RBE550: Motion Planning

Project 1: Fundamentals

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1. Question 1:

   We have a point (x, y) in H. If rotate point (x, y) a θ degree with the origin we got (x', y'). Then consider the below matrix.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xcos(\theta) - ysin(\theta) \\ xsin(\theta) + ycos(\theta) \end{bmatrix}$$

   For the new point (x', y') satisfy $x'^2 + y'^2 \leq 1$

   Since then, according to the above matrix, we have

   $(x\cos\theta - y\sin\theta)^2 + (x\sin\theta + y\cos\theta)^2 \leq 1$ then we have $x^2(\sin^2\theta + \cos^2\theta) + y^2(\sin^2\theta + \cos^2\theta) \leq 1$

   Since we know that $\sin^2\theta + \cos^2\theta = 1$ So $x^2 + y^2 \leq 1$

   Then, we proved that the transformed primitive is unchanged.

2. Question 2:

Algorithm 1 segment pair intersection

function SEGMENT PAIR INTERSECTION($line1$, $line2$)
  $((x1, y1), (x2, y2)) \leftarrow line1$
  $((x3, y3), (x4, y4)) \leftarrow line2$
  $denom \leftarrow (y4 - y3) \times (x2 - x1) - (x4 - x3) \times (y2 - y1)$
  if $denom = 0$ **then return** `None`
  $ua \leftarrow \frac{(x4-x3)\times(y1-y3)-(y4-y3)\times(x1-x3)}{denom}$
  $ub \leftarrow \frac{(x2-x1)\times(y1-y3)-(y2-y1)\times(x1-x3)}{denom}$
  if $0 \leq ua \leq 1$ & $0 \leq ub \leq 1$ **then**
    $ix \leftarrow x1 + ua \times (x2 - x1)$
    $iy \leftarrow y1 + ua \times (y2 - y1)$ **return** (ix, iy)
  return `None`

   Explanation:
   First, we assign coordinates to line 1 and line 2.

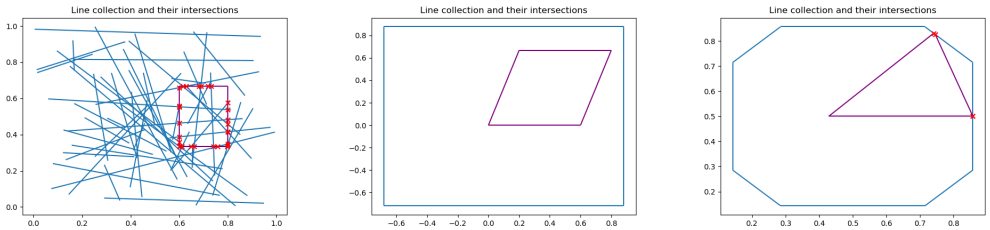
   Second, we compute the cross product of two lines, which is the function's denominator.

   Third, determine whether parallel. If the denominator is zero from the product theorem, we can conclude that two lines are parallel to each other.

   Fourth, calculate the distance between two line segments. If results lie in $[0 - 1]$, then we can conclude that the line segments intersect with each other.

3. Question 3:

   (a) Q3 (1)



(a) Test1 (b) Test2 (c) Test3

Figure 1: Test Results

(b) Q3 (2)

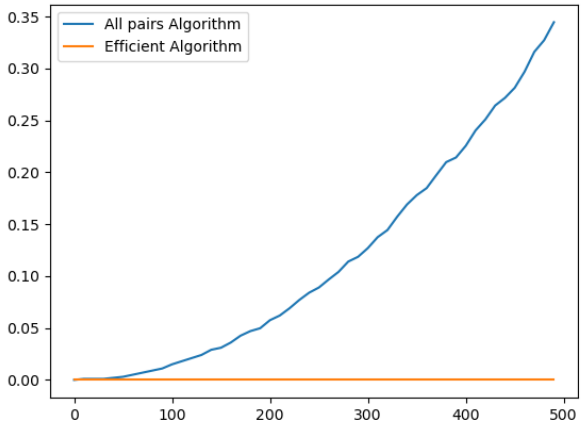


Figure 2: Time Complexity

The time complexity is $O(N^2)$

Explanation: To check each segment we have $N-1$ which is $N \times (N-1)$ which is $O(N^2)$