Amath 482 homework 4: Music Classification

Andy Zhang

March 6, 2020

Abstract

This report demonstrates the idea of Machine Learning on music classification, which is separated into 3 cases: Band Classification with different music genres, Band Classification with the same genre, and Genre Classification. To accomplish such goal, we are going to do spectrogram transform to our data first, then apply Principal Component Analysis (PCA) for dimension reduction and Linear Discriminant Analysis (LDA) to statistically determine threshold values. Finally, the result will be applied to new data to test the algorithm.

1 Introduction

As described in the report from Homework 3, PCA is an useful technique to extract lower-dimension data from the dynamic. Since machine learning commonly deals with a bunch of similar data, applying PCA makes it much more efficient (referring to energy captured by each mode) and provides a matrix U that we can project our data onto.

LDA is based on the idea of projecting two sets of data onto a new bases that separate the inter-class distance to the max extent while making the intra-class data as compact as possible. The corresponding eigenvector of the maximum eigenvalue between the within-class variance S_W and between-class variance S_B will give us a good one to project onto.

In this report, we are going to apply these techniques to several 5-seconds music clips for classification. In order to guarantee the success rate, different amounts of clips might be chosen for different cases, since it would definitely be harder to classify three artists performing the same genre than those performing different genres.

2 Theoretical Background

2.1 Spectrogram transform

The spectrogram is the visual representation of the spectrum of frequencies along the signal verses time. We can extract time-frequency component of the music clips by simply using the Matlab built-in function:

$$spectrogram(data(:,k))$$
 (1)

and then rearranging it into a single column vector that represents one specific music clip.

2.2 Principal Component Analysis (PCA)

Since we're dealing with similar spectrogram data that represent the frequency-time components of music clips, it's necessary and feasible to do PCA that helps dimension reduction, and thus making the algorithm much more efficient. PCA is based on the idea of singular value decomposition, as described in the report of **Homework 3**.

2.3 Linear Discriminant Analysis (LDA)

LDA is a statistic method that can classify different groups to the max extent. In this case of music classification, we have three groups of data, thus, we should apply formula of multi-group between-class variance S_B by:

$$S_B = \sum_{j=1}^n mj(\vec{\mu_j} - \vec{\mu})(\vec{\mu_j} - \vec{\mu})^T$$
 (2)

, where n denotes the number of groups and $\vec{\mu}$ is the mean of all all $\vec{\mu_1}$ to $\vec{\mu_n}$. mj is the size of each group. In this case, we're going to use same amount of clips from each artist, so we can omit it when calculating S_B .

The within-class variance, S_W can be computed by:

$$S_W = \sum_{i=1}^n \sum_{\vec{x}} (\vec{x} - \vec{\mu_j}) (\vec{x} - \vec{\mu_j})^T$$
 (3)

, where \vec{x} represents each column vector of the projection onto principal component matrix for each group of the data.

The projection vector w is defined as:

$$w = \arg\max_{w} \frac{w^T S_B w}{w^T S_W w} \tag{4}$$

We can interpret it as the vector w that maximizes the relation of S_B over S_W . To find what the projection basis, w exactly is, we can simply find the maximum eigenvalue and the corresponding eigenvector between $S_B w$ and $S_W w$, computed by:

$$S_B \vec{w} = \lambda S_W \vec{w} \tag{5}$$

Finally, we project the data onto vector w and then need to find threshold values that help to distinguish between groups. We are going to choose the boundary values from each group and taking average of the closest two as thresholds. Since we are dealing with three data groups, two threshold values are then needed.

3 Algorithm Implementation and Development

Prepare data

To start with each case, I first load the .wav files as data for machine learning. Since the clips I choose are stereos, I then take the average and make them mono types. I set up a function called **prepareTest** that's able to do the procedures above.

