BTP 1 - 200260006

Entanglement in Quantum Algorithms

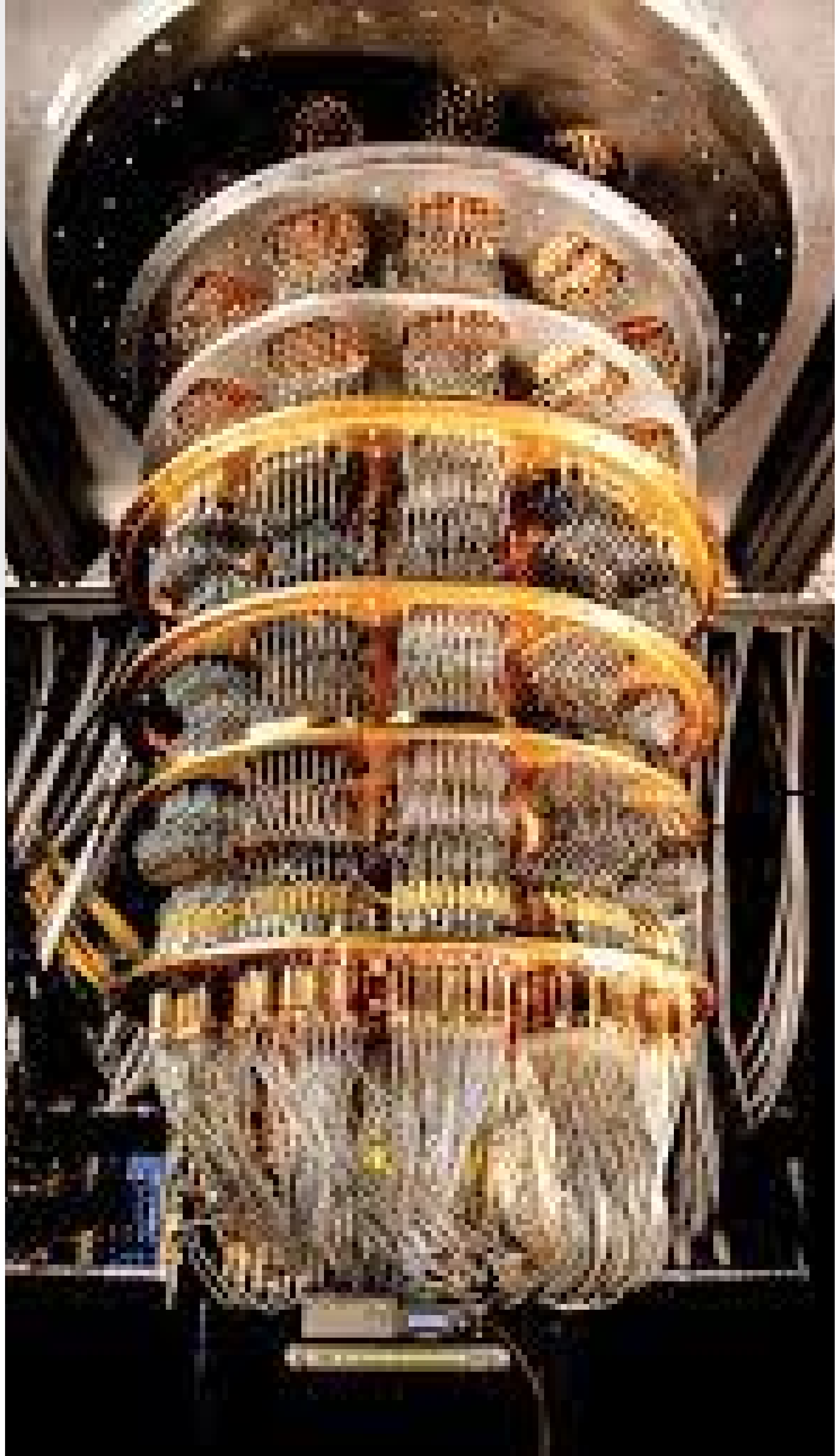
The QAOA Perspective

PRESENTED BY
Aneesh Anand Kamat

Agenda

Introduction

- Why Quantum Computers?
- Entanglement as a quantum resource
- What is Quantum Supremacy?
- Why look at QAOAs for MaxCut?
- What has already been established?
- Potential impact on future research



Introduction

Measures of Entanglement

All composite pure states can be partitioned into two subsystems A and B. We obtain the reduced density matrix ρ_A by tracing out subsystem B,

$$\rho_A(|\Psi\rangle) = \mathrm{Tr}_B(|\Psi\rangle\langle\Psi|),$$

The von Neumann entropy generalizes the concept of Shannon entropy to quantum states and is given by $S(\rho_A) = -(\rho_A \log \rho_A)$.

The Schmidt-rank entanglement is equal to the rank of the reduced density matrix.

Role of Entanglement

Introduction

1) As the system size increases, growing amounts of entanglement should be generated in order for a quantum algorithm to achieve an exponential quantum speed-up. [Jozsa]
2) Universal quantum computation is possible in the standard circuit model even when the entanglement entropy of every bipartition is at most δ. [Maarten]
3) Large amounts of Schmidt-rank entanglement must be generated in a universal quantum computer for quantum supremacy. [Vidal]
4) Operations from the Clifford group composed of Pauli measurement in a computational basis can be efficiently simulated on a classical computer. [Knill-Gottesman]

Entanglement is a necessary but not a sufficient indicator of the possible quantum advantage provided by an algorithm.

Introduction

What is Quantum Supremacy?

QUANTUM COMPUTING AND THE ENTANGLEMENT FRONTIER

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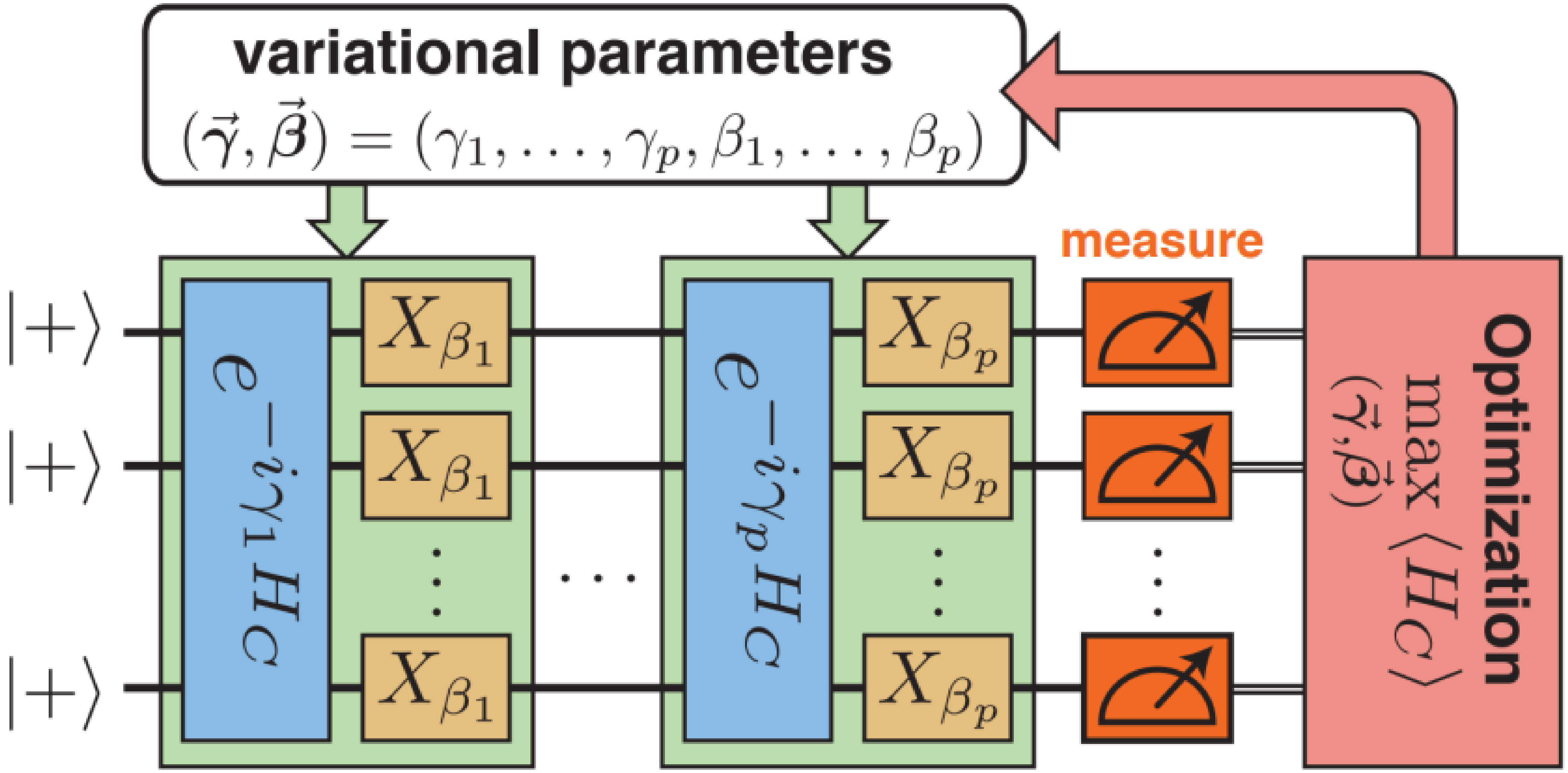
1) What quantum tasks are feasible?
2) What quantum tasks are hard to simulate classically?

