# Internship Presentation

Numerical Estimation of Fission Gas Release Fraction

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# Overview

## Problem Description

- Cylindrical Fuel Pin - 1.1cm diameter
- Radial heat conduction in $UO_2$ matrix
- Surface temperature held at 675K
- Linear heat rating (element power) varies from 300 to 700 W/cm
- Find areal average of release fraction

# Overview

**Approach**

- Consider 100 annular rings
- Conductivity is T-dependent
- Determine q''' from given q'
- Apply cylindrical thermal analysis
- Find T at each ring and thus f using
  - (I) empirical relationships
  - (II) diffusion models
- Calculate areal average of f over the pellet's cross-section

# Phase 1 - Modelling Temperature Distribution in Fuel Pin

**UO2 Conductivity Expression**

$$k = \frac{38.24}{129.4+T} + 6.1256 \times 10^{-13} \left(T^3\right)$$

$$k = \mathrm{UO_2\ conductivity\ (W/cm\ K)}$$

$$T = \mathrm{local\ UO_2\ temperature\ (K)}$$

**100 Annular Rings (n: 1 -> 100)**

$$r_n = \frac{n+10}{200}\,cm,\ a_n = \pi(r_n + r_{n-1})(r_n - r_{n-1})\,cm^2$$

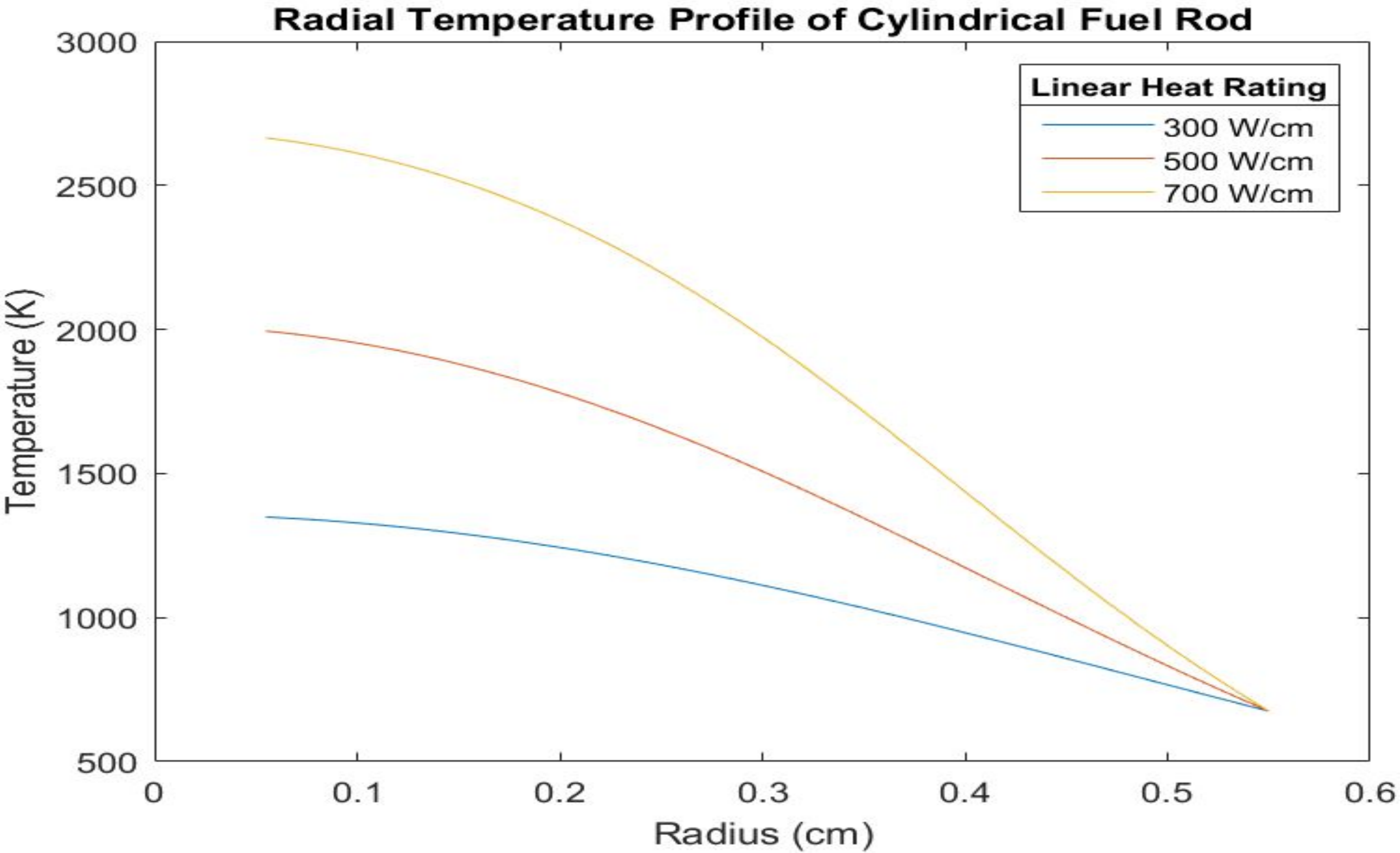
# Phase 1 - Modelling Temperature Distribution in Fuel Pin

