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TIME DEPENDENT DIFFUSION MODEL FOR THE FISSION GAS RELEASE IN FUEL RODS

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The fission gas release is analyzed on the basis of the equivalent-sphere model for time-dependent conditions. For this case the general solution of the diffusion equation with a time-dependent diffusion coefficient and production rate is given. As an application the technically important case of piecewise constant diffusion coefficients and production rates is treated.

1. Introduction

In the behaviour of fuel rods, the fission gas release into the free volume of the fuel rod is an extremely important process, reducing the heat conductance through the gas gap, and thereby changing the temperature of the fuel and numerous different processes. During the time that the reactor operates the local power density of a fuel rod is a function of time. Generally, the power density of the rod is approximately a piecewise constant function of time, in which the steps are connected by linear ascents and descents. The production rate of the gaseous fission products is proportional to the power density and, hence, time dependent in the same manner. Moreover, the fuel temperature is a function of the power density. By Arrhenius' law this leads to a time dependence of the diffusion constant which, in the diffusion model, is the driving force of the fission gas release.

In this paper we present a solution of the mathematical problem arising in this context. To this end we solve the diffusion equation and calculate the release of the stable gaseous fission products on the basis of the equivalent-sphere model [1,2]. The novel feature of this study is that the production rate and, most importantly, the diffusion constant are taken to be functions of time. The details of the model are described in the next section. In section 3 the diffusion equation is solved in the short time approximation, demonstrating that the important point is a special transformation of the time coordinate. The last section deals with the solution for piecewise constant production rates and diffusion coefficients, which is directly relevant to the evaluation of fuel rod experiments and the modelling of fuel rods.

2. The equivalent-sphere model

Among the processes which contribute to the release of noble gas atoms, produced by fission, from the UO_2 fuel into the free volume of the rod the volume diffusion of fission gas atoms in a porous fuel is considered only within a diffusion model of fission gas release. The sinter pores of the fuel are, hereby, assumed to be connected with the free volume of the rod, hence, the fission gas is released at free surfaces inside the fuel.

The mathematical treatment of the model now starts with the assumption, that the fuel consists of equal spheres whose effective radius a is defined by

$$F_g/V = 3/a \ ,$$

where V is the volume of the fuel and F_g the free surface which can be measured by gas adsorption experiments. Within the model the fission gas is considered released as soon as it arrives at the surface of the spheres. Summing over all equivalent spheres one obtains the total amount of released fission gas. It is further assumed that the production rate of fission gas, as well as the temperature are uniform throughout a sphere. However, extending the work of other authors [3] we allow the production rate and the diffusion coefficient to be time dependent. In the following we focus attention to the behaviour of a single sphere, under time dependent conditions.

The diffusion equation for the concentration i.e. the number per unit of volume, of fission gas atoms in the sphere is given by

$$\frac{\partial c}{\partial t} = D(t) \frac{1}{r} \frac{\partial^2}{\partial r^2} (rc) + Q(t) , \tag{1}$$

with the boundary conditions

$$c(0, t) = \text{finite}, \quad c(a, t) = 0 , \tag{2}$$

and the initial condition

$$c(r, 0) = 0 . \tag{3}$$

$D(t)$ is the diffusion coefficient and $Q(t)$ the production rate, i.e. the number per unit of volume and time of gas atoms produced by fission. Putting

$$v(r, t) = r\, c(r, t) , \tag{4}$$

and

$$\xi = r/a , \tag{5}$$

one obtains

$$\frac{\partial v}{\partial t} = D'(t) \frac{\partial^2 v}{\partial \xi^2} + a\xi\, Q(t) , \tag{6}$$

where

$$v(0, t) = v(1, t) = 0 , \tag{7}$$

$$v(\xi, 0) = 0 , \tag{8}$$

and

$$D' = D/a^2 . \tag{9}$$

The number (per unit time) of fission gas atoms released from the sphere at time t is then given by

$$\hat{r}(t) = 4\pi a^2 j(a, t) , \tag{10}$$

where $j(r, t)$ is the radial component of the flux density:

$$j(r, t) = -D(t) \frac{\partial c}{\partial r} = -\frac{D'(t)}{\xi} \left(\frac{\partial v}{\partial \xi} - \frac{v}{\xi} \right) . \tag{11}$$

