Course **Artificial Intelligence Techniques (IN4010)**
Part *Game Theory*

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Exercise Sheet **Tutorial 5: Normal Form and Extensive Form Games**
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Exercise 1 (Guessing game)**.** There are two agents a and b in separate rooms. Both agents choose a number from the set $\{2, 3, 4\}$, independently from the other agent. If both agents choose the same number, say, x, then both agents get x EUR. However, if the agents chose different numbers, say x, y, then they have to pay x and y EUR respectively.

(a) Model this as a normal form game.

(b) Next, we consider it as an extensive form game. Suppose the two agents choose their numbers subsequently, first a and then b. Player b sees the number agent a has chosen.

1. Model this as a perfect information extensive form game
2. Compute the subgame perfect equilibria by backwards induction and explain each reasoning step.

Exercise 2 (Nash equilibrium in Normal Form Game)**.** Each of the 100 players in $N = \{1, \ldots, 100\}$ *announces a natural number* from 1 to 100. A prize of *10 EUR* is split equally between *all the players whose number is closest to* $\frac{2}{3}$ *of the average* number of the numbers chosen by all players. (In this exercise we only consider pure strategies.)

1. Model this as a normal form game.
2. Compute a pure Nash equilibrium of the game.
3. Show that this Nash equilibrium is unique, i.e. that there is no other Nash equilibrium. Give an intuitive and formal argument.

Exercise 3 (Zero sum game)**.** Consider the following game (Bombers and Fighters):

		Bomber Crew	
		Look Up	Look Down
Fighter Pilots	Hun-in-the-Sun	*0.95,0.05*	*1,0*
	Ezak-Imak	*1,0*	*0,1*

1. Compute the maxmin strategies of both players.
2. Compute all (mixed) Nash equilibria deriving the answer from your solution to 1.
3. Show that the value of the "Fighters and Bombers" game (i.e. the maxmin value of the Fighter Pilots) is *strictly greater than* 0.95 (and thus that there is a mixed strategy that is better than the pure Hun-in-the-sun strategy).

Exercise 4 (Elimination method)**.** To which final game does the following game reduce? Give the Nash equilibria.

	L	**C**	**R**
U	$\langle 3, 1\rangle$	$\langle 0, 1\rangle$	$\langle 0, 0\rangle$
M	$\langle 1, 1\rangle$	$\langle 1, 1\rangle$	$\langle 5, 0\rangle$
D	$\langle 0, 1\rangle$	$\langle 4, 1\rangle$	$\langle 0, 0\rangle$

Exercise 5 (Pirates and Gold)**.** We are given the following puzzle. Five rational *pirates* a, b, c, d, e negotiate about how to share 100 *gold coins*. The pirates are ranked from a *(highest)* to e *(lowest)* The task is to distribute the coins such that:

1. the highest ranked pirate proposes a distribution.
2. each pirate can *accept* or *reject* the proposal, *majority* decides (highest ranked pirate *breaks ties*, otherwise the pirate takes part in the negotiation as any other pirate).
3. if the proposal is rejected the proposing pirate is *killed* and the next highest ranked pirate makes a proposal, and so forth.

The *preferences* of the pirates (in this order and additive) are as follows: (i) stay alive, (ii) maximize the number of gold coins the pirate gets, and (iii) kill other pirates.

1. Model this as an extensive form game (informally) and explain how strategies are defined.
2. Compute the unique subgame perfect Nash equilibrium.