Do Spectrogram transform

Another function called **doSpec** is set up for this part. First, it will spectrogram transform each column vector of the given data, taking the absolute value, and then resize the result from a m*n matrix into a m*n by 1 column vector. This part ends up by arranging each column vector into a matrix called **spec**, with the size of m*n by the amount of .wav files chosen.

Training the given data

First, we are going to choose an appropriate amount of features for dimension reduction by referring to the Energy-Mode plots for each of the three tests. Using the matrix **spec** and given feature number, we then do SVD to **spec** and choose feature amounts of columns of U and feature amounts of rows of the principal component projection matrix $S*V^T$. 1/3 of the columns of $S*V^T$ are then distributed to corresponding artist. Using the mean-of-rows vector $\vec{\mu_j}$ of each matrix, we are able to perform LDA that generate three sorted projection vectors that are used for finding 2 threshold values. This part is also stored in a function called **trainer**.

Test the algorithm on new data

To test the success rate of the algorithm, I choose one more clip of each used music for machine learning and two new clips that's not used for the algorithm. I then compute the exact labels for each of the music clip (3 being the artist with the largest mean value of the sorted projection vector, and 1 being the one with the least mean). To have the label generated by the algorithm, I first compute the spectrogram and project it onto matrix U and vector w following then. Naming the projection vector \mathbf{pval} , I then call label = (pval > threshold1) + (pval > threshold2) + 1. Subtracting this label vector from correct labels gives us a vector of either 0 or not. If an entry is not 0, that means we fail to classify that clip. Using functions \mathbf{check} generates the label and \mathbf{pval} , and $\mathbf{checkRate}$ gives us the success rate in each test.

4 Computational Results

Test 1 - Band classification with different genres

Three artists, Dan Bodan, Density & Time, Dan Lebowitz are chosen for this test, each performing classical, electronics, and folk musics. 9 clips are chosen from each artist (3 clips per music piece). While for the test data, 5 clips are chosen from each, with a fourth clip from each of the three pieces used as algorithm data and 2 clips from new pieces. 8 features are applied, which results in a success rate of 0.8667, that is, 13/15 being correct. What's wrongly classified are two clips from Dan Bodan's "National Express" and "Kallimachou", and they are mis-classified to be folk music from Dan Lebowitz. The reason might be that, these two pieces are composed with instruments that generate high-frequency sounds. Since I decrease the sampling rate to 11025 Hz, they may then have similar pitches as folk musics that are mostly played by guitars.

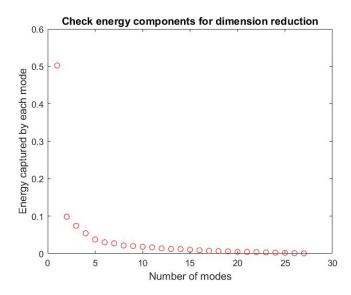


Figure 1: Energy captured by each mode in Test 1

Test 2 - Band classification with the same genre

The jazz & blues music pieces, from Quincas Moreira, Chris Haugen, and Freedom Trail Studio are used for this test. 4 clips from each of 4 pieces are chosen from each artist, which makes a total of 48 algorithm data. 6 clips from each artist (another 4 from the music pieces that are used as algorithm data and 2 from new pieces) are used for testing. In this case, I apply 20 features and the success rate is 0.7222, which means 13 out of 18 are rightly classified. The result tells us that 3 clips from Quincas Moreira and 2 from Freedom trauk

studio are thought to be performed by Chris Haugen. In this case, this makes sense since jazz or blues are composed with similar chords.

