared to be randomly scattered around zero, suggesting that there were possible violations of the independence assumption, which were present. Simultaneously, we applied the QQ plot to check the normality of the residuals. The results showed minor deviations from normality, particularly for the heavy tail. So we conducted a Shapiro-Wilk test with a p-value of 0.0421, suggesting some degree of non-normality. Lin et al. (2002) emphasize that slight deviations do not necessarily compromise the validity of a regression model as long as the residuals do not exhibit strong systematic patterns. With the subsequent permutation test with a value of 0, we can say that the minor deviations in residuals are unlikely to affect the overall conclusions of the study.

4.3 Limitation of the experiment

While our factorial design experiment gives us insight into the effect of the clip placement on paper airplane flight distance, we can see there are some outliers in our experiment, which makes our residuals not normal. After that, I analyzed some factors that led to these results, which created some limitations. For example, to make the experiment result more accurate, the ideal place to test should be an indoor environment without any wind interference. Though I chose the underground parking lot as the field, it is still not a perfect environment since we cannot prevent all the air currents, no matter how strong or weak. Thus, the outliers may exist in this situation and the limitation leads to that.

The other possible situation is that, though we tried to standardize the throwing position, and the angle of each airplane launch, minor variations inevitably existed between throws, like the initial velocity, and the force driven from me. etc These small inconsistencies, introduced by human operators, could slightly influence flight distances, adding some random variability to the measurements. Hence the outlier can exist.

Last but not least, we employed paper airplanes constructed from a single type of paper with uniform weight and dimensions in this study. While this choice effectively eliminated variability arising from material differences, the limitation of our experiment exists since we can only conclude about the type of paper we choose and cannot extend the scope. Different paper materials (varying thickness, rigidity, or texture) or structural designs could exhibit different aerodynamic properties, potentially altering the influence of clip placements (Puspita et al., 2019)

To conclude, if we have further study in the future we should consider testing a broader range of environmental conditions and applied mechanical launching devices for better control. Also, we may also explore different materials and airplane structures to improve the robustness and generalizability of findings. Through these further approaches, we can have a broad conclusion to our study, and may contribute not only to the statistical field but also to aerodynamic designs.

A Reference

Das, Sabyasachi, Koel Mitra, and Mohanchandra Mandal. Sample Size Calculation: Basic Principles. Indian Journal of Anaesthesia 60, no. 9 (2016): 652. https://doi.org/10.4103/0019-5049.190621.

Lin, D. Y., L. J. Wei, and Z. Ying. Model-Checking Techniques Based on Cumulative Residuals. Biometrics 58, no. 1 (March 2002): 1–12. https://doi.org/10.1111/j.0006-341X.2002.00001.x.

Make: DIY Projects and Ideas for Makers. "World's Best Paper Airplane - Make:," June 8, 2020. https://makezine.com/projects/worlds-best-paper-airplane/.

Puspita, Angella Natalia Ghea, Laksmi Ambarwati, Eki Ludfiyanti, Dimas Prabu Tejonugroho, and Yunizar Zen. Effect of Paper Weight, Paper Length, and Nose of Paper Plane on Aircraft Mileage in Paper Airplane Game (2019).

van Teijlingen, Edwin, and Vanora Hundley. The Importance of Pilot Studies. Nursing Standard 16, no. 40 (2002): 33.

B Experiment photos

Figure 10: Folded Suzzane paper plane

Figure 11: Paper plane test

C R code

PSTAT 122 Final project

Andy Chen

2025-03-19

```r
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
source("C:/Users/yican/Downloads/power_factorial_23.R")
library(ggplot2)
sample_df <- read.csv("final_project_r.csv", stringsAsFactors = TRUE)

sample_df <- sample_df %>%
  mutate(nose = factor(nose, levels = c("no", "yes")),
         middle = factor(middle, levels = c("no", "yes")),
         rear = factor(rear, levels = c("no", "yes")))

model <- lm(distance ~ nose * middle * rear, data = sample_df)
coeff_table <- summary(model)$coefficients[-1, ]

# Sample size calcuation
beta_mean <- coef(model)
beta_mean
```

```
##               (Intercept)                   noseyes                 middleyes
##                     234.0                      37.5                     -29.5
##                   rearyes         noseyes:middleyes           noseyes:rearyes
##                    -140.5                      27.0                      54.0
##         middleyes:rearyes noseyes:middleyes:rearyes
##                     166.0                    -140.5
```

```r
coef <- summary(model)$coefficients[, "Std. Error"]
coef
```

```
##               (Intercept)                   noseyes                 middleyes
##                  20.49619                  28.98599                  28.98599
##                   rearyes         noseyes:middleyes           noseyes:rearyes
##                  28.98599                  40.99238                  40.99238
##         middleyes:rearyes noseyes:middleyes:rearyes
##                  40.99238                  57.97198
```

```r
beta_se <- rep(mean(summary(model)$coefficients[, "Std. Error"]),8)
beta_se
```

```
## [1] 36.05041 36.05041 36.05041 36.05041 36.05041 36.05041 36.05041 36.05041
```