Quantum Approximate Optimization Algorithm (QAOA)

Introduction



Introduction

Quantum Approximate Optimization Algorithm (QAOA)

$$|s\rangle = |+\rangle^{\otimes n} = \frac{1}{2^n} \sum_{z \in \{0,1\}^n} |z\rangle.$$

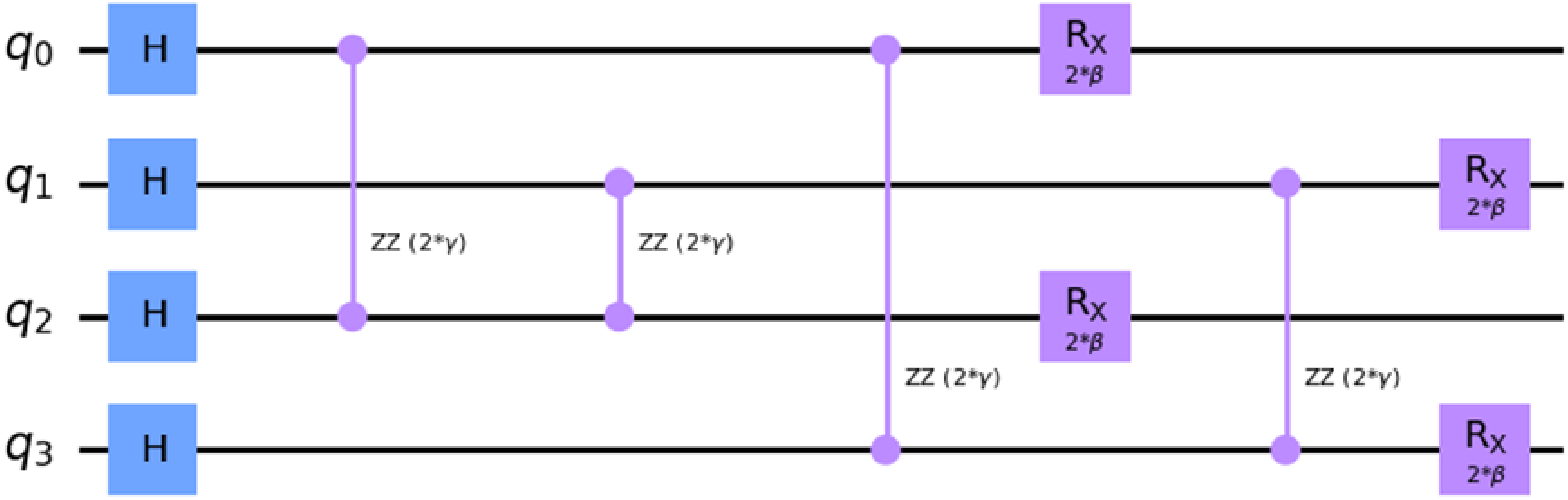
$$|\vec{\gamma}, \vec{\beta}\rangle = U(H^B, \beta_p) U(H^C, \gamma_p) \ldots U(H^B, \beta_1) U(H^C, \gamma_1) |s\rangle,$$

$$U(H^C, \gamma) = e^{-i\gamma H^C} = \prod_{\alpha=1}^{m} e^{-i\gamma H^C_\alpha}, U(H^B, \beta) = e^{-i\beta H^B} = \prod_{j=1}^{m} e^{-i\beta H^B_j}$$

with $H^B = \sum_{j=1}^{n} X_j$ as the *mixing Hamiltonian.*

Quantum Approximate Optimization Algorithm (QAOA)

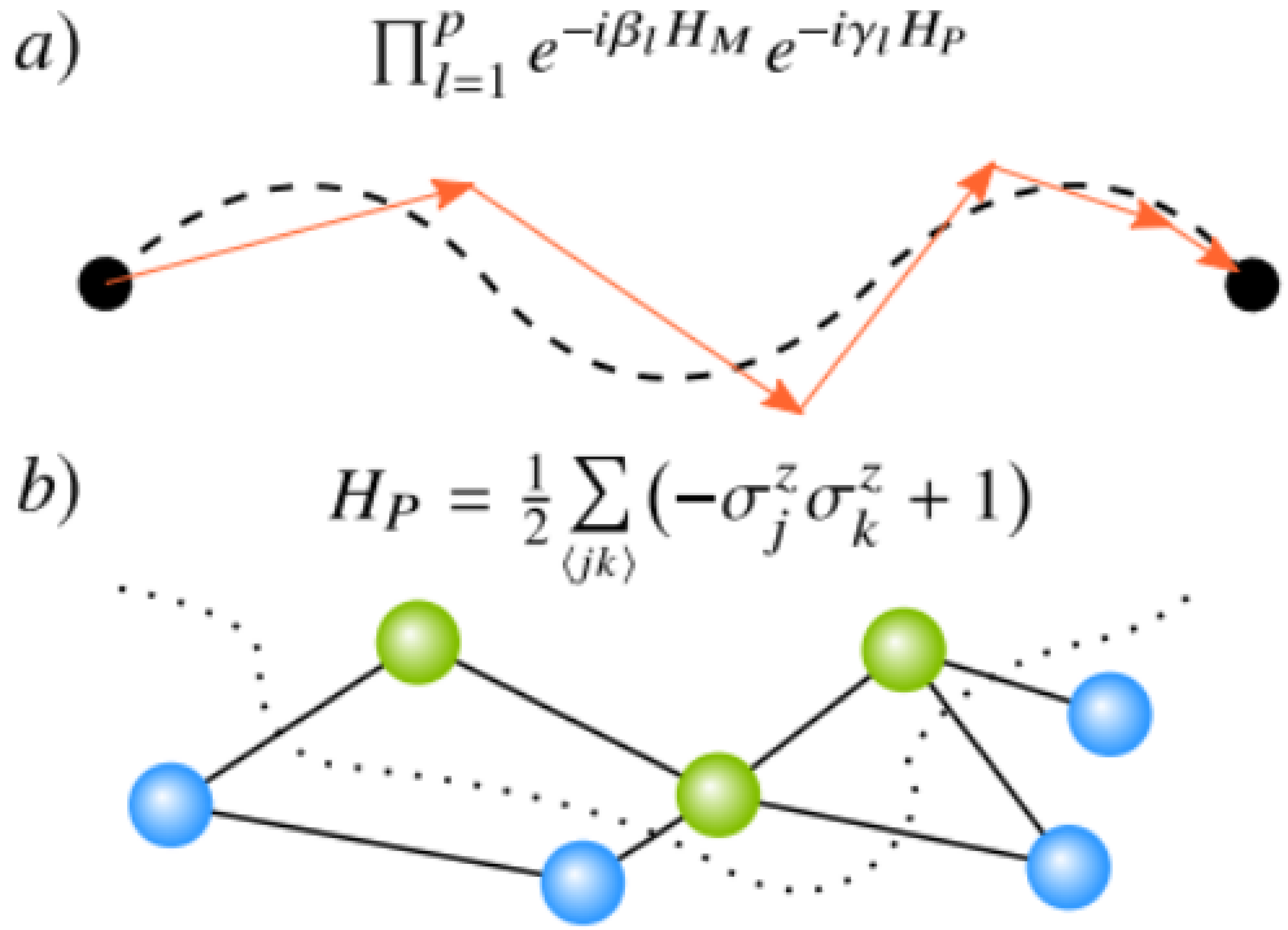
Introduction



Introduction

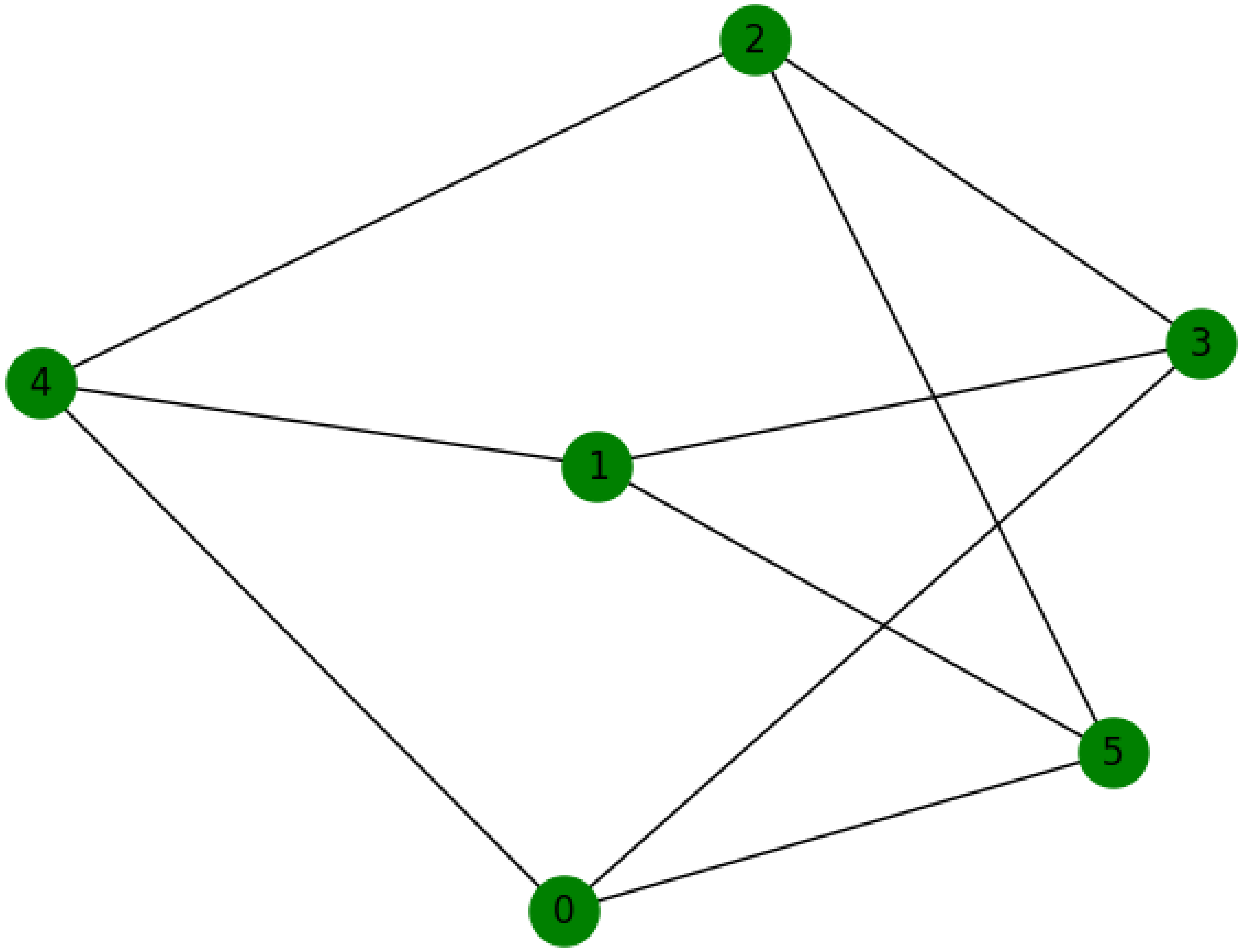
MaxCut Problem (NP Complete)

Given a graph G = (V,E) with n = |V| vertices and |E| edges, the objective is to partition the graph vertices into two sets such that the number of edges connecting vertices in different sets is maximized.