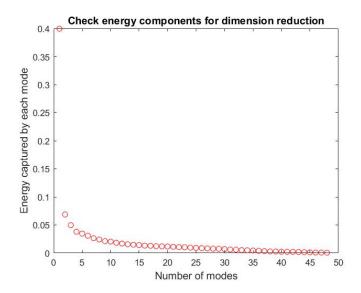


Figure 2: Energy captured by each mode in Test 2

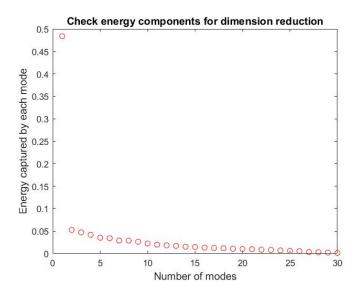


Figure 3: Energy captured by each mode in Test 3

Test 3 - Genre classification

In this case, 10 music clips are randomly chosen in the fields of Jazz & Blues, Electronics, and Punk. The test data are 4 new clips from each of the three genres. 8 features are used for the algorithm, yielding 0.8333 success rate, 10/12 being correct. One of jazz and one of electronics are thought to be punk, these two clips might also be wrongly classified due to the decreased sampling rate.

5 Summary and Conclusion

To sum up, we would see a much higher success rate when dealing with Band Classification (different genres) and Genre Classification than classifying different bands performing same genre of musics. We may also see a increase in all of the tests if we apply more input data and a higher sampling rate, however, this would also make our algorithm much slower, i.e, the tradeoff between efficiency and accuracy should be considered in the field of machine learning.

6 Appendix A

function spec = doSpec(data) returns the spectrogram of data with the right size.

function file = prepareTest(clip) reads stereo audio files and converts them to monos as a column vector.

function sucRate = checkRate(M1, M2, M3, label) checks means of each projection basis onto w and provides us a correct label. It also gives us a success rate determined by correct label and label determined by the algorithm.

function [label, pval] = check(U, w, threshold1, threshold2, spec) takes two threshold values, U and w for projections, and the spectrogram of test data and returns generated label and pval.

function [U, S, V, threshold1, threshold2, w, M1, M2, M3] = trainer(sp, feature) is the training function for machine learning by performing PCA and LDA.

S = spectrogram(data) produces the spectrogram by using STFT.

B = sort(A) sorts A in ascending order.

C = setdiff(A,B, 'rows') returns the values in A that are not in B with no repetitions and C will be sorted in rows.