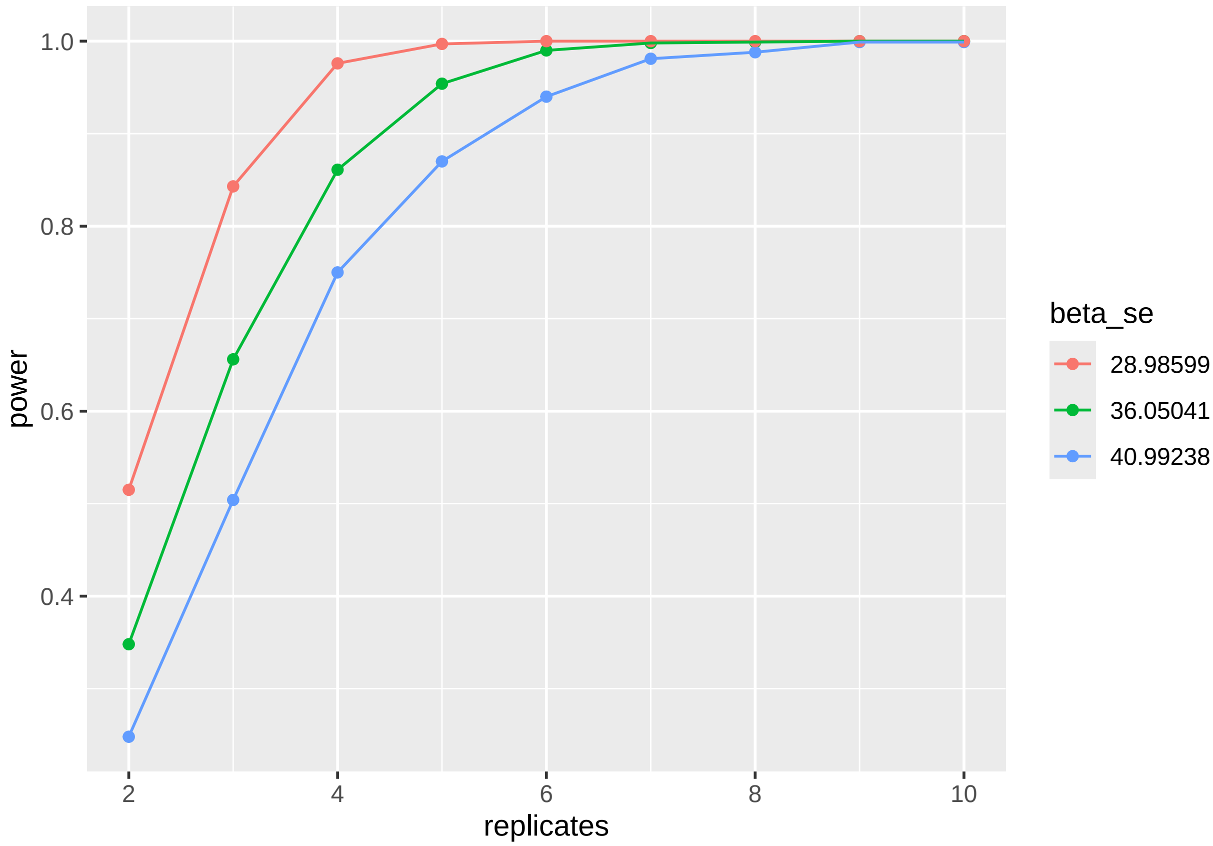
```r
coeff_table
```

```
##                           Estimate Std. Error   t value    Pr(>|t|)
## noseyes                       37.5   28.98599  1.293729 0.231858868
## middleyes                    -29.5   28.98599 -1.017733 0.338592599
## rearyes                     -140.5   28.98599 -4.847170 0.001276438
## noseyes:middleyes             27.0   40.99238  0.658659 0.528611813
## noseyes:rearyes               54.0   40.99238  1.317318 0.224208560
## middleyes:rearyes            166.0   40.99238  4.049533 0.003686599
## noseyes:middleyes:rearyes   -140.5   57.97198 -2.423585 0.041616678
```

```r
set.seed(123)
replicates = 2:10
power1 <- NA
for(i in 1:length(replicates)){
  power1[i] <- power_factorial_23(beta_mean, beta_se, replicates[i])
}
power1
```

