Drive strain displacement ~~corr~~ relation from cylindrical coordinate to spherical coordinate.

$$u(x, y, z) \rightarrow u(r, \theta, z).$$

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

$$U_r = \frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial r}$$

$$U_r = U_x \cos\theta + U_y \sin\theta.$$

$$U_\theta = \frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial \theta} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial \theta}.$$
$$= -U_x\, r\sin\theta + U_y\, r\cos\theta$$

$$U_z = \frac{\partial u}{\partial z} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial z} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial z} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial z}$$

$$U_z = U_z$$

$$u(r,\theta,z) \rightarrow u(R,\theta,\phi)$$

$$x = R\cos\theta\sin\phi$$
$$r\cos\theta = R\cos\theta\sin\phi$$
$$\boxed{r = R\sin\phi}$$

$$y = R\sin\theta\sin\phi$$
$$r\sin\theta = R\sin\theta\sin\phi$$
$$\boxed{r = R\sin\phi}$$

$$\boxed{z = R\cos\phi}$$

As we known

$$U_R = \frac{\partial U_r}{\partial r}\cdot\frac{\partial r}{\partial R} + \frac{\partial U_r}{\partial \theta}\,\frac{\partial \theta}{\partial R} + \frac{\partial U_r}{\partial z}\cdot\frac{\partial z}{\partial R}$$

$$U_\theta = \frac{\partial U_r}{\partial r}\cdot\frac{\partial r}{\partial \theta} + \frac{\partial U_r}{\partial \theta}\cdot\frac{\partial \theta}{\partial \theta} + \frac{\partial U_r}{\partial z}\cdot\frac{\partial z}{\partial \theta}$$

$$U_\phi = \frac{\partial U_r}{\partial r}\cdot\frac{\partial r}{\partial \phi} + \frac{\partial U_r}{\partial \theta}\,\frac{\partial \theta}{\partial \phi} + \frac{\partial U_r}{\partial z}\cdot\frac{\partial z}{\partial \phi}$$

As

$$\frac{\partial U_r}{\partial r} = \frac{\partial}{\partial r}\left(U_x\cos\theta + U_y\sin\theta\right)$$

$$\frac{\partial U_r}{\partial r} = \cos\theta \frac{\partial}{\partial r} U_x + \sin\theta \frac{\partial}{\partial r} U_y$$

we have

$$r = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1} y/x$$

$$z = z$$

$$U_x = \frac{\partial U}{\partial x} = \frac{\partial U}{\partial r}\cdot\frac{\partial r}{\partial x} + \frac{\partial U}{\partial \theta}\cdot\frac{\partial \theta}{\partial x} + \frac{\partial U}{\partial z}\cdot\frac{\partial z}{\partial x}$$

$$= U_r\left(\frac{1}{2\sqrt{x^2+y^2}}(2x)\right) + U_\theta \frac{1}{1+(y/x)^2}(y)\left(-\frac{1}{x^2}\right) + 0.$$

$$= U_r \frac{x}{\sqrt{x^2+y^2}} - U_\theta \frac{y}{x^2+y^2}$$

$$U_y = \frac{\partial U}{\partial y} = \frac{\partial U}{\partial r}\cdot\frac{\partial r}{\partial y} + \frac{\partial U}{\partial \theta}\cdot\frac{\partial \theta}{\partial y} + \frac{\partial U}{\partial z}\cdot\frac{\partial z}{\partial y}$$

$$= U_r \frac{1}{2\sqrt{x^2+y^2}}(2y) + U_\theta\left(\frac{1}{1+(y/x)^2}\right)\frac{1}{x} + 0.$$

$$= U_r \frac{y}{\sqrt{x^2+y^2}} + U_\theta \frac{x}{x^2+y^2}.$$

$$\frac{\partial U_r}{\partial r} = \cos\theta \frac{\partial}{\partial r}\left[U_r \frac{x}{\sqrt{x^2+y^2}} - U_\theta \frac{y}{\sqrt{x^2+y^2}} \right]$$

$$+ \sin\theta \frac{\partial}{\partial r}\left[U_r \frac{y}{\sqrt{x^2+y^2}} + U_\theta \frac{x}{x^2+y^2} \right]$$

$$\frac{\partial U_r}{\partial \theta} = \frac{\partial}{\partial \theta}\left[U_x \cos\theta + U_y \sin\theta \right]$$

$$\frac{\partial U_r}{\partial z} = \frac{\partial}{\partial z}\left[U_x \cos\theta + U_y \sin\theta \right]$$

$$\frac{\partial U_r}{\partial \theta} = U_x \frac{\partial}{\partial \theta}\cos\theta + U_y \frac{\partial}{\partial \theta}\sin\theta$$

$$= U_x(-\sin\theta) + U_y(\cos\theta).$$

$$\frac{\partial U_r}{\partial z} = 0.$$