7 Appendix B

```
1 %%
2 % Test 1: Band Classification (different genres)
  clear; close all; clc;
  % Dan Bodan
  x1 = audioread('Anton1.wav');
  x2 = audioread('Anton2.wav');
x3 = audioread('Anton4.wav');
  x4 = audioread('City4.wav');
  x5 = audioread('City2.wav');
  x6 = audioread('City3.wav');
  x7 = audioread('National1.wav');
  x8 = audioread ('National2.wav');
  x9 = audioread('National3.wav');
  x = [x1, x2, x3, x4, x5, x6, x7, x8, x9];
   for i = 1 : 9
       x(:, i) = (x(:, 2*i - 1) + x(:, 2*i)) / 2;
  x = x(:, 1 : end/2);
21
  % Density & Time
  y1 = audioread('Clean1.wav');
  y2 = audioread('Clean2.wav');
y3 = audioread('Clean3.wav');
  y4 = audioread('Dream4.wav');
  y5 = audioread ('Dream2.wav');
  y6 = audioread ('Dream3.wav');
  y7 = audioread('Day4.wav');
  y8 = audioread('Day1.wav');
  y9 = audioread('Day3.wav');
  y = [y1, y2, y3, y4, y5, y6, y7, y8, y9];
  for i = 1 : 9
       y(:, i) = (y(:, 2*i - 1) + y(:, 2*i)) / 2;
38
  y = y(:, 1 : end/2);
  % Dan Lebowitz
  z1 = audioread('Rag3.wav');
z2 = audioread ('Rag1.wav');
```

```
z3 = audioread('Rag4.wav');
  z4 = audioread('Opening4.wav');
  z5 = audioread('Opening2.wav');
  z6 = audioread('Opening1.wav');
  z7 = audioread('Birds1.wav');
  z8 = audioread ('Birds2.wav');
  z9 = audioread ('Birds3.wav');
51
  z = [z1, z2, z3, z4, z5, z6, z7, z8, z9];
   for i = 1 : 9
       z(:, i) = (z(:, 2*i - 1) + z(:, 2*i)) / 2;
55
  z = z(:, 1 : end/2);
56
57
   Test1 = [x, y, z];
   spec = doSpec(Test1);
59
   feature = 8;
61
   [U, S, V, threshold1, threshold2, w, M1, M2, M3] =
      trainer (spec, feature);
   file (:, 1) = prepareTest ('Anton3.wav');
   file(:, 2) = prepareTest('City1.wav');
   file (:, 3) = prepareTest('National4.wav');
   file (:, 4) = prepareTest('Kallimachou1.wav');
   file (:, 5) = prepareTest('Kallimachou2.wav');
   file (:, 6) = prepareTest('Clean4.wav');
   file (:, 7) = prepareTest ('Dream1.wav');
   file (:, 8) = prepareTest('Day2.wav');
   file(:, 9) = prepareTest('Castlevania1.wav');
   file (:, 10) = prepareTest('Castlevania2.wav');
   file (:, 11) = prepareTest('Rag2.wav');
   file (:, 12) = prepareTest('Opening3.wav');
   file (:, 13) = prepareTest('Birds4.wav');
   file (:, 14) = prepareTest('Cliffsides1.wav');
77
   file (:, 15) = prepareTest('Cliffsides2.wav');
79
   spec = doSpec(file);
81
   [label, pval] = check(U, w, threshold1, threshold2, spec)
   sucRate = checkRate(M1, M2, M3, label);
83
84
  %%
  % Test 2: Band Classification (same genre, Jazz & Blues)
```

```
clear all; close all; clc;
88
   % Quincas Moreira
90
   file (:, 1) = prepareTest('Brooklin1.wav');
   file (:, 2) = prepareTest('Brooklin2.wav');
   file (:, 3) = prepareTest('Brooklin3.wav');
   file (:, 4) = prepareTest('Brooklin4.wav');
   file(:, 5) = prepareTest('Infusion1.wav');
file(:, 6) = prepareTest('Infusion2.wav');
   file (:, 7) = prepareTest('Infusion3.wav');
   file (:, 8) = prepareTest ('Infusion4.wav');
   file(:, 9) = prepareTest('Malan1.wav');
   file(:, 10) = prepareTest('Malan2.wav');
   file (:, 11) = prepareTest('Malan3.wav');
101
   file(:, 12) = prepareTest('Malan4.wav');
   file (:, 13) = prepareTest('Paradox1.wav');