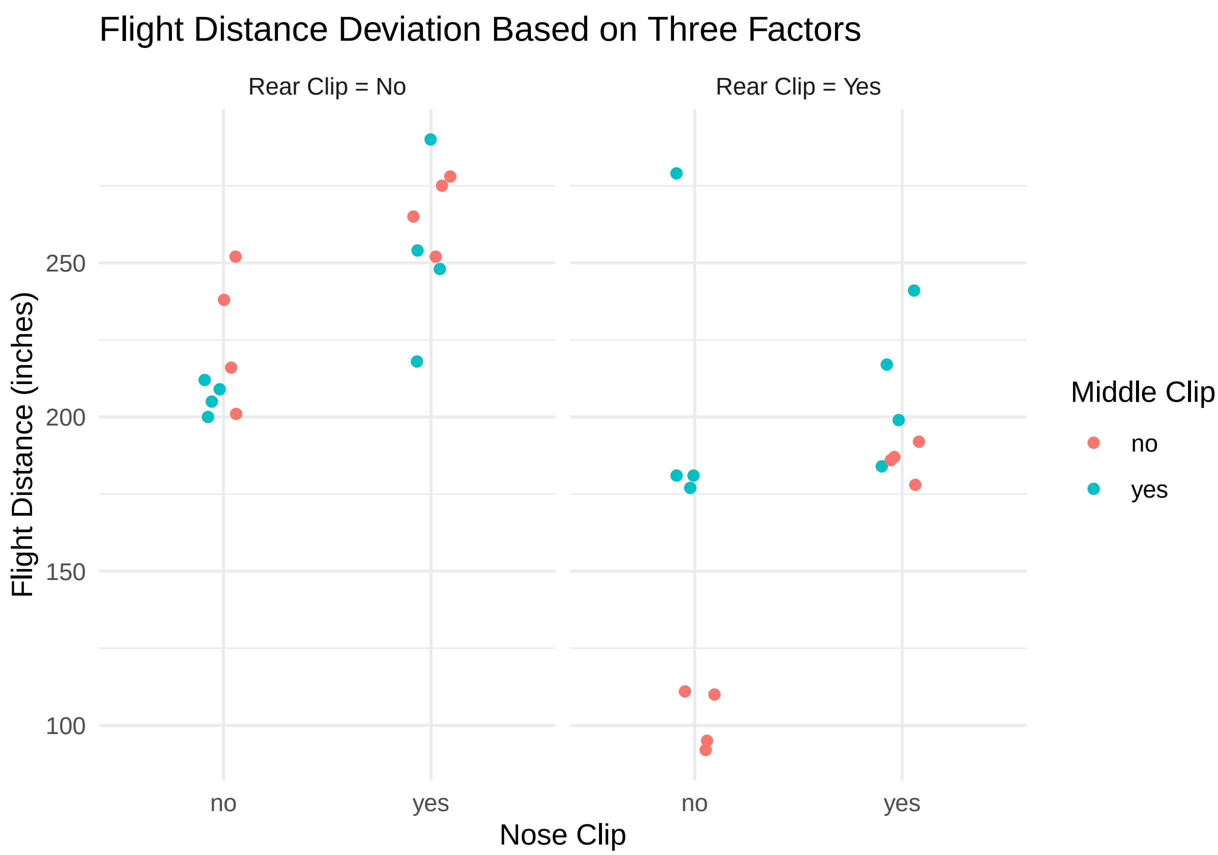
```
## [1] 0.348 0.656 0.861 0.954 0.990 0.998 0.999 1.000 1.000
```

```r
beta_se <- rep(40.99238,8)
power2 <- NA
for(i in 1:length(replicates)){
  power2[i] <- power_factorial_23(beta_mean, beta_se, replicates[i])
}

beta_se <- rep(28.98599, 8)
power3 <- NA
for(i in 1:length(replicates)){
  power3[i] <- power_factorial_23(beta_mean, beta_se, replicates[i])
}

all_power <- data.frame(
  power = c(power1, power2, power3),
  beta_se = c(rep("36.05041", length(power1)),
              rep("40.99238", length(power2)),
              rep("28.98599", length(power3))),
  replicates = rep(replicates, 3)
)

ggplot(data = all_power, mapping = aes(x=replicates, y = power, group = beta_se, color=beta_se)) +geom_
```