- Steady state radial heat conduction equation: $\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(kr\frac{dT}{dr}\right) + q''' = 0$
- After integration and rearrangement: $k\frac{\mathrm{d}T}{\mathrm{d}r} + q'''\frac{r^2}{2} + \frac{C_1}{r} = 0$
- Solid cylinder has no heat flux at the centreline giving us: $q'' = -k\frac{\mathrm{d}T}{\mathrm{d}r} = 0$

  $\Rightarrow C_1 = 0$
- After integration we get, $\int_{T}^{T_{\max}} kdT = \frac{q'''r^2}{4} \ldots (1)$
- Substituting limit at fuel-outer (fo), $\int_{T_{fo}}^{T_{\max}} kdT = \frac{q'''R_{fo}^2}{4}$
- The linear heat rate is given by, $q' = \pi R_{fo}^2 q''' \ldots (2)$

# Phase 1 - Modelling Temperature Distribution in Fuel Pin

- Differentiating (1) and rearranging gives us: $dT = -\frac{q'''r}{2k(T)}dr$
- Working from out to in, and assuming constant conductivity for every annulus, we get a range of temperature values from the outer ring to the centreline.
- The temperature profile for three distinct linear heat ratings, is given in the next slide. $q' = 300\,W/cm,\ 500\,W/cm,\ 700\,W/cm$



Radial Temperature Profile of Cylindrical Fuel Rod
Temperature (K)
3000
2500
2000
1500
1000
500
Radius (cm)
0
0.1
0.2
0.3
0.4
0.5
0.6
Linear Heat Rating
300 W/cm
500 W/cm
700 W/cm

## Phase 2 - Estimating Release Fraction of Fission Gas (Empirical)

We use the following empirical relationship to estimate the fraction of fission gases released on the basis of the temperature in the fuel pin.

$$
\begin{aligned}
f &= 0.05 & T &< 1400^\circ\mathrm{C} \\
f &= 0.10 & 1500 > T &> 1400^\circ\mathrm{C} \\
f &= 0.20 & 1600 > T &> 1500^\circ\mathrm{C} \\
f &= 0.40 & 1700 > T &> 1600^\circ\mathrm{C} \\
f &= 0.60 & 1800 > T &> 1700^\circ\mathrm{C} \\
f &= 0.80 & 2000 > T &> 1800^\circ\mathrm{C} \\
f &= 0.98 & T &> 2000^\circ\mathrm{C}
\end{aligned}
$$