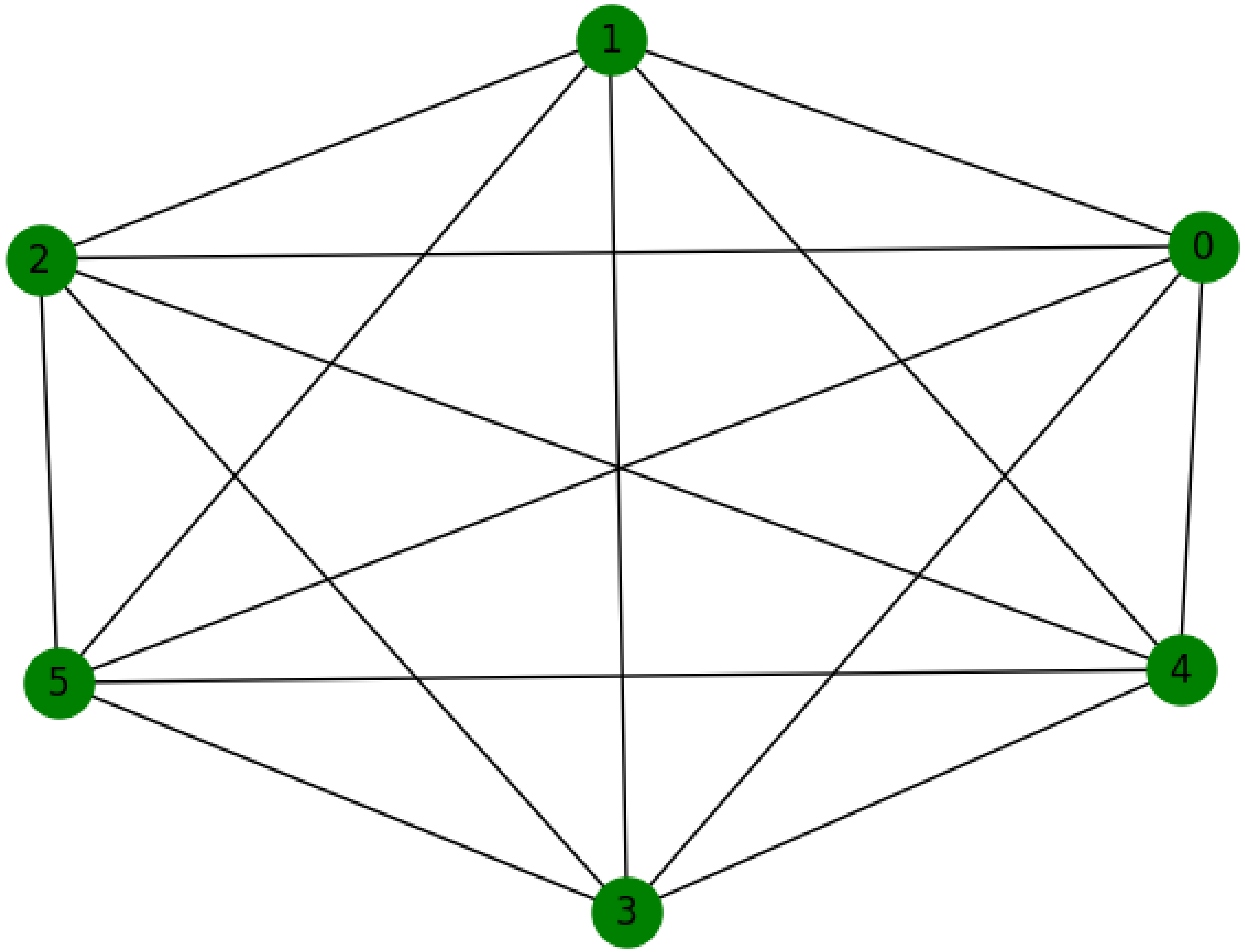
Complete Bipartite Graph

Introduction



Complete Graph

Introduction



Introduction

Random Erdos-Renyi Graph

Random MAX SAT, Random MAX CUT, and Their Phase Transitions

Don Coppersmith,[1] David Gamarnik,[1] MohammadTaghi Hajiaghayi,[2] Gregory B. Sorkin[1]
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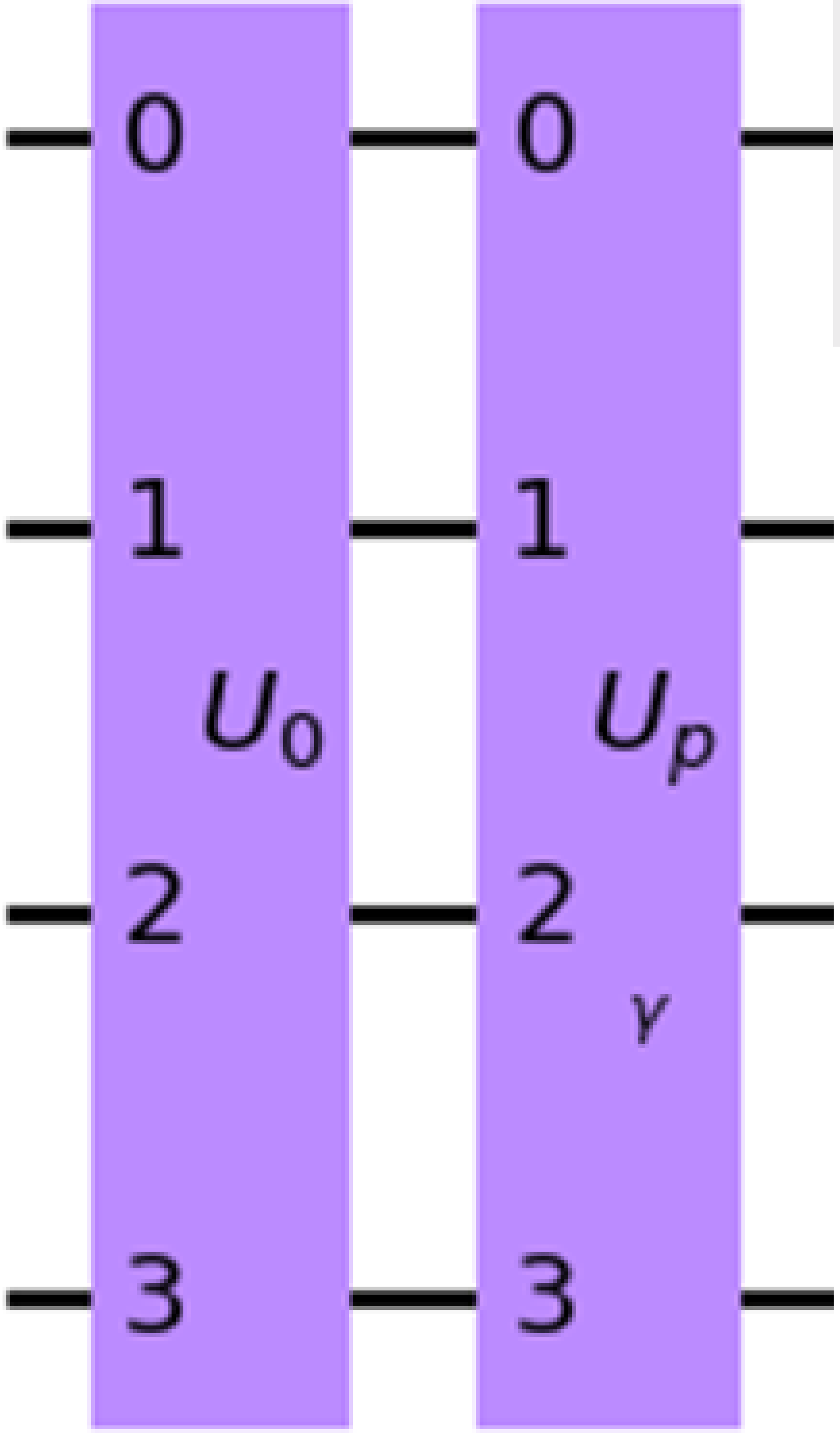
e-mail: {copper,gamarnik,sorkin}@watson.ibm.com
[2] *Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139;*
email: hajiagha@math.mit.edu

In the G(n,m) model, n is the number of vertices and m is the number of edges.
There is a complexity phase transition at m = n/2, below which efficient classical solutions to the MaxCut exist.