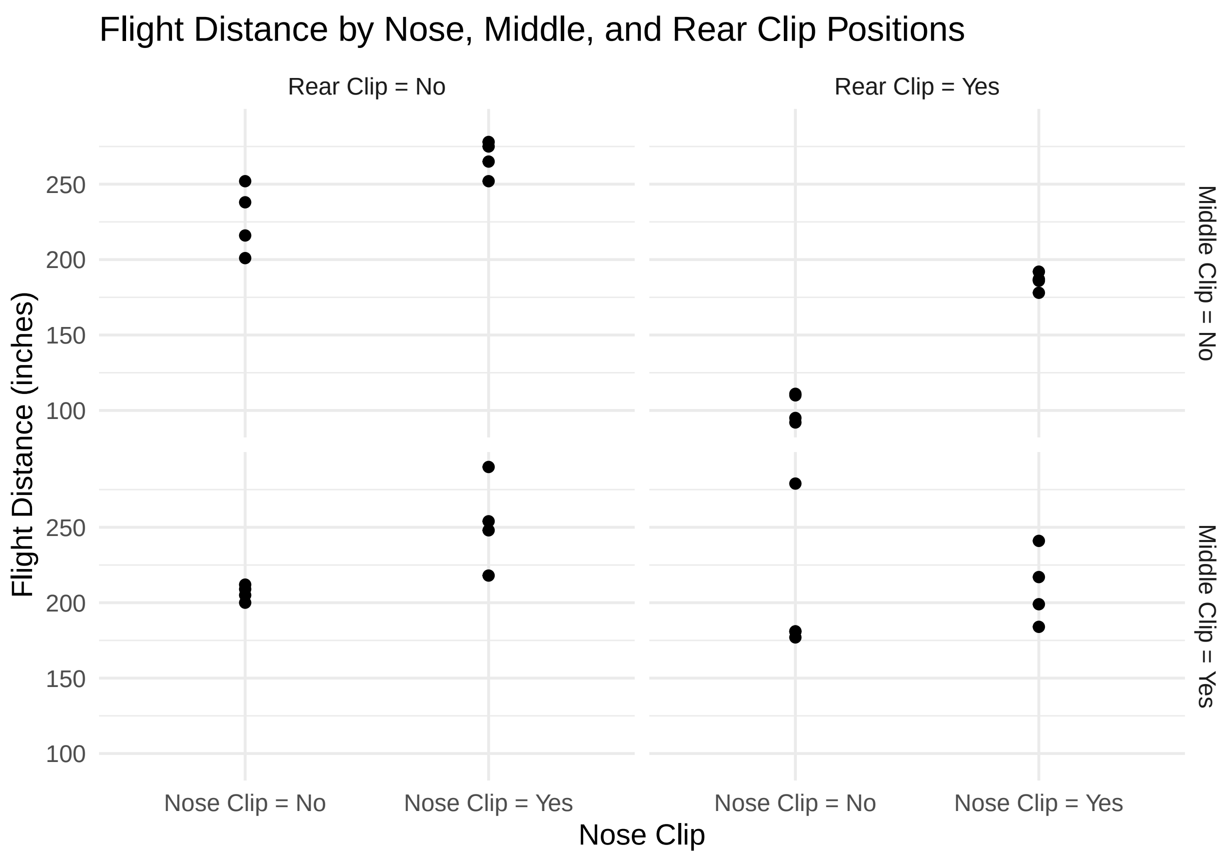



```r
## Full data
df <- read.csv("final_project_adjusted_r.csv", stringsAsFactors = TRUE)

df <- df %>%
  mutate(nose = factor(nose, levels = c("no", "yes")),
         middle = factor(middle, levels = c("no", "yes")),
         rear = factor(rear, levels = c("no", "yes")))
```