103
   file (:, 14) = prepareTest('Paradox2.wav');
   file (:, 15) = prepareTest('Paradox3.wav');
105
   file (:, 16) = prepareTest('Paradox4.wav');
107
   i = 16;
109
   % Chris Haugen
   file (:, 1+i) = prepareTest('Bleeker1.wav');
   file(:, 2+i) = prepareTest('Bleeker2.wav');
   file(:, 3+i) = prepareTest('Bleeker3.wav');
   file (:, 4+i) = prepareTest('Bleeker4.wav');
   file(:, 5+i) = prepareTest('Et1.wav');
   file(:, 6+i) = prepareTest('Et2.wav');
   file(:, 7+i) = prepareTest('Et3.wav');
117
   file (:, 8+i) = prepareTest('Et4.wav');
118
   file(:, 9+i) = prepareTest('Old1.wav');
119
   file(:, 10+i) = prepareTest('Old2.wav');
120
   file (:, 11+i) = prepareTest('Old3.wav');
121
   file (:, 12+i) = prepareTest('Old4.wav');
122
   file (:, 13+i) = prepareTest('Sunshine1.wav');
   file (:, 14+i) = prepareTest('Sunshine2.wav');
124
   file (:, 15+i) = prepareTest('Sunshine3.wav');
   file (:, 16+i) = prepareTest('Sunshine4.wav');
126
128
   % Freedom trauk studio
   i = 32;
130
   file(:, 1+i) = prepareTest('Crazy1.wav');
   file(:, 2+i) = prepareTest('Crazy2.wav');
   file (:, 3+i) = prepareTest('Crazy3.wav');
```

```
file (:, 4+i) = prepareTest('Crazy4.wav');
   file (:, 5+i) = prepareTest('Current1.wav');
   file(:, 6+i) = prepareTest('Current2.wav');
136
   file (:, 7+i) = prepareTest('Current3.wav');
   file (:, 8+i) = prepareTest('Current4.wav');
138
   file (:, 9+i) = prepareTest('Elder1.wav');
   file (:, 10+i) = prepareTest('Elder2.wav');
140
   file (:, 11+i) = prepareTest('Elder3.wav');
   file (:, 12+i) = prepareTest ('Elder4.wav');
   file (:, 13+i) = prepareTest('Mix1.wav');
   file (:, 14+i) = prepareTest('Mix2.wav');
   file(:, 15+i) = prepareTest('Mix3.wav');
145
   file(:, 16+i) = prepareTest('Mix4.wav');
146
147
148
   spec = doSpec(file);
149
   feature = 20;
150
151
   [U, S, V, threshold1, threshold2, w, M1, M2, M3] =
       trainer (spec, feature);
153
   file 2 (:, 1) = prepare Test ('Brooklin5.wav');
154
   file 2 (:, 2) = prepareTest('Infusion5.wav');
155
   file2(:, 3) = prepareTest('Malan5.wav');
   file 2 (:, 4) = prepareTest('Paradox5.wav');
   file 2 (:, 5) = prepareTest('Miles1.wav');
   file 2 (:, 6) = prepareTest(', Miles2.wav');
159
160
   file 2 (:, 7) = prepareTest('Bleeker5.wav');
161
   file2(:, 8) = prepareTest('Et5.wav');
162
   file 2 (:, 9) = prepareTest('Front1.wav');
163
   file 2 (:, 10) = prepareTest('Front2.wav');
   file 2 (:, 11) = prepare Test ('Old5.wav');
165
   file 2 (:, 12) = prepareTest('Sunshine5.wav');
166
167
   file2(:, 13) = prepareTest('Crazy5.wav');
   file 2 (:, 14) = prepareTest('Current5.wav');
169
   file 2 (:, 15) = prepareTest('Elder5.wav');
   file2(:, 16) = prepareTest('Mix5.wav');
171
   file 2 (:, 17) = prepareTest('Swing1.wav');
   file 2 (:, 18) = prepareTest('Swing2.wav');
173
   spec2 = doSpec(file 2);
175
   [label, pval] = check(U, w, threshold1, threshold2, spec2
       );
   sucRate = checkRate(M1, M2, M3, label);
```

```
178
179
180
182
183
   % Test 3: Genre Classification
184
   clear; close all; clc;
185
186
   % Jazz & Blues
   file(:, 1) = prepareTest('Et5.wav');
188