Problem Statement

- Scope of the study

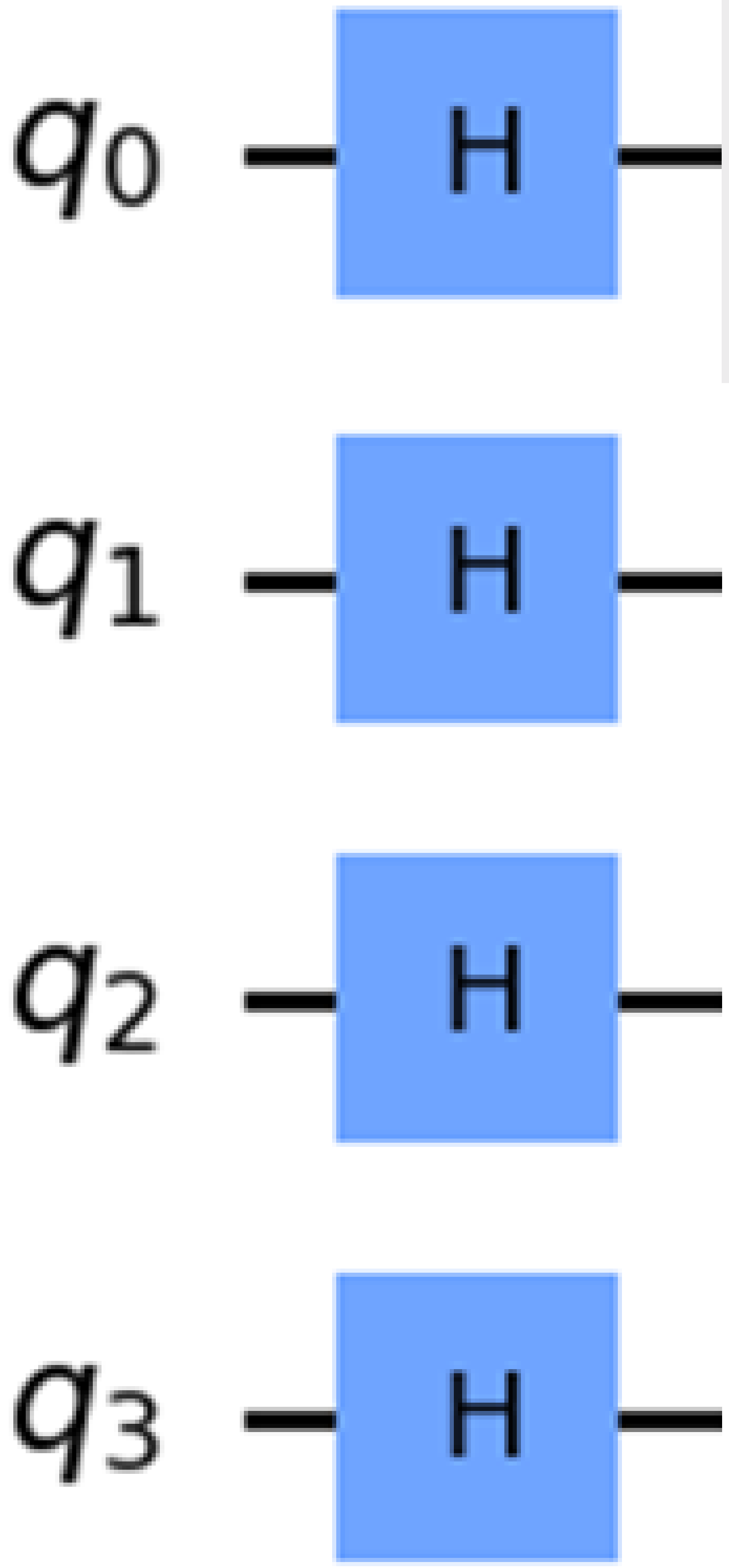
  Bipartite entanglement across a central bipartition in QAOA solving MaxCut for 4 to 10-node graphs of differing topologies.
  All parameters are initialized at 1.0.

- Relevance of the study

  Understand the dynamics of entanglement under continuous and discrete measures, and its behaviour under increasingly complex problems.

- Research Questions

  How does the entanglement behave in between initial and final states?
  What is the impact of problem size and circuit depth on the entanglement?
  How does the complexity of a problem affect the entanglement generated?



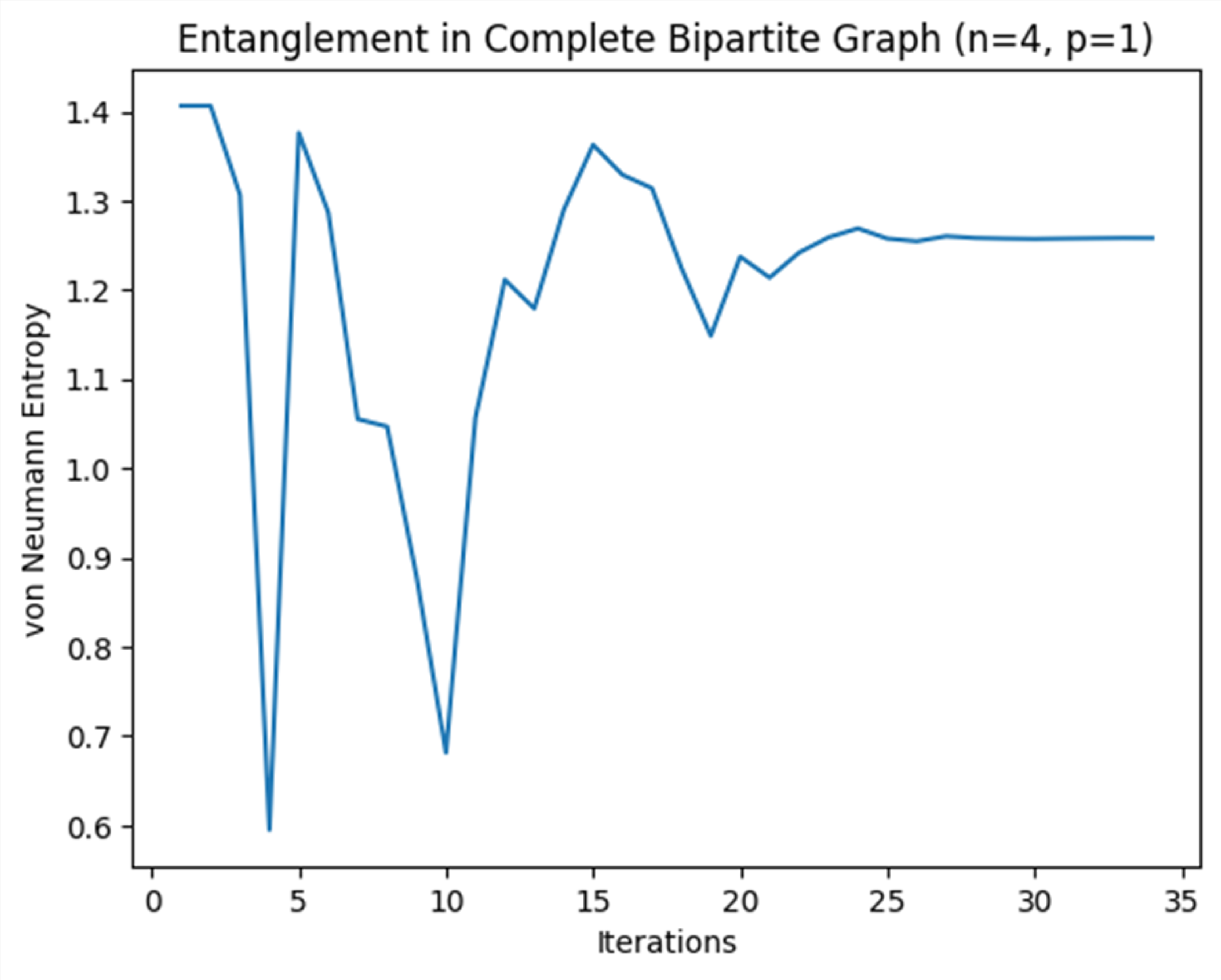
Hypothesis

- Non-trivial evolution of entanglement during the algorithm
  Despite target states potentially being very weakly entangled.
- Schmidt-rank entanglement grows unfavourably with 'n'
  Classical simulation will be hard, leading to potential quantum supremacy
- Classical hardness and quantum hardness are correlated
  High values of Schmidt-rank entanglement should be observed near the phase transition for random Erdos-Renyi graphs

Analysis

4-node Complete Bipartite Graph (p=1 layer)

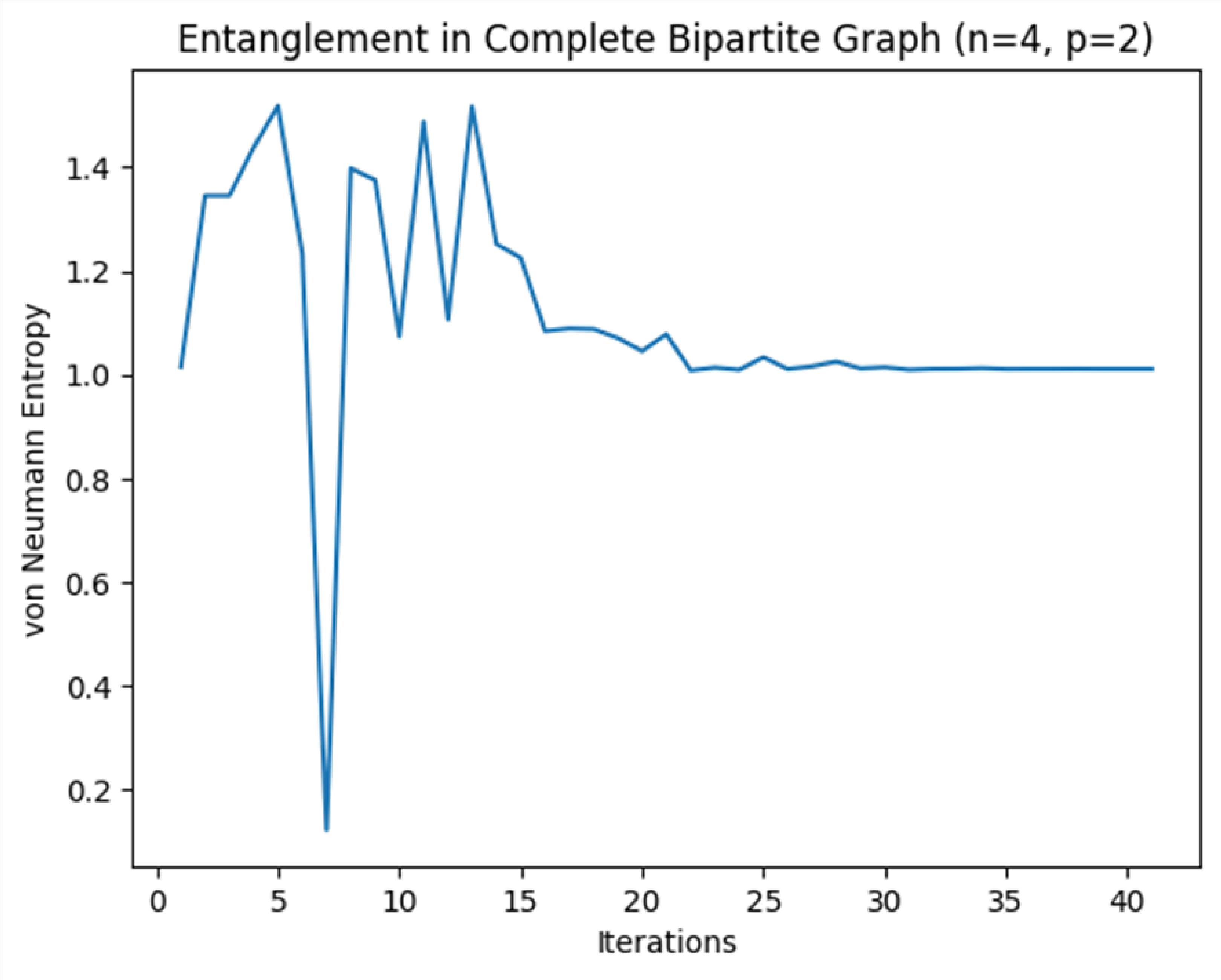
Schmidt Number = 3
Final vN Entropy = 1.26
Iterations to convergence = 34