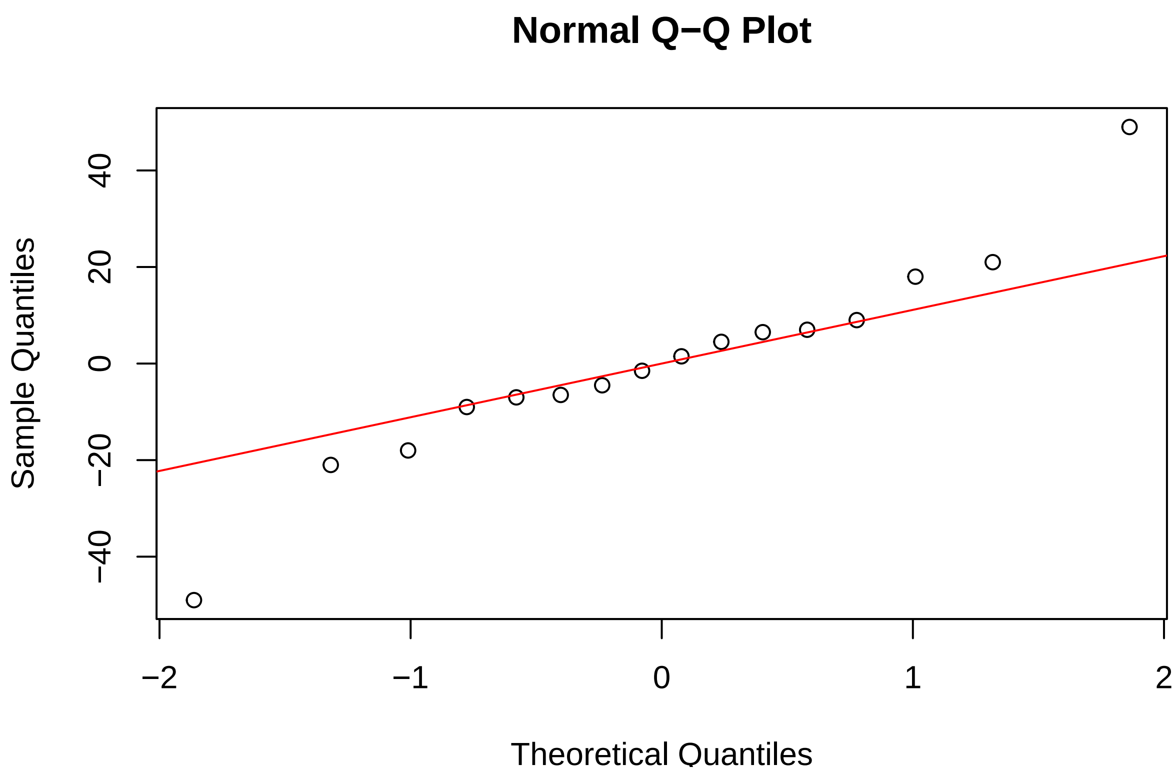
```r
library(ggplot2)
library(dplyr)

# Interaction plot
ggplot(df, aes(x = nose, y = distance, color = middle)) +
  geom_jitter(width = 0.1, height = 0) +
  facet_grid(cols = vars(rear),
             labeller = labeller(rear = c("no" = "Rear Clip = No",
                                          "yes" = "Rear Clip = Yes"))) +
  labs(title = "Flight Distance Deviation Based on Three Factors",
       x = "Nose Clip", y = "Flight Distance (inches)", color = "Middle Clip") +
  theme_minimal()
```



