RBE550: Motion Planning

Project 3: Sampling-Based Planners

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Question 1

Visibility Graph Consider an environment with a limited number of polygonal obstacles where the objective is to find the shortest path between two points. Here, the visibility graph will excel because it guarantees optimality in such settings.

PRM: Consider a high-dimensional robot arm in a cluttered environment. Here, PRM would be more suitable because constructing a visibility graph for each vertex of such a complex configuration space would be computationally prohibitive. The probabilistic nature of PRM allows it to capture the connectivity of such complex spaces without exhaustively analyzing every possible configuration.

Conclusion: If task is in a relatively simple environment with polygonal obstacles and want to find the shortest path, the visibility graph is a better choice. But if task is in a more complex or high-dimensional environment where optimality is a secondary concern, PRM is often more practical.

Question 2

Definitions

Prismatic Joint (P):
Its configuration space is represented by an interval on the real line:

$$C_P = [a, b]$$

where a and b are the limits of the linear motion.

Revolute Joint (R):
Its configuration space is represented by a circle, typically parameterized by an angle θ:

$$C_R = \{\theta \mid 0 \leq \theta < 2\pi\}$$

Manipulators

Here each θ represents an angle for the revolute joints, and x represents the linear position for the prismatic joint. The intervals $[a_1, b_1]$ and $[a_2, b_2]$ represent the linear motion limits for the prismatic joints in the first manipulator, while $[a, b]$ represents the limit for the prismatic joint in the third manipulator.

Manipulator with Two Prismatic Joints:
Its configuration space will be a Cartesian product of two intervals:

$$C_{P1,P2} = [a_1, b_1] \times [a_2, b_2]$$

Manipulator with Three Revolute Joints:
Its configuration space will be a Cartesian product of three circles:

$$C_{R1,R2,R3} = \{(\theta_1, \theta_2, \theta_3) \mid \theta_i \in [0, 2\pi) \text{ for } i = 1, 2, 3\}$$

Manipulator with Two Revolute Joints and a Prismatic Joint:
Its configuration space combines the characteristics of both joint types:

$$C_{R1,R2,P} = \{(\theta_1, \theta_2, x) \mid \theta_1, \theta_2 \in [0, 2\pi) \text{ and } x \in [a, b]\}$$

Question 3

If $A \cap B \neq \emptyset$

Claim:
$Q_A \cap Q_B \neq \emptyset$

Justification:
Given: $A \cap B \neq \emptyset$. This implies that there exists some point p such that: $p \in A$ and $p \in B$. Now, the configuration space obstacle for A and B, represented by Q_A and Q_B respectively, is the set of all configurations of the robot that collide with A and B. Thus, for any configuration c that collides with the overlapping region of A and B, c belongs to both Q_A and Q_B. Hence: $c \in Q_A$ and $c \in Q_B$ which implies: $Q_A \cap Q_B \neq \emptyset$

If $A \cap B = \emptyset$

Claim:
It is possible that $Q_A \cap Q_B \neq \emptyset$ even if $A \cap B = \emptyset$

Justification:
Given: $A \cap B = \emptyset$ Even if A and B are disjoint in the workspace, consider a robot configuration space C. Let's say the robot has a certain configuration c such that part of the robot intersects with A and another part with B simultaneously. This means: $c \in Q_A$ and $c \in Q_B$. Hence: $Q_A \cap Q_B \neq \emptyset$. This overlap of Q_A and Q_B in C is possible due to the robot's geometry and its potential configurations in relation to A and B.

Summarize:

- When workspace obstacles overlap, their respective C-space obstacles always overlap.
- When workspace obstacles do not overlap, it is still possible, depending on the robot's geometry and the environment, for their C-space obstacles to overlap.

Question 4

Given:

- T: Translation space in $\mathbb{R}^3$
- R: Rotation space, represented by $SO(3)$
- n: Number of polyhedral bodies

Single polyhedral body in C_{body}:
$C_{\text{body}} = T \times R$
$C_{\text{body}} = \mathbb{R}^3 \times SO(3)$

The $C_{composite}$ for n polyhedral bodies
$C_{\text{composite}} = (C_{\text{body}})^n$

Given $n = 5$:
$C_{\text{composite}} = (\mathbb{R}^3 \times SO(3))^5$
Dimension of $C_{\text{composite}}$ is: $dim(C_{\text{composite}}) = n \times dim(C_{\text{body}}) = 5 \times 6 = 30$

Programming Component

PRM:

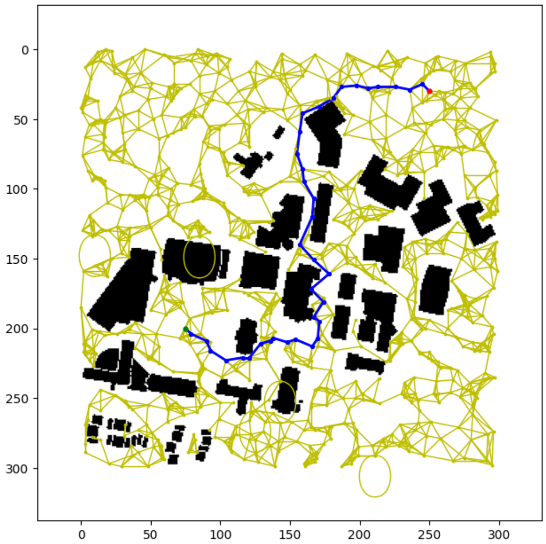

Figure 1: PRM Figure1: n pts = 1000; nodes = 826; edges = 2385; path length = 405.06

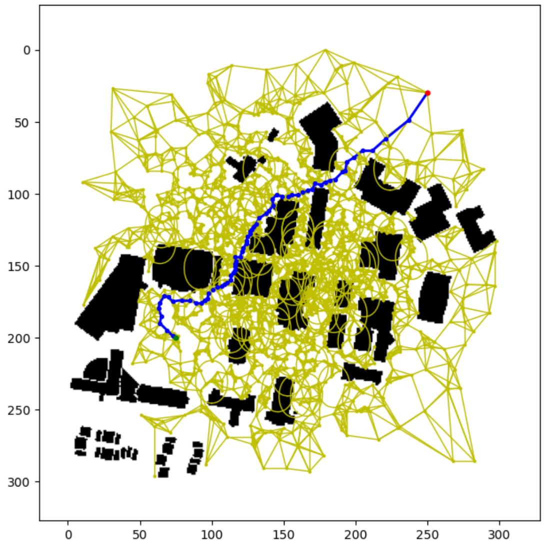

Figure 2: PRM Figure2: n pts = 2000; nodes = 1996; edges = 6020; path length = 301.99

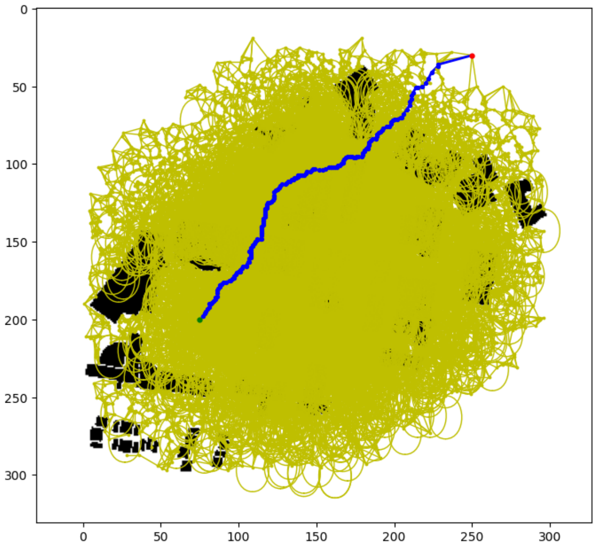

Figure 3: PRM Figure3: n pts = 20000; nodes = 19995; edges = 65380; path length = 284.51

RRT:

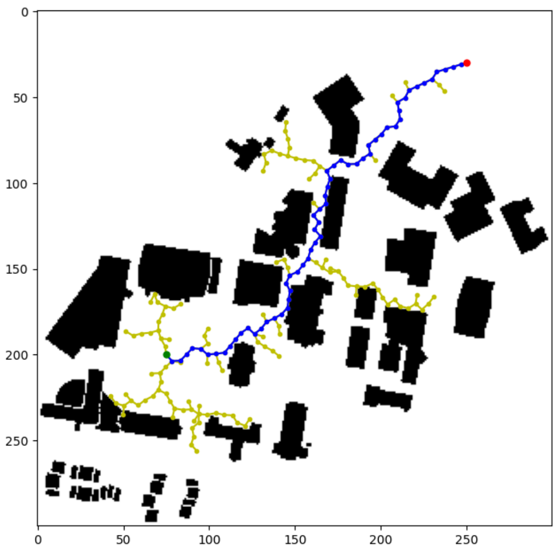

Figure 4: RRT Figure1: n pts = 1000; nodes = 176; path length = 303.08