Analysis

4-node Complete Bipartite Graph
(p=2 layers)

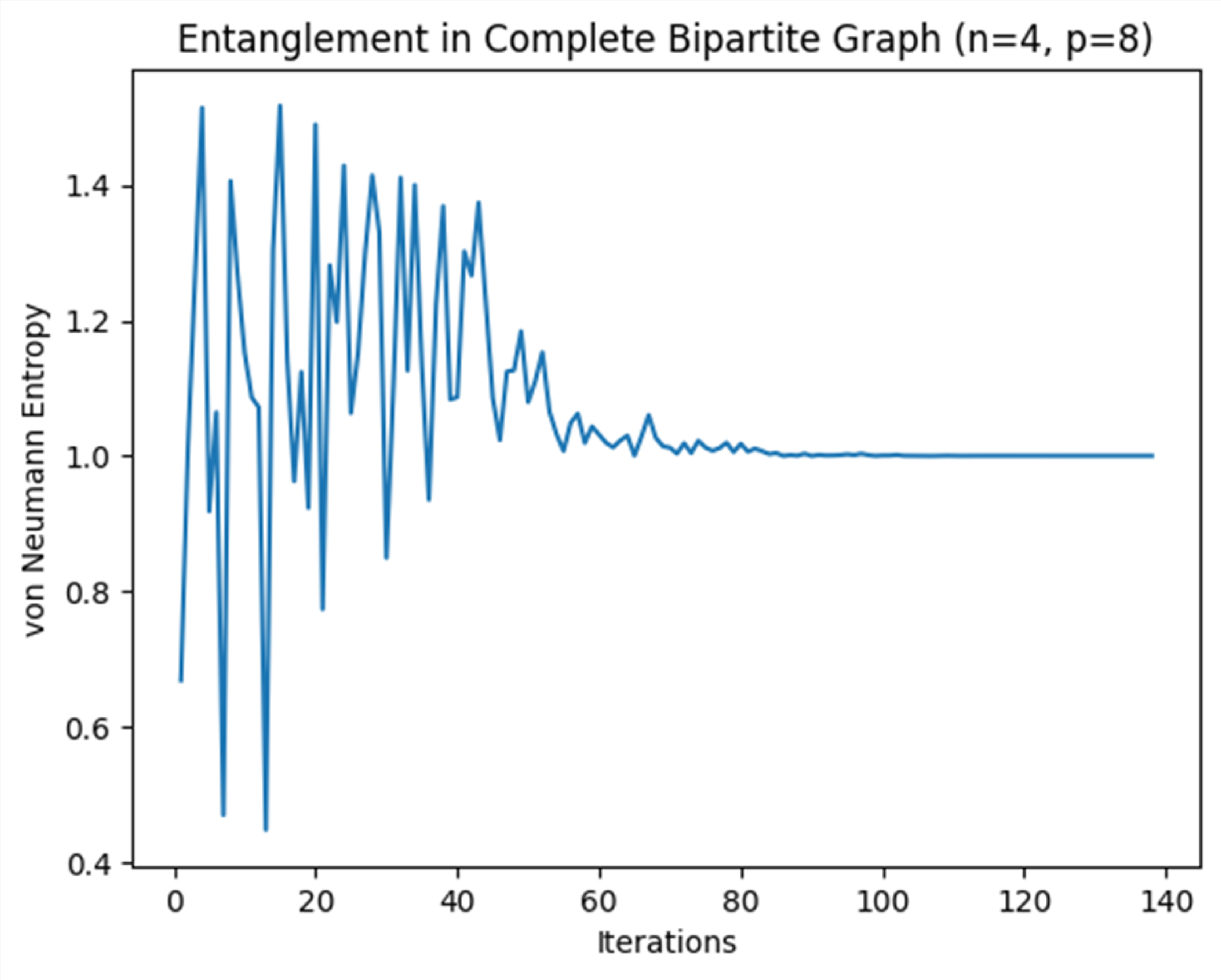
Schmidt Number = 3
Final vN Entropy = 1.01
Iterations to convergence = 41



Analysis

4-node Complete Bipartite Graph
(p=8 layers)

Schmidt Number = 3
Final vN Entropy = 1.00
Iterations to convergence = 138

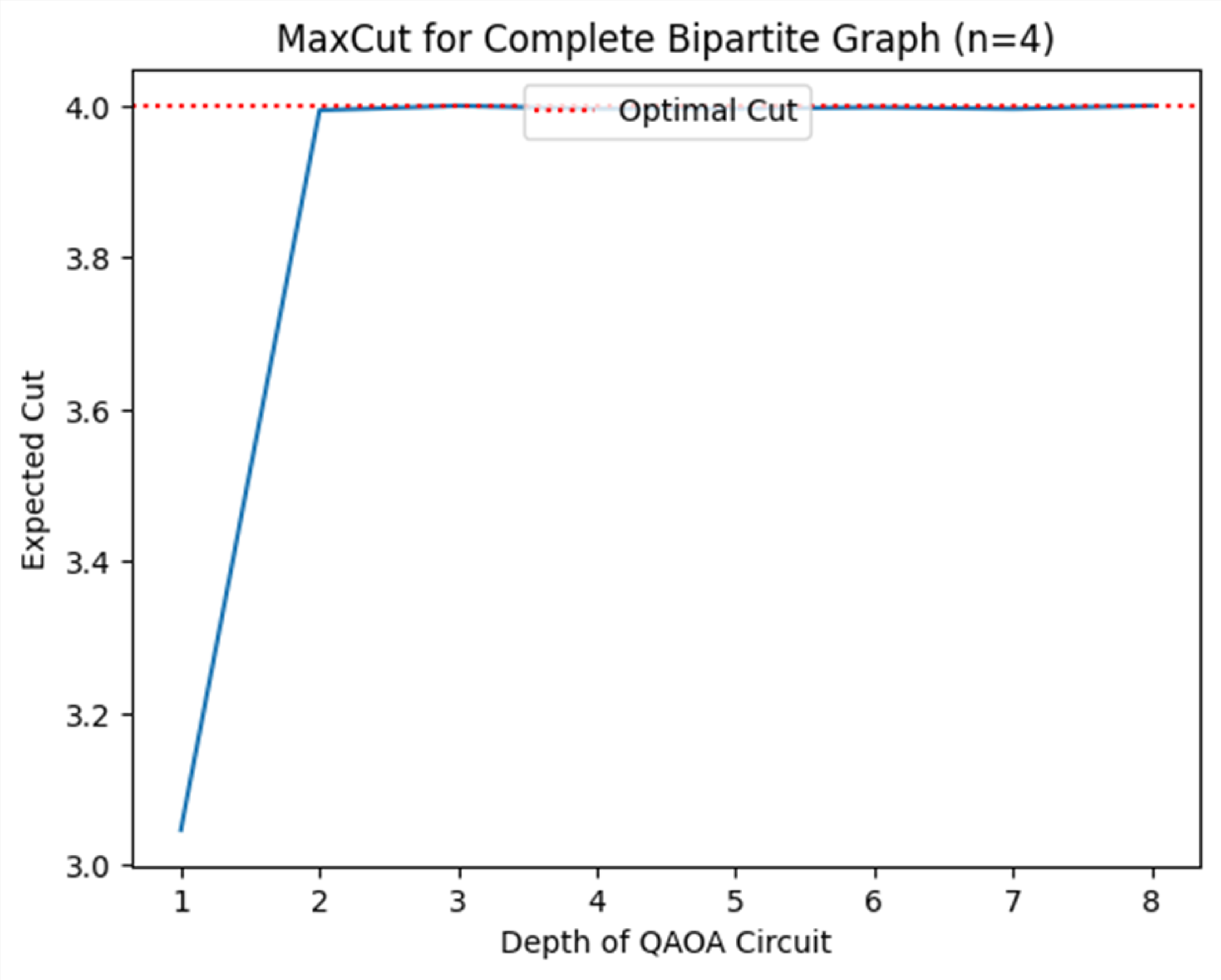


Analysis

4-node Complete Bipartite Graph

Expected Cut is the expectation value of the Cut function by taking the average over the histogram generated by 1024 shots.