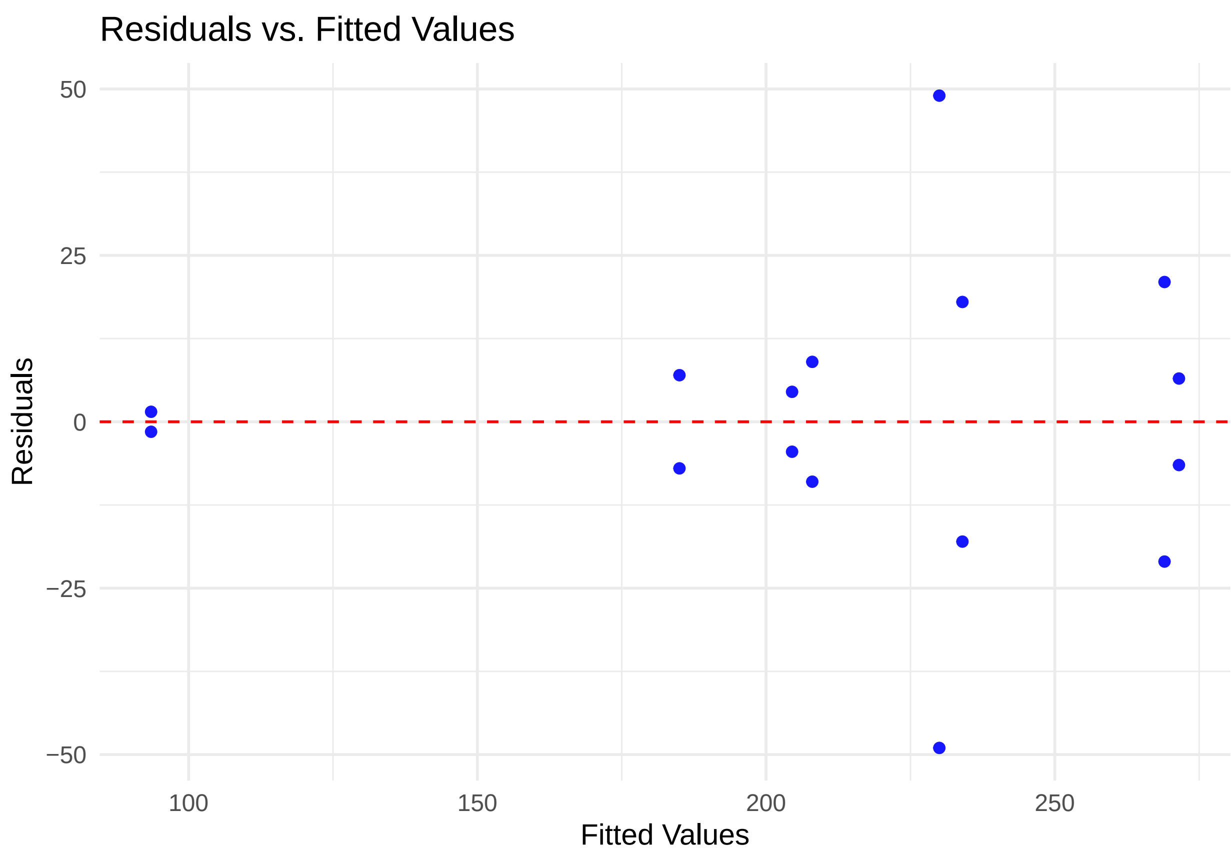

```r
# Facet Grid
ggplot(df, aes(x = nose, y = distance)) +
  geom_point() +
  facet_grid(middle ~ rear,
             labeller = labeller(middle = c("no" = "Middle Clip = No",
                                            "yes" = "Middle Clip = Yes"),
                                 rear = c("no" = "Rear Clip = No",
                                          "yes" = "Rear Clip = Yes"))) +
  labs(title = "Flight Distance by Nose, Middle, and Rear Clip Positions",
       x = "Nose Clip", y = "Flight Distance (inches)") +
  scale_x_discrete(labels = c("no" = "Nose Clip = No", "yes" = "Nose Clip = Yes")) +
  theme_minimal()
```




```r
# Residuals test
#QQ plot
qqnorm(resid(model))
qqline(resid(model), col = "red")
```




```r
df$residuals <- resid(model)
df$fitted_values <- fitted(model)

# Residuals vs. Fitted Value Plot
ggplot(df, aes(x = fitted_values, y = residuals)) +
  geom_point(alpha = 0.7, color = "blue") +
  geom_hline(yintercept = 0, linetype = "dashed", color = "red") +
  labs(title = "Residuals vs. Fitted Values",
       x = "Fitted Values",
       y = "Residuals") +
  theme_minimal()
```




```r
# Shapiro-Wilk test
shapiro_test <- shapiro.test(df$residuals)
shapiro_test
```

```
##
##  Shapiro-Wilk normality test
##
## data:  df$residuals
## W = 0.93115, p-value = 0.04216
```

```r
# Permutation test
set.seed(123)
perm_f <- NA
reps <- 100000

for(i in 1:reps){
  perm_data <- df
  perm_data$distance <- sample(perm_data$distance)

  perm_model <- lm(distance ~ nose * middle * rear, data = perm_data)
  perm_f[i] <- summary(perm_model)$fstatistic[1]
}

# Permutation p-value
model <- lm(distance ~ nose * middle * rear, data = df)
F <- summary(model)$fstatistic[1]
perm_p_value <- sum(perm_f >= F) / reps
perm_p_value
```

```
## [1] 0
```

```r
summary(model)
```

```
##
## Call:
## lm(formula = distance ~ nose * middle * rear, data = df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -34.500 -10.875  -0.625   7.625  74.500
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)                  226.75      12.19  18.599 9.24e-16 ***
## noseyes                       40.75      17.24   2.364   0.0265 *
## middleyes                    -20.25      17.24  -1.175   0.2517
## rearyes                     -124.75      17.24  -7.236 1.78e-07 ***
## noseyes:middleyes              5.25      24.38   0.215   0.8313
## noseyes:rearyes               43.00      24.38   1.764   0.0905 .
## middleyes:rearyes            122.75      24.38   5.034 3.81e-05 ***
## noseyes:middleyes:rearyes    -83.25      34.48  -2.414   0.0238 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 24.38 on 24 degrees of freedom
## Multiple R-squared:  0.8316, Adjusted R-squared:  0.7825
## F-statistic: 16.93 on 7 and 24 DF,  p-value: 7.177e-08
```

```r
summary_model <- summary(model)

coeff_table <- as.data.frame(summary_model$coefficients)

library(knitr)
coeff_table <- cbind(Variable = rownames(coeff_table), coeff_table)
rownames(coeff_table) <- NULL

coeff_table$`Pr(>|t|)` <- ifelse(coeff_table$`Pr(>|t|)` < 0.001,
                                  formatC(coeff_table$`Pr(>|t|)`, format = "e", digits = 2),
                                  round(coeff_table$`Pr(>|t|)`, 4))

kable(coeff_table, digits = 4, caption = "Result of linear regression model of paper airplane")
```