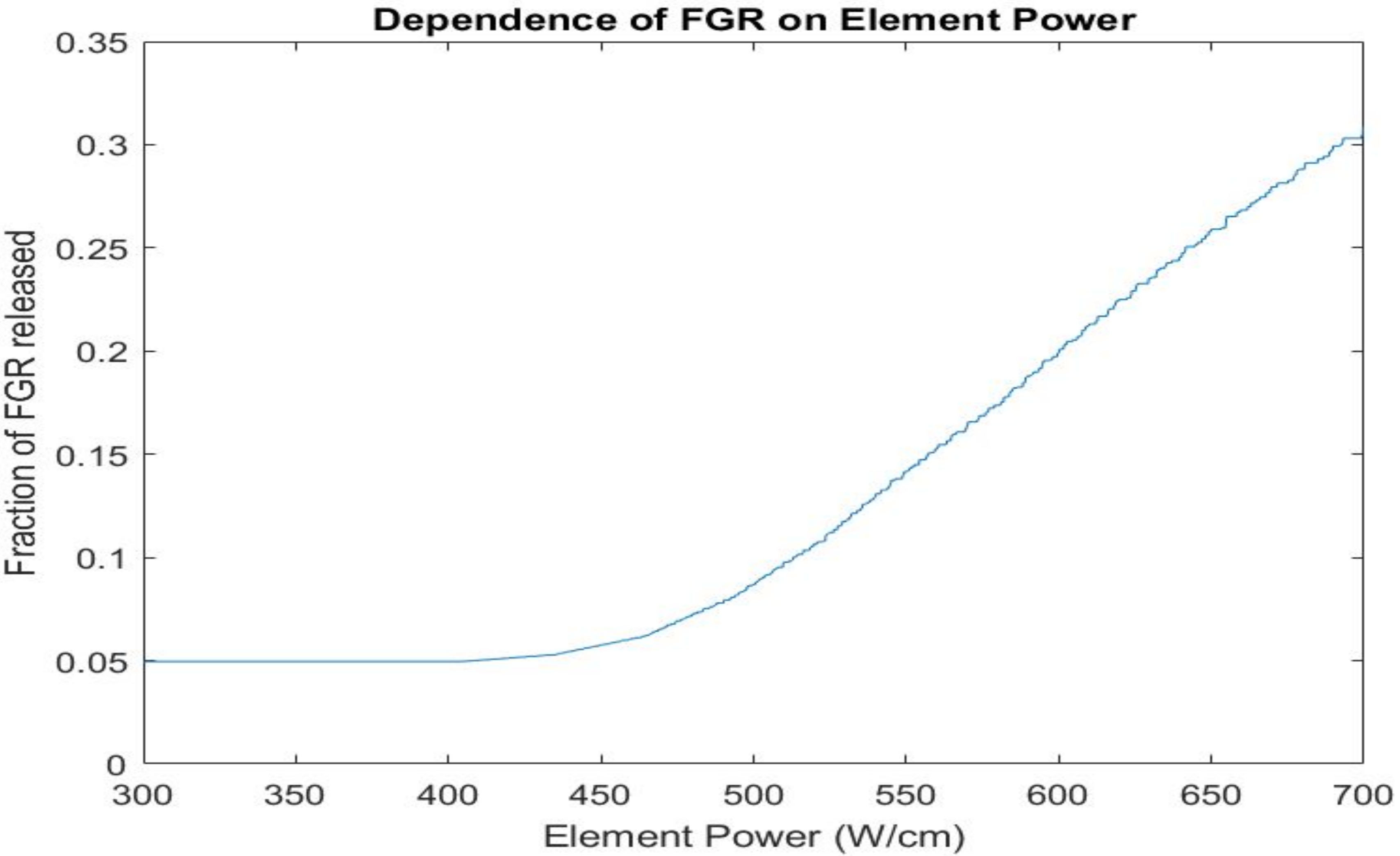
# Average Release Fraction

## for Linear Power Ratings of 300 W/cm, 500 W/cm, 700 W/cm

1. P = 300 W/cm - 5%
2. P = 500 W/cm - 9.19%
3. P = 700 W/cm - 31.66%

# Phase 2 - Estimating Release Fraction of Fission Gas (Empirical)

- Now, consider 10,000 linear heat ratings between 300 W/cm and 700 W/cm.
- The dependence of fission gas release fraction on the element power is plotted next.
- This empirical model depends exclusively on the temperature, determined by q'.
- We need better models that can account for factors such as burn-up and grain radius .
- A few such examples, modelling the diffusion of fission gases through the fuel material after their generation, are discussed next.