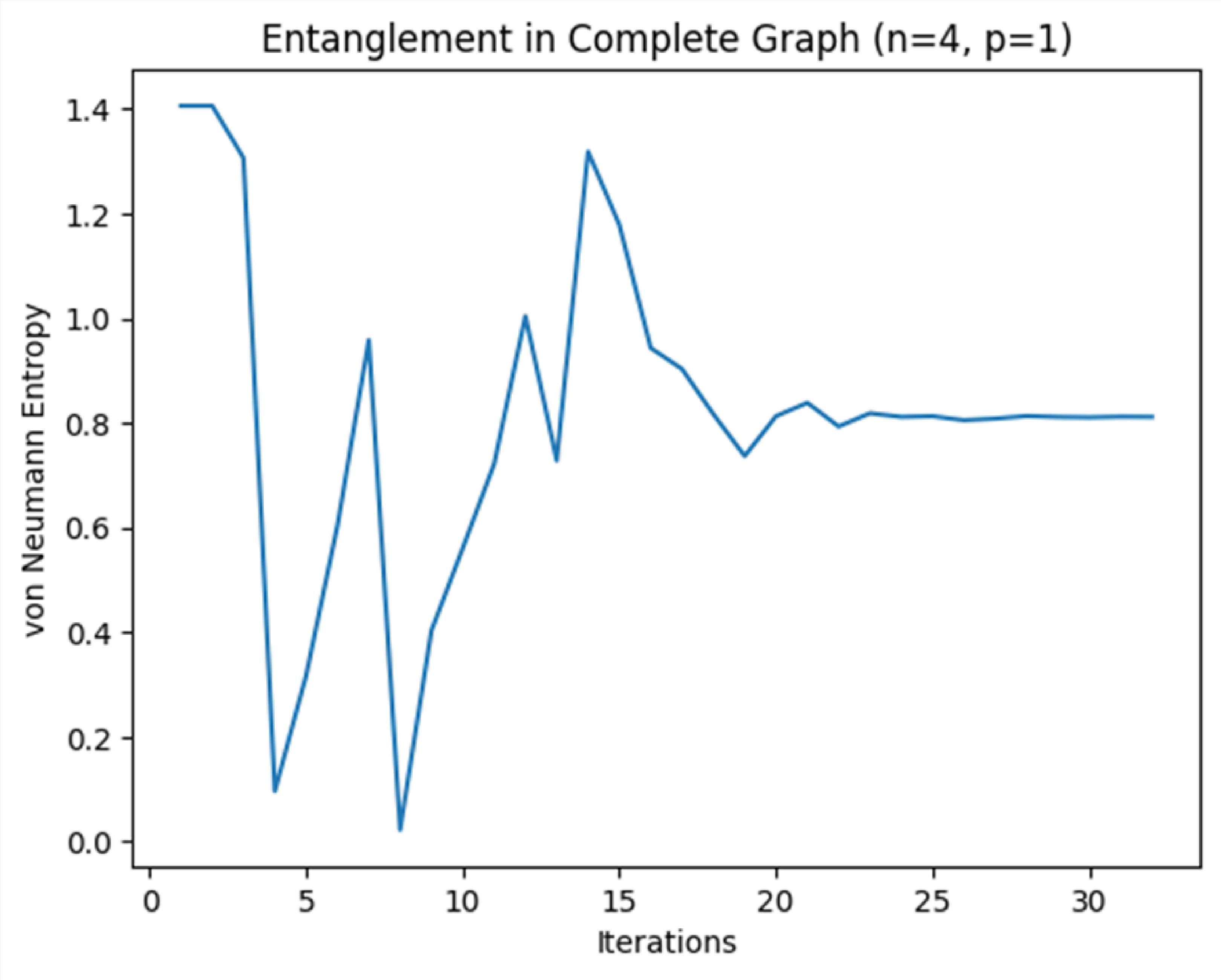
Optimal Cut is the theoretical maximum value of the Cut function for the given graph.



Analysis

4-node Complete Graph
(p=1 layer)

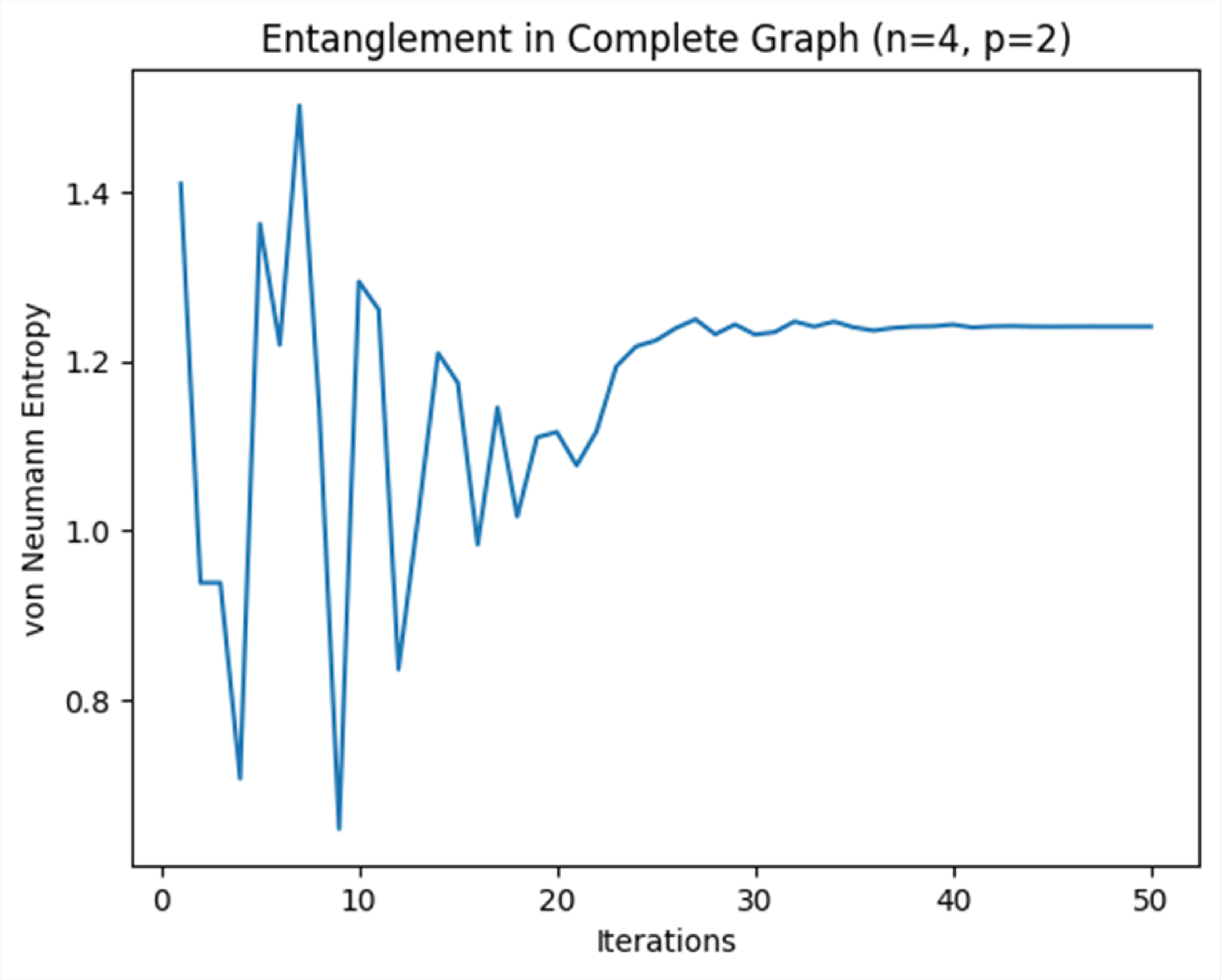
Schmidt Number = 3
Final vN Entropy = 0.81
Iterations to convergence = 32



Analysis

4-node Complete Graph
(p=2 layers)

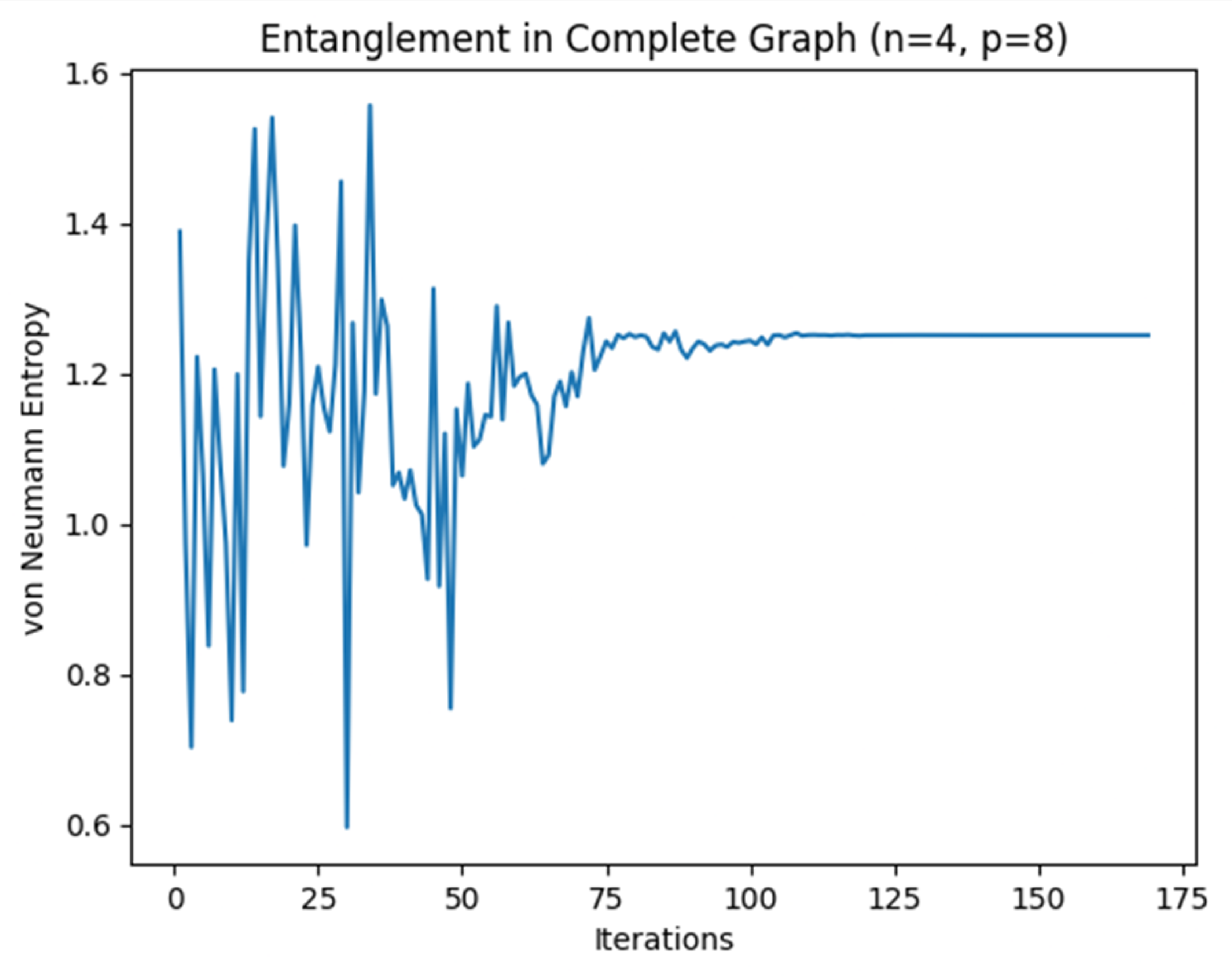
Schmidt Number = 3
Final vN Entropy = 1.24
Iterations to convergence = 50