With the boundary condition (8) we obtain

$$j(a, t) = -D'(t)\, (\partial v/\partial \xi)_{\xi=1} . \tag{12}$$

Hence, the number of fission gas atoms released in the time interval (t_1, t_2) is

$$R(t_1, t_2) = -4\pi a^2 \int_{t_1}^{t_2} D'(t)\, (\partial v/\partial \xi)_{\xi=1}\, \mathrm{d}t . \tag{13}$$

3. General solution in short time approximation

The crucial step in solving the diffusion equation (6) is the transformation of the time coordinate according to

$$\tau(t) = \int_0^t D'(t')\, \mathrm{d}t' . \tag{14}$$

With the physically evident supposition $D'(t) > 0$ for every point t this is a one-to-one transformation. By using eq. (14), one obtains for $v(\xi, \tau)$ the differential equation

$$\partial v/\partial \tau = (\partial^2 v/\partial \xi^2) + a\xi\, F(\tau) , \tag{15}$$

where

$$F(\tau) = Q(\tau)/D'(\tau) , \tag{16}$$

and for the boundary conditions

$$v(0, \tau) = v(1, \tau) = 0, \quad v(\xi, 0) = 0 . \tag{17}$$

From eq. (13) it follows directly that

$$R(t_1, t_2) = -4\pi a^2 \int_{\tau_1}^{\tau_2} (\partial v/\partial \xi)_{\xi=1}\, \mathrm{d}\tau , \tag{18}$$

where $\tau_1 = \tau(t_1)$ and $\tau_2 = \tau(t_2)$.

The problem is most easily solved by using the Laplace transform. Let

$$\bar{v}(\xi, p) = \int_0^\infty \mathrm{e}^{-p\tau}\, v(\xi, \tau)\, \mathrm{d}\tau , \tag{19}$$

and

$$\bar{F}(p) = \int_0^\infty \mathrm{e}^{-p\tau}\, F(\tau)\, \mathrm{d}\tau \tag{20}$$

be the Laplace transforms of $v(\xi, \tau)$ and $F(\tau)$. By inserting in eq. (15) we obtain

$$\bar{v}(\xi, p) = a\left(-\frac{\sinh(\sqrt{p}\xi)}{\sinh(\sqrt{p})} + \xi\right)\frac{\bar{F}(p)}{p}. \tag{21}$$

After transforming back into the τ-space and applying the Faltung theorem the solution for

$(\partial v(\xi, \tau)/\partial\xi)_{\xi=1}$

is given by

$$(\partial v/\partial\xi)_{\xi=1} = a\int_0^\tau H(\tau - \tau')\, F(\tau')\, \mathrm{d}\tau', \tag{22}$$

where $H(\tau)$ is defined by its Laplace transform

$$\bar{H}(p) = -\frac{\coth(\sqrt{p})}{\sqrt{p}} + \frac{1}{p}. \tag{23}$$

In most cases of practical interest the diffusion coefficient D' turns out to be small so that

$$\tau = \int_0^t D'(t')\, \mathrm{d}t' << 1 .$$

It is, therefore, sufficient to consider the Laplace transform for values of p for which $p \gg 1$. In this case eq. (23) reduces to

$$\bar{H}(p) = -\frac{1}{\sqrt{p}} + \frac{1}{p}; \tag{24}$$

hence,

$$H(\tau) = -\frac{1}{\sqrt{(\pi\tau)}} + 1, \tag{25}$$

and

$$\left(\frac{\partial v}{\partial\xi}\right)_{\xi=1} = a\int_0^\tau \left(-\frac{1}{\sqrt{[\pi(\tau-\tau')]}} + 1\right) F(\tau')\, \mathrm{d}\tau'. \tag{26}$$

A general expression for $(\partial v/\partial\xi)_{\xi=1}$ which is valid for any value of τ is easily obtained by applying the theory of Theta functions and is given by [4]

$$\left(\frac{\partial v}{\partial\xi}\right)_{\xi=1} = -2a\sum_{\nu=1}^{\infty}\int_0^\tau \mathrm{e}^{-\nu^2\pi^2(\tau-\tau')} F(\tau')\, \mathrm{d}\tau'. \tag{27}$$

4. Fission gas release for piecewise constant production rates and diffusion coefficients [6]

Generally the local power density of a reactor fuel rod will be a function of time, made up by a sequence of constant pieces which are connected by linear ascents and descents. During the periods of constant power density the fission gas production rate is a constant of time as well as approximately the diffusion coefficient, since for constant power density the temperature is a slowly varying function of time. In the latter case the approximation for the diffusion coefficient can evidently be improved by further subdividing the periods of constant power density. The intervals connecting the periods of constant power density are very short compared to the operating time of the reactor and can therefore be neglected in calculating the fission gas release.