   file(:, 2) = prepareTest('Miles2.wav');
189
   file (:, 3) = prepareTest('Malan2.wav');
   file (:, 4) = prepareTest ('Malan3.wav');
191
   file (:, 5) = prepareTest('Bleeker3.wav');
   file (:, 6) = prepareTest ('Swing2.wav');
193
   file (:, 7) = prepareTest ('Current3.wav');
   file(:, 8) = prepareTest('Front2.wav');
195
   file (:, 9) = prepareTest('Sunshine1.wav');
   file (:, 10) = prepareTest('Swing1.wav');
197
   i = 10;
199
   % Electronics
201
   file(:, 1+i) = prepareTest('Day3.wav');
   file (:, 2+i) = prepareTest('Dream4.wav');
203
   file (:, 3+i) = prepareTest('Clean2.wav');
   file(:, 4+i) = prepareTest('Macaw1.wav');
205
   file (:, 5+i) = prepareTest('Dragonfly1.wav');
206
   file(:, 6+i) = prepareTest('Stranger2.wav');
207
   file (:, 7+i) = prepareTest('Tea1.wav');
208
   file (:, 8+i) = prepareTest('Tea2.wav');
   file (:, 9+i) = prepareTest('Clean4.wav');
210
   file (:, 10+i) = prepareTest('Macaw2.wav');
211
212
   i = 20;
213
214
   % Punk
   file (:, 1+i) = prepareTest('Beach.wav');
   file (:, 2+i) = prepareTest('Fail1.wav');
   file (:, 3+i) = prepareTest('Fail2.wav');
218
   file (:, 4+i) = prepareTest('Girl1.wav');
   file (:, 5+i) = prepareTest('Girl2.wav');
   file (:, 6+i) = prepareTest('Runner2.wav');
   file (:, 7+i) = prepareTest('Pumps1.wav');
   file (:, 8+i) = prepareTest('Radio1.wav');
```

```
file (:, 9+i) = prepareTest('Radio2.wav');
   file (:, 10+i) = prepareTest('Runner1.wav');
226
   spec = doSpec(file);
   feature = 8;
228
229
   [U, S, V, threshold1, threshold2, w, M1, M2, M3] =
230
       trainer (spec, feature);
231
   file 2 (:, 1) = prepareTest('Infusion2.wav');
   file 2 (:, 2) = prepareTest('Paradox5.wav');
233
   file2(:, 3) = prepareTest('Elder3.wav');
234
   file2(:, 4) = prepareTest('Old4.wav');
235
236
   file2(:, 5) = prepareTest('Firefly1.wav');
237
   file2(:, 6) = prepareTest('Cubic.wav');
238
   file 2 (:, 7) = prepareTest('Operatic2.wav');
   file 2 (:, 8) = prepareTest('Anniversary2.wav');
240
   file 2 (:, 9) = prepareTest('Boy.wav');
242
   file2(:, 10) = prepareTest('Give1.wav');
   file 2 (:, 11) = prepareTest('Grassy.wav');
   file2(:, 12) = prepareTest('Outlet1.wav');
246
247
   spec2 = doSpec(file 2);
248
   [label, pval] = check(U, w, threshold1, threshold2, spec2
   sucRate = checkRate(M1, M2, M3, label);
250
251
252
253
254
255
   function spec = doSpec(data)
256
        for k = 1 : size(data, 2)
257
            temp = spectrogram (data(:, k));
258
            temp = abs(temp);
            [m, n] = size(temp);
260
            spec(:, k) = reshape(temp, m * n, 1);
261
        end
262
   end
264
   function file = prepareTest(clip)
        temp = audioread(clip);
266
        file = (\text{temp}(:, 1) + \text{temp}(:, 2))/2;
267
```

```
end
268
269
270
    function sucRate = checkRate(M1, M2, M3, label)
        Mean = [M1, M2, M3];
272
        Mean = sort(Mean);
273
        for i = 1 : 3
274
             if Mean(i) == M1
275
                 ind1 = i;
276
277
             end
             if Mean(i) = M2
278
                 ind2 = i;
279
             end
280
             if Mean(i) == M3
281
                 ind3 = i;
