Course **Artificial Intelligence Techniques (IN4010)**
Part *Game Theory*
Contact Lecturer Dr. Nils Bulling
Teaching Assistant Pietro Pasotti
Course e-mail pa.ai-ewi@lists.tudelft.nl
Exercise Sheet **Tutorial 6: Incomplete Information and Mechanism Design**
Release date 19.10.2016
Tutorial date 20.10.2016

Exercise 1 (Extensive form game with incomplete information, 4 points)**.** Consider the following setting: Player 1 receives a card that is either H or L with equal probabilities. Player 2 does not see the card. Player 1 may announce that the card is L (then she has to pay 1 Euro to player 2). Or she claims it is H. In that case, player 2 can either concede (and pay 1 Euro to player 1) or insist to see the card. In the latter case, if the card is L, then player 1 has to pay 4 Euro to player 2, if the card is H, then player 2 has to pay 4 Euro to player 1.

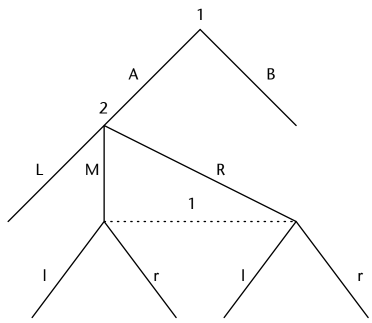
(a) Formulate it as a 2-person extensive form game with imperfect information and a chance moves.

Hint: A *chance move* is to be used for the initial situation. Accordingly, the payoff functions have to be adopted to this situation: They have to be replaced by *expected* payoffs.

(b) Determine the Nash equilibria in mixed strategies.

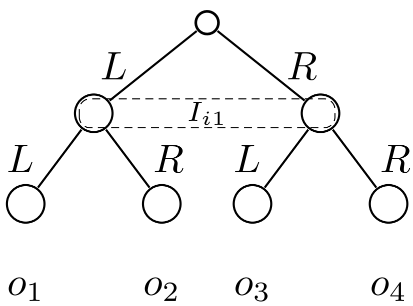
Exercise 2 (Extensive form game with incomplete information, 3 points)**.**

Consider the following imperfect information game. Player 1 has the following mixed strategy: $\langle B, r\rangle$ with probability 0.4, $\langle B, l\rangle$ with probability 0.1, $\langle A, l\rangle$ with probability 0.5. Is there an equivalent behavioural strategy? If there is, how is it defined?

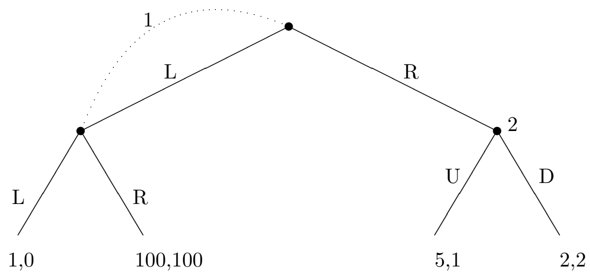


Exercise 3 (Mixed and behavioural strategies, 8 points)**.** This exercise shows that the concepts of mixed and behavioural strategies are not in general comparable.

(a) Prove that in the following one-player imperfect information game (cf. Example 2.44 from the lecture), there exists no *behavioural* strategy which is **outcome equivalent** to the *mixed* strategy $\langle LL : \frac{1}{2}, RR : \frac{1}{2}\rangle$.



(b) Consider the following 2-player game with *imperfect recall* (cf. Example 2.44 from the lecture):



1. Prove that the strategy profile $\langle R, D\rangle$ is the *unique* mixed Nash equilibrium in mixed strategies of the game (i.e. 1 plays R and 2 plays D).
2. Let $I_1 = \{\epsilon, L\}$ and $I_2 = \{R\}$ denote the only information set of player 1 and 2, respectively. Show that the behavioral strategy β_1 with $\beta_1(I_1)(L) = \frac{98}{198}$ and $\beta_1(I_1)(R) = \frac{100}{198}$ of player 1 gives a better outcome against player 2 playing the behavioral strategy β_2 with $\beta_2(I_2)(D) = 1$ and $\beta_2(I_2)(U) = 0$ (i.e. player 2 plays D with probability 1) than the outcome of the mixed Nash equilibrium (R, D).
3. Prove that (β_1, β_2) is a Nash equilibrium in behavioral strategies. That is, player 1 has no better behavioral strategy response than β_1 to β_2.

Exercise 4 (Auctions, 3 points)**.** Show that bidding truthfully is *not* a dominant strategy in Vickrey auctions in case the value of items is correlated. Give a concrete example.

Exercise 5 (Lying and mechanisms, 3 points)**.** Suppose there are three candidates $Out = \{A, B, C\}$. The possible preferences of the candidates are strict total orders over X. For example, $A \succ B \succ C$ denotes that A is strictly preferred over B, and B is strictly preferred over C. The order $\succ$ is transitive.

A type of an agent is such a strict total order. Θ_i defines the set of all types (i.e. the set of all strict orderings over O) of player i.

We suppose that there are five voters. Consider a direct mechanism mapping a tuple of types $(\theta_1, \ldots, \theta_3)$ to an outcome Out defined as follows:

1. For each candidate the mechanism counts how many voters rank the candidate highest (i.e. ranked top).
2. If one candidate is ranked top by more voters than any other candidate, this candidate is chosen.
3. If there is a tie between A and B, or A and C, then A is chosen.
4. If there is a tie between B and C, B is chosen.

Suppose Player 1 knows the preferences of the Players 2 to 4. Is it always best for Player 1 to announce its true preference, or can there be situations in which the player is better off lying about its true preference?