Linear Algebra, 1131 Homework 1 113.09.25

※請保留計算過程，作業務必寫上姓名學號並準時繳交

■ **1.** Prove that the set of $m \times n$ upper triangular matrices is a subspace of $M_{m\times n}(\mathbb{F})$.

■ **2.** Let $V = \{(a_1, a_2) | (a_1, a_2) \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in V$ and $\alpha \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$$
$$\alpha(a_1, a_2) = (\alpha a_1, \alpha a_2)$$

Is V a vector space over $\mathbb{R}$?

■ **3.** Let

$$W_1 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 | a_1 = 3a_2 \text{ and } a_3 = -a_2\}$$
$$W_2 = \{(a_1, a_2, a_3) \in \mathbb{R}^3 | 2a_1 - 7a_2 + a_3 = 0\}$$

Show that W_1 and W_2 are subspaces of $\mathbb{R}^3$. Find $W_1 \cap W_2$.

■ **4.** Let $W_1, W_2, \ldots$ be subspaces of a vector space V for which $W_1 \subseteq W_2 \subseteq \cdots$. Let

$$W = \bigcup_{i=1}^{\infty} W_i = W_1 \cup W_2 \cup \ldots$$

Prove that W is a subapace of V.

■ **5.** Let $V = \{r \in \mathbb{R} | r > 0\}$. For each $x, y \in V$ and $c \in \mathbb{R}$, define

$$x \oplus y = xy \text{ and } c \odot x = x^c$$

Prove that $(V, \oplus, \odot)$ a vector space over $\mathbb{R}$.

■ **6.** Let $\mathbb{F} = \mathbb{Z}_2$. Consider the vector space $\mathbb{F}^n$ over $\mathbb{F}$. For each $v \in \mathbb{F}^n$, defined the **weight** $\omega(v)$ to be the number of nonzero coordinates in v. For instance, $\omega(1,0,1,0,1,0) = 3$ and $\omega(1,1,0,0,0) = 2$. Let V be the subset of $\mathbb{F}^n$ such that $v \in V$ if and only if $\omega(v)$ is even. Show that V is a vector space over $\mathbb{F}$.

繳交時間：請在 10/2(三) 13:00 (課輔時間)繳交紙本。