Dependence of FGR on Element Power
Fraction of FGR released
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
300
350
400
450
500
550
600
650
700
Element Power (W/cm)

# Phase 3 - Diffusion-based Modelling of FGR Fraction

- Now, consider 100 linear heat ratings between 300 W/cm and 700 W/cm

- Determine the temperature profile for each rating and find the diffusion constant for every annulus using $D = 7.8 \times 10^{-2} \exp\left(-\frac{288\,\mathrm{kJ\,mole}^{-1}}{R(T/10^3)}\right) m^2 s^{-1}$

- Assuming a burnup (B) of 50 MWh/kg U, and metal density (d) in UO2 as 9.67e3 kg/m3, we obtain the lifetime (t) of the fuel rod for a particular linear heat rating or element power (P) using $B \times V \times d = P \times t$

# Phase 3 - Diffusion-based Modelling of FGR Fraction

- We now consider two diffusion-based models
  - Notley and Hastings
  - White and Tucker
- These models consider the release of fission gases to be controlled by atomic diffusion to the fuel grain boundaries. These grains are assumed to be spherical with a radius of the scale of 10-100 microns.
- We assume that we are beginning our analysis with a fresh fuel rod with no stored fission gas reservoir (i.e., only “new” gas is being formed and released).
- While the grains in the fuel matrix may not be spherical, this approach drastically simplifies the model

# Phase 3 (A) - Notley and Hastings Model

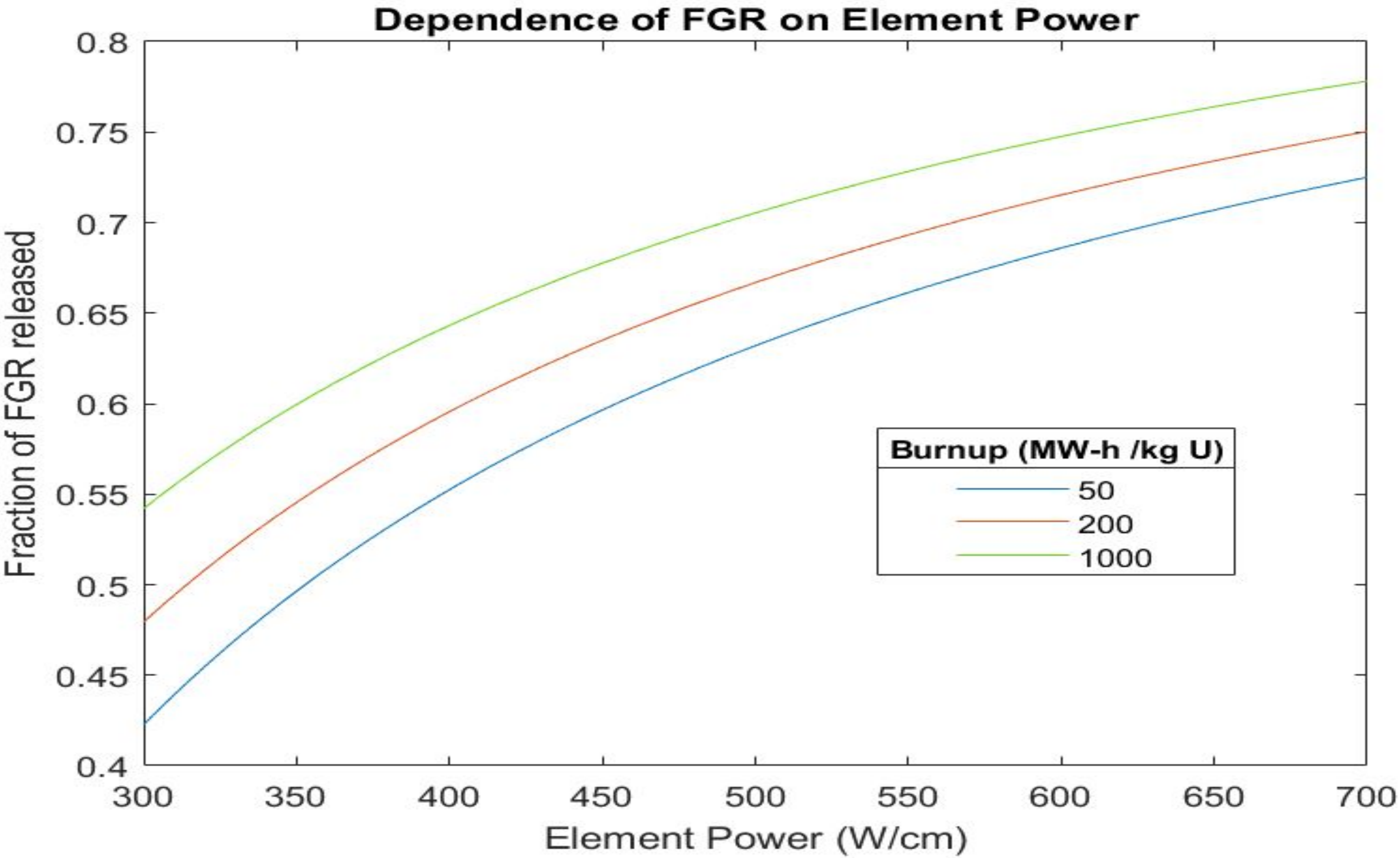
- This model has a short-time and long-time approximation as described below.