Analysis

4-node Complete Graph
(p=8 layers)

Schmidt Number = 3
Final vN Entropy = 1.25
Iterations to convergence = 169



Analysis

4-node Complete Graph

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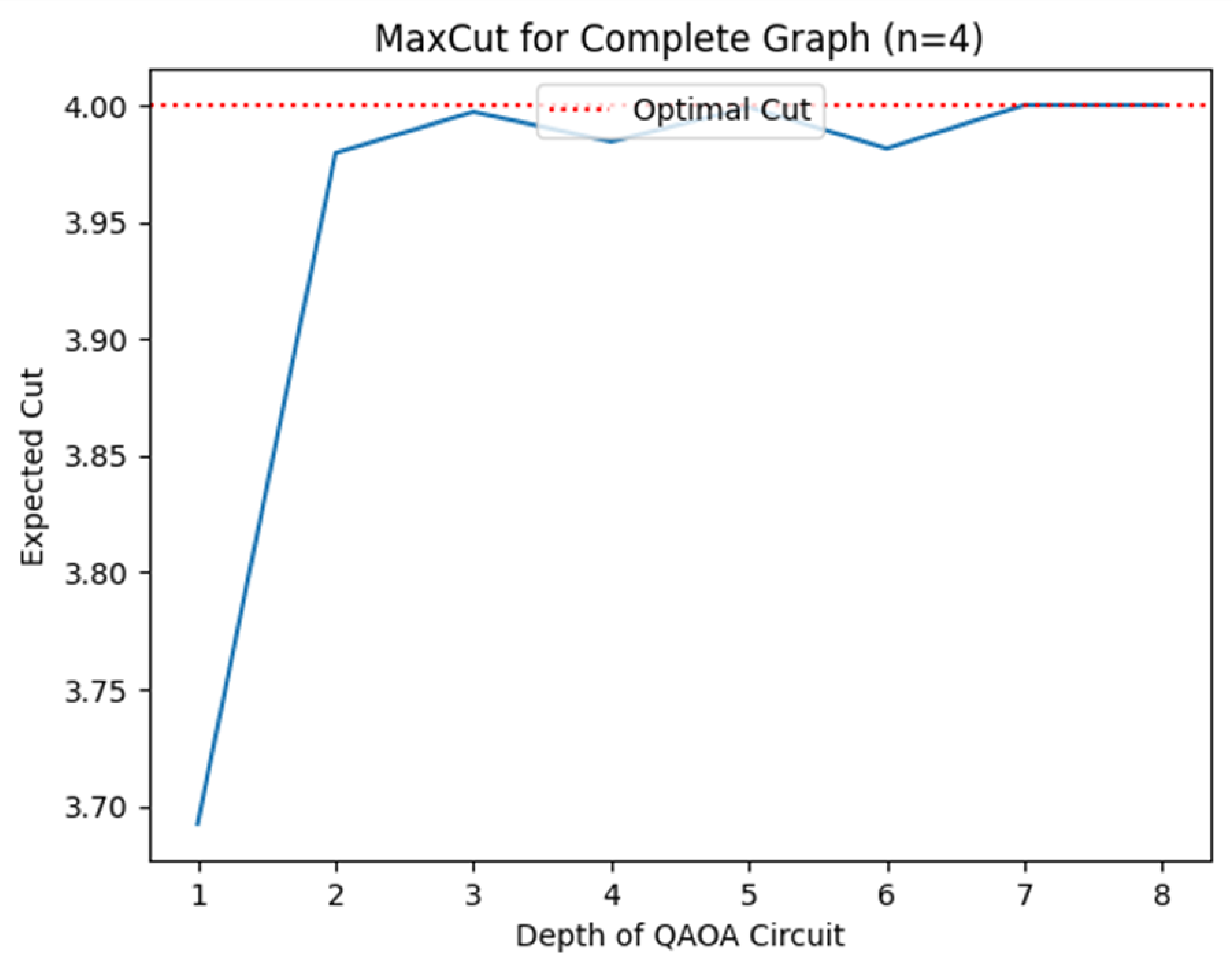
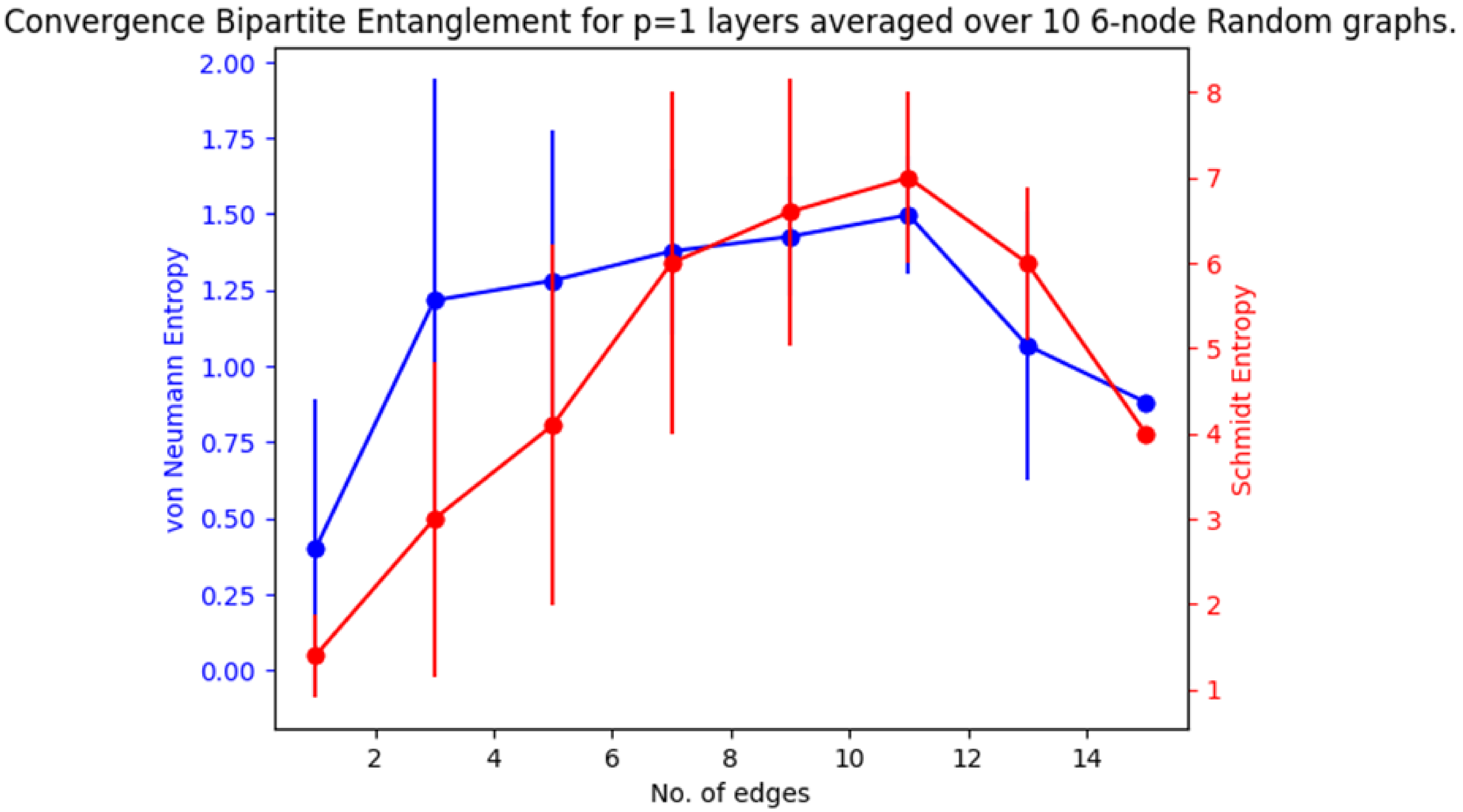
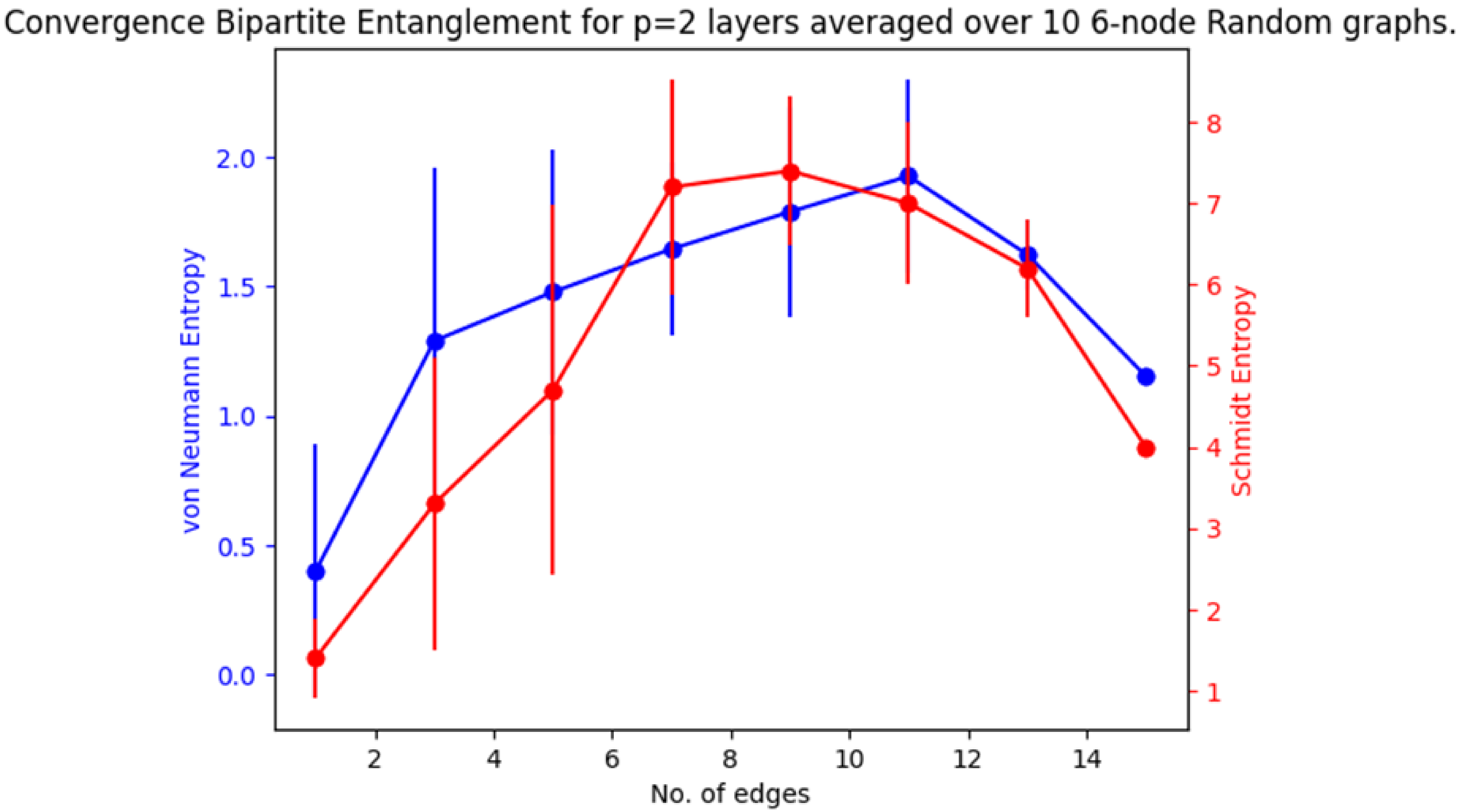