We define Q_k to be the production rate and D'_k the diffusion constant in the kth time interval $t_{k-1} < t \leqslant t_k$, where $k = 1, 2, ..., n$ and $t_0 \equiv 0$. According to eq. (14) we have for $\tau_k = \tau(t_k)$

$$\tau_k = D'_1 t_1 + D'_2(t_2 - t_1) + ... + D'_k(t_k - t_{k-1}), \tag{28}$$

and for $\tau(t)$, where $t_{n-1} < t$,

$$\tau(t) = D'_1 t_1 + D'_2(t_2 - t_1) + ... + D'_{n-1}(t_{n-1} - t_{n-2}) + D'_n(t - t_{n-1}). \tag{29}$$

In order to calculate the number of fission gas atoms released in the time interval $t_{n-1} < t \leqslant t_n$ we proceed by using eq. (26)

$$\left(\frac{\partial v}{\partial\xi}\right)_{\xi=1} = a\left[\sum_{k=1}^{n-1} F_k \int_{\tau_{k-1}}^{\tau_k} \left(-\frac{1}{\sqrt{[\pi(\tau-\tau')]}} + 1\right) \mathrm{d}\tau' + F_n \int_{\tau_{n-1}}^{\tau} \left(-\frac{1}{\sqrt{[\pi(\tau-\tau')]}} + 1\right) \mathrm{d}\tau'\right], \tag{30}$$

where $F_k = Q_k/D'_k$, and obtain

$$\left(\frac{\partial v}{\partial\xi}\right)_{\xi=1} = a\sum_{k=1}^{n} (F_k - F_{k-1}) \times\left(-\frac{2}{\sqrt{\pi}}\sqrt{(\tau-\tau_{k-1})} + (\tau-\tau_{k-1})\right), \tag{31}$$

where F_0 has to be put equal to zero. Substituting this result into eq. (18) and defining $R_n = R(t_{n-1}, t_n)$

one finally obtains

$$R_n = V_\kappa \sum_{k=1}^{n} \left(\frac{Q_k}{D'_k} - \frac{Q_{k-1}}{D'_{k-1}} \right) \times \left(\frac{4}{\sqrt{\pi}} \left[(\tau_n - \tau_{k-1})^{3/2} - (\tau_{n-1} - \tau_{k-1})^{3/2} \right] - \tfrac{3}{2} \left[(\tau_n - \tau_{k-1})^2 - (\tau_{n-1} - \tau_{k-1})^2 \right] \right) \qquad (32)$$

Here $V_\kappa = \frac{4}{3}\pi a^3$ is the volume of the equivalent sphere and $F_0 = Q_0/D'_0 \equiv 0$. This is the general result for the number of fission gas atoms released in the nth time interval. For production rate and diffusion coefficient being constant over the whole operating time of the reactor, it reduces to the familiar result

$$R_n = V_\kappa \frac{Q}{D'} \left(\frac{4}{\sqrt{\pi}} (\tau_n^{3/2} - \tau_{n-1}^{3/2}) - \tfrac{3}{2} (\tau_n^2 - \tau_{n-1}^2) \right). \qquad (33)$$

Eq. (32) has been used to calculate the fission gas release of the fuel rod with maximium linear heat rating of the FDR-2 core of the NS Otto Hahn. The results of this investigation are published elsewhere [5].

5. Summary

On the basis of the equivalent-sphere model the fission gas release in fuel rods has been analyzed by solving the diffusion equation for a sphere with production rate and diffusion coefficient being functions of time. It is shown that a special transformation of the time coordinate reduces the diffusion equation to a form which can easily be solved by Laplace transform. The general results have been specialized for the useful short time approximation and applied to the technically important case of piecewise constant production rates and diffusion coefficients.

References

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[5] P. Anuschewski, H.-J. Lahann and H. Mertins, GKSS 80/E/10.

[6] This case has been studied for the members of a radioactive chain in a recent publication by G.V. Kidson, J. Nucl. Mater. 88 (1980) 299, which has just come to our attention. Kidson's results for the case of one stable fission gas product are in agreement with the results of section 4. Note, however, that our formalism is not restricted to the special case of piecewise constant production rate and diffusion coefficient. Moreover, our method can readily be generalized to allow inclusion of non-stable fission gas products.