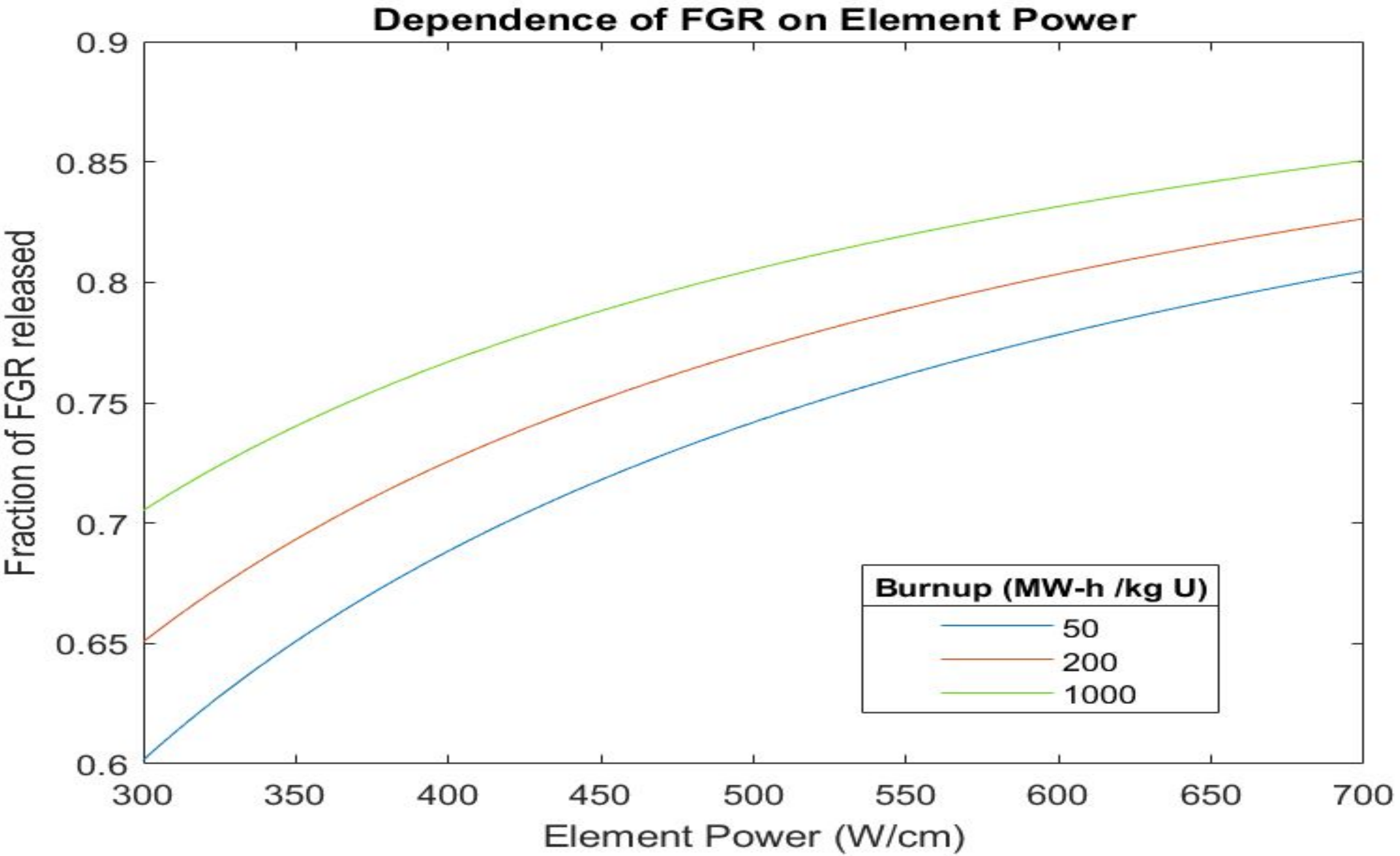
- If $\frac{\pi^2 Dt}{a^2} < 1,\ f = 4\left(\frac{Dt}{a^2\pi}\right)^{1/2} - 1.5\frac{Dt}{a^2}$