             end
283
        end
284
        correctLabel = [];
285
        siz = length(label)/3;
        for i = 1 : siz
287
             correctLabel(i) = ind1;
288
             correctLabel(i + siz) = ind2;
289
             correctLabel(i + 2*siz) = ind3;
        end
291
292
        fail = 0;
293
        result = correctLabel - label;
        for i = 1 : length(result)
295
             if result(i) = 0
296
                 fail = fail + 1;
297
             end
298
        end
299
300
        sucRate = 1 - fail/length(result);
301
302
303
    function [label, pval] = check(U, w, threshold1,
304
       threshold2, spec)
        TestMat \, = \, U' \ * \ spec \, ;
305
        pval = w' * TestMat;
306
307
        label = (pval > threshold1) + (pval > threshold2) +
            1;
   end
309
310
    function [U, S, V, threshold1, threshold2, w, M1, M2, M3]
```

```
= trainer(sp, feature)
    [U, S, V] = svd(sp, 'econ');
313
   PCA_proj = S * V';
   U = U(:, 1 : feature);
315
316
    \operatorname{plot}(\operatorname{diag}(S).^2/\operatorname{sum}(\operatorname{diag}(S).^2), 'ro')
317
    xlabel ('Number of modes', 'FontSize', 12)
    ylabel ('Energy captured by each mode', 'FontSize', 12)
    title ('Check energy components for dimension reduction',
        'FontSize', 12)
321
    Art1 = PCA_proj(1 : feature, 1 : end/3);
322
    Art2 = PCA_proj(1 : feature, end/3 + 1 : end * 2/3);
323
    Art3 = PCA_proj(1 : feature, end * 2/3 + 1 : end);
325
   m1 = mean(Art1, 2);
   m2 = mean(Art2, 2);
327
   m3 = mean(Art3, 2);
329
   Sw = 0:
    for k = 1: size (Art1, 2)
331
        Sw = Sw + (Art1(:, k) - m1) * (Art1(:, k) - m1)';
   end
333
    for k = 1: size (Art2, 2)
334
        Sw = Sw + (Art2(:, k) - m2) * (Art2(:, k) - m2)';
335
336
    for k = 1 : size(Art3, 2)
337
        Sw = Sw + (Art3(:, k) - m3) * (Art3(:, k) - m3)';
338
   end
339
340
   mu = (m1 + m2 + m3) / 3;
   Sb = 0:
342
   Sb = Sb + (m1 - mu) * (m1 - mu) ';
   Sb = Sb + (m2 - mu) * (m2 - mu) ';
   Sb = Sb + (m3 - mu) * (m3 - mu) ';
346
    [V2, D] = eig(Sb, Sw);
    [\tilde{\ }, \operatorname{ind} 1] = \max(\operatorname{diag}(D));
348
349
   w = V2(:, ind1);
350
   w = w / norm(w, 2);
351
352
   vArt1 = w' * Art1;
   vArt2 = w' * Art2;
   vArt3 = w' * Art3;
```

```
356
   M1 = mean(vArt1);
   M2 = mean(vArt2);
   M3 = mean(vArt3);
360
   sortArt1 = sort(vArt1);
361
   sortArt2 = sort(vArt2);
362
   sortArt3 = sort(vArt3);
   sortArt = [sortArt1; sortArt2; sortArt3];
364
365
   Mean = [M1, M2, M3];
366
   Mmax = max([M1, M2, M3]);
367
368
   IndMax = Mean == Mmax;
369
   sortMax = sortArt(IndMax, :);
370
371
   sortArt = setdiff(sortArt, sortMax, 'rows');
372
373
374
   sortNext = sortArt(2, :);
375
   sortLeast = sortArt(1, :);
377
   sizeMax = length(sortArt1);
   t1 = sizeMax;
   t2 = 1;
381
   while sortLeast(t1) > sortNext(t2)
        t1 = t1 - 1;
383
        t2 = t2 + 1;
384
385
   threshold1 = (sortLeast(t1) + sortNext(t2)) / 2;
386
387
   t3 = sizeMax;
388
   t4 = 1;
389
   while sortNext(t3) > sortMax(t4)
390
        t3 = t1 - 1;
391
        t4 = t4 + 1;
392
   end
   threshold2 = (sortNext(t3) + sortMax(t4)) / 2;
394
   end
396
```