Table 1: Result of linear regression model of paper airplane

Variable	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	226.75	12.1913	18.5993	9.24e-16
noseyes	40.75	17.2411	2.3635	0.0265
middleyes	-20.25	17.2411	-1.1745	0.2517
rearyes	-124.75	17.2411	-7.2356	1.78e-07
noseyes:middleyes	5.25	24.3826	0.2153	0.8313
noseyes:rearyes	43.00	24.3826	1.7636	0.0905
middleyes:rearyes	122.75	24.3826	5.0343	3.81e-05
noseyes:middleyes:rearyes	-83.25	34.4822	-2.4143	0.0238

```
p_value <- pf(summary_model$fstatistic[1],
              summary_model$fstatistic[2],
              summary_model$fstatistic[3],
              lower.tail = FALSE)

formatted_p_value <- ifelse(p_value < 0.001, formatC(p_value, format = "e", digits = 2), round(p_value,

overall_stats <- data.frame(
  `Statistic` = c("Multiple R-squared", "Adjusted R-squared", "Overall p-value"),
  `Value` = c(round(summary_model$r.squared, 4),
              round(summary_model$adj.r.squared, 4),
              formatted_p_value)
)


kable(overall_stats, digits = 4, caption = "Overall result for the paper airplane data")
```

Table 2: Overall result for the paper airplane data

Statistic	Value
Multiple R-squared	0.8316
Adjusted R-squared	0.7825
Overall p-value	7.18e-08

```
no_interaction_model <- lm(distance ~ nose + middle + rear, data = df)
anova_results <- anova(no_interaction_model, model)

kable(anova_results,digits = 5, caption = "ANOVA result for the interaction coefficients")
```

Table 3: ANOVA result for the interaction coefficients

Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
28	33546.12	NA	NA	NA	NA
24	14268.25	4	19277.88	8.10662	0.00028