  else, $f = 1 - \frac{a^2}{15Dt} + \frac{6a^2}{\pi^4 Dt}\exp\left(\frac{-\pi^2 Dt}{a^2}\right)$

- The fraction obtained is plotted with respect to the element power for a grain radius of 100 micron and 10 micron respectively.

- Both plots model the process at burnups of 50, 200 and 1000 MWh/kg U.


Dependence of FGR on Element Power
Fraction of FGR released
Element Power (W/cm)
Burnup (MW-h /kg U)
50
200
1000
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
300
350
400
450
500
550
600
650
700

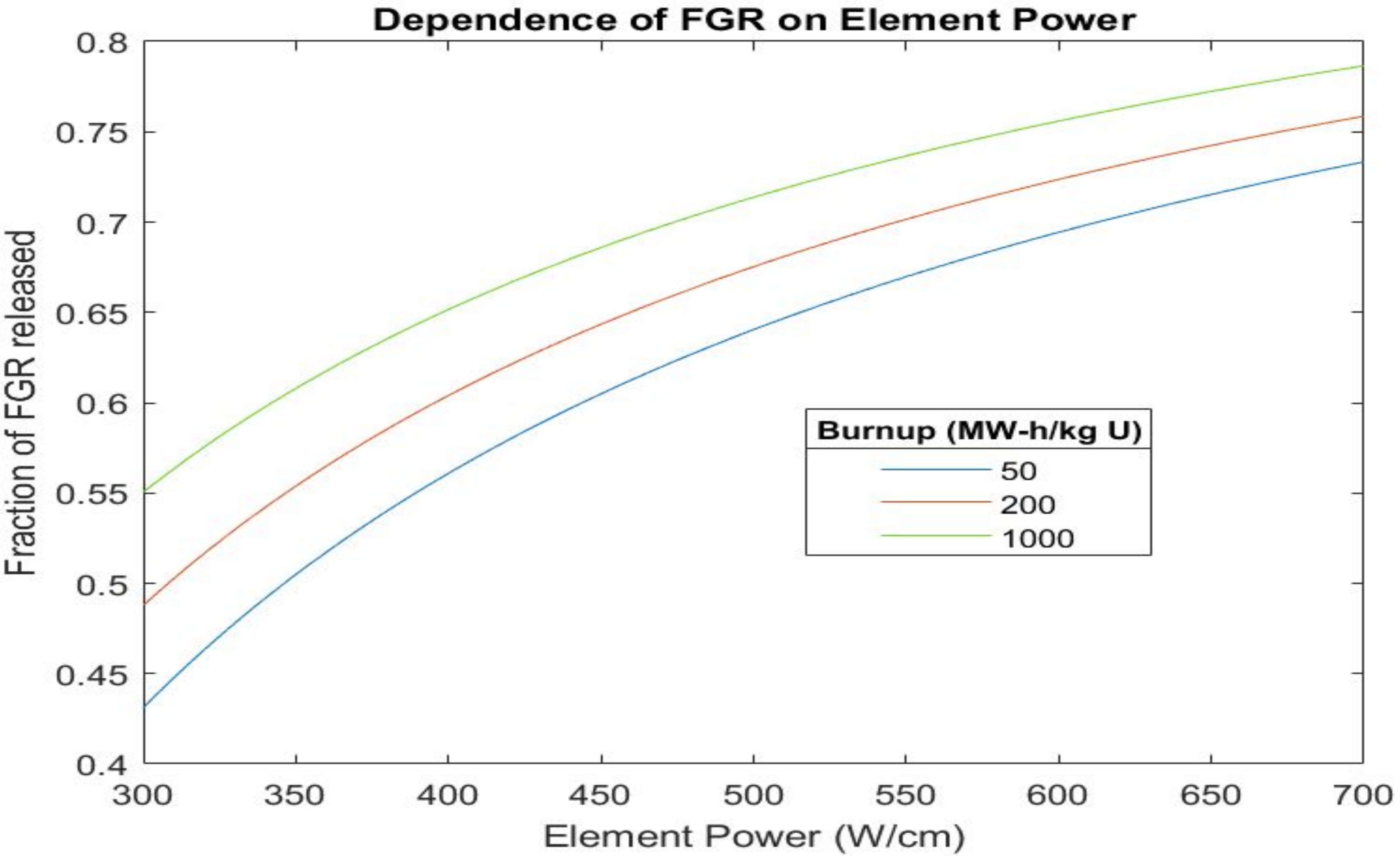

Dependence of FGR on Element Power
Fraction of FGR released
0.9
0.85
0.8
0.75
0.7
0.65
0.6
300
350
400
450
500
550
600
650
700
Element Power (W/cm)
Burnup (MW-h /kg U)
50
200
1000

# Phase 3 (B) - White and Tucker Model

- The spherical diffusion equation for a uniform generation rate (G) is $\frac{\partial c}{\partial t} = D\nabla^2 c + G$
- The constraint imposed on it are: $c = 0 \text{ at } r = a, 0 \leq t \text{ and } 0 \leq r \leq a, t = 0$

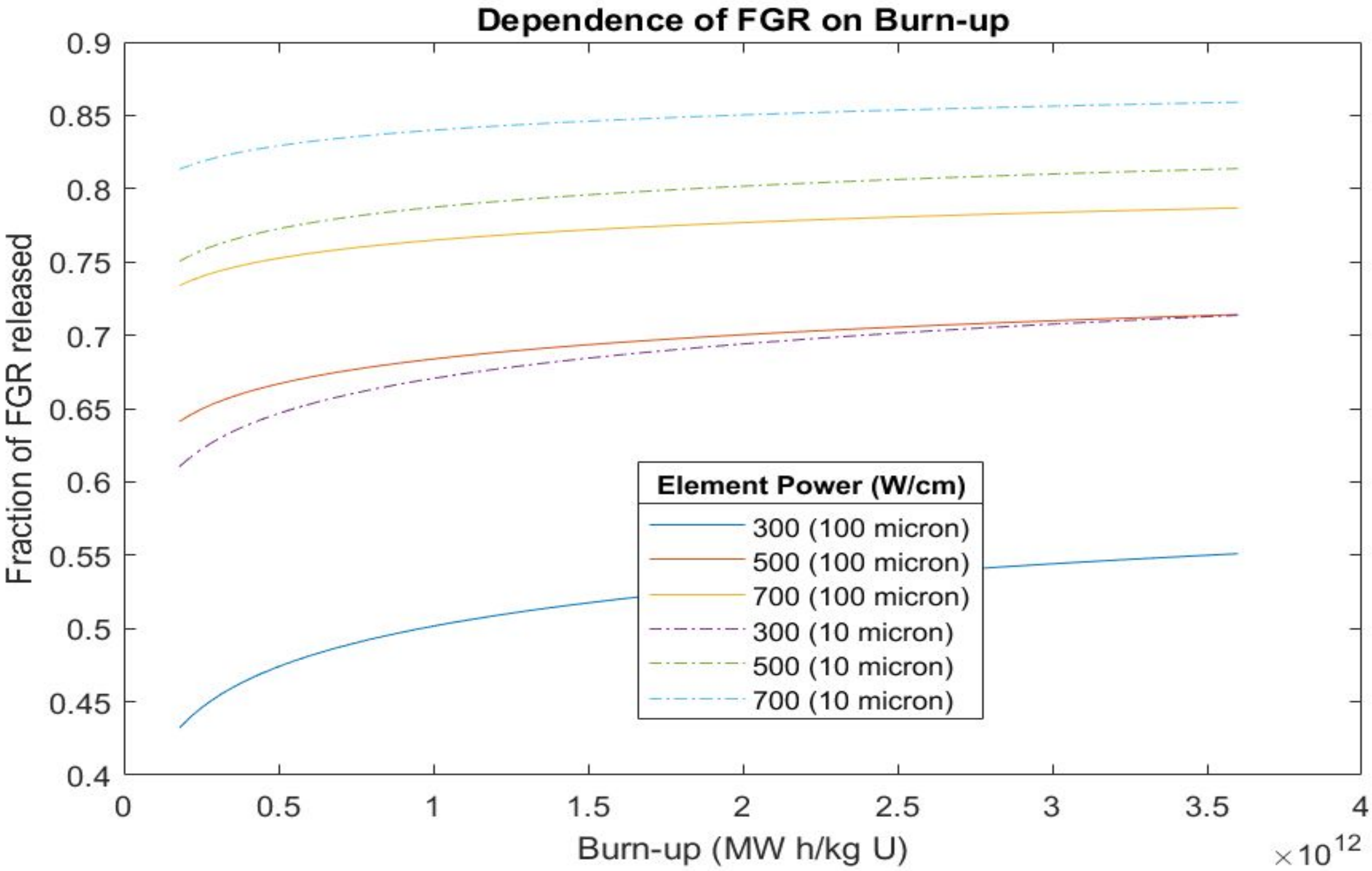
$$\frac{\partial c}{\partial r} = 0 \text{ at } r = 0$$

- We can integrate the concentration profile to the calculated time 't' to get the fraction:
- This is plot $$f = 1 - \frac{6}{\omega} \sum_{n=1}^{\infty} \frac{1 - \exp\left(-\pi^2 n^2 \omega\right)}{(n\pi)^4}$$ rain radius of 100 microns and models the process at burnups of 50, 200 and 1000 MWh/kg U.


Dependence of FGR on Element Power
Fraction of FGR released
Element Power (W/cm)
Burnup (MW-h/kg U)
50
200
1000
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
300
350
400
450
500
550
600
650
700

# Phase 3 (B) - White and Tucker Model

- The fraction is also plotted against the burnup for a grain radius of 100 and 10 microns, modelling the process at element powers of 300, 500 and 700 W/cm.
- Note that increasing the grain radius, decreases the fraction of FGR released.
- The fraction has a high dependence on temperature (and therefore element power), and a moderate dependence on burnup.



Dependence of FGR on Burn-up
Fraction of FGR released
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0
0.5
1
1.5
2
2.5
3
3.5
4
Burn-up (MW h/kg U)
×10^12
Element Power (W/cm)
300 (100 micron)
500 (100 micron)
700 (100 micron)
300 (10 micron)
500 (10 micron)
700 (10 micron)

# Summary

- Numerically determined temperature profile of fuel pin
- Empirically estimated FGR fraction based on temperature
- Observed sensitivity to element power fluctuations
- Explored diffusion models to extend consideration of factors to burnup and grain radius

# The End