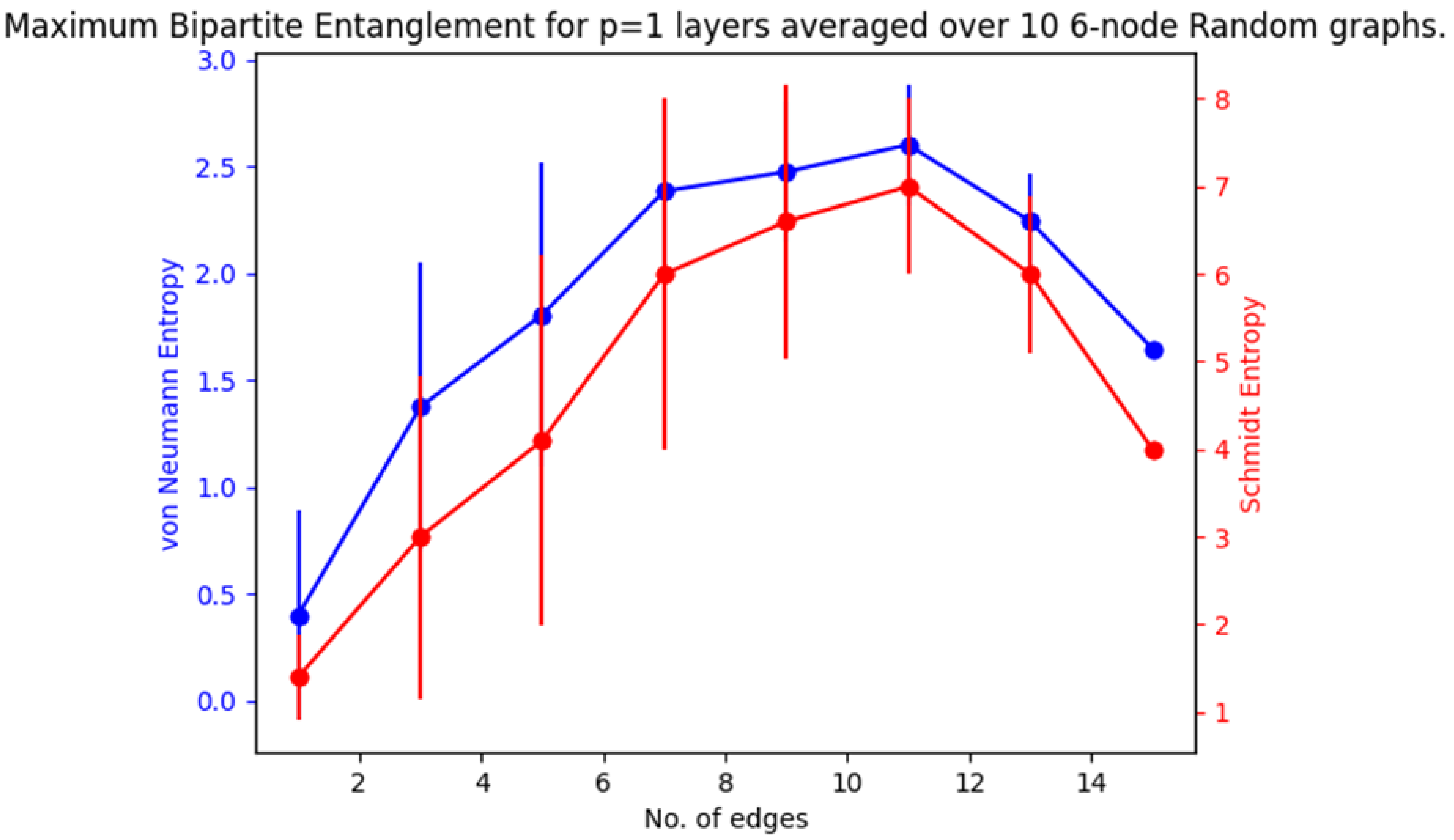
Analysis of 6-node Random Graphs



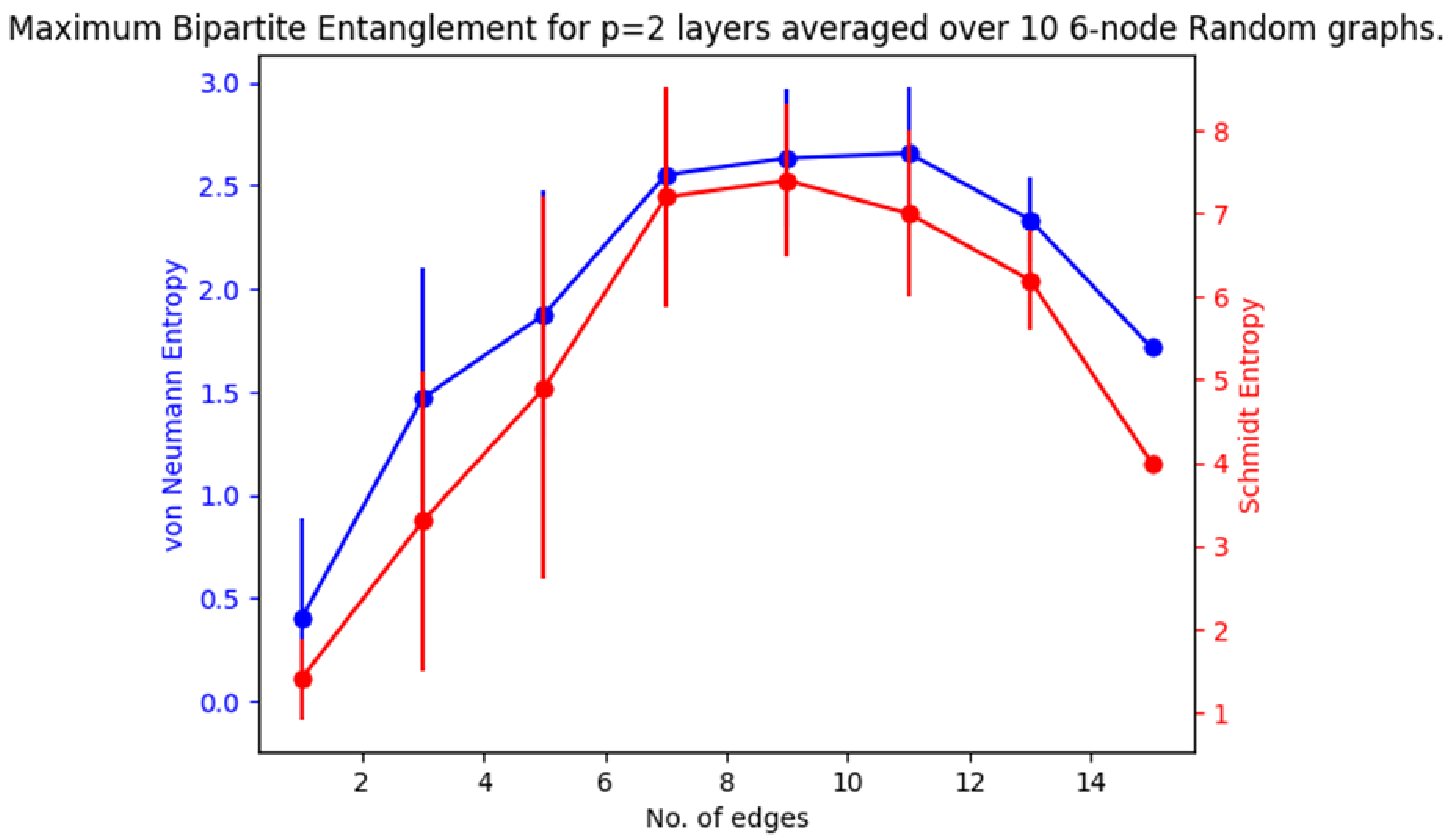
Analysis of 6-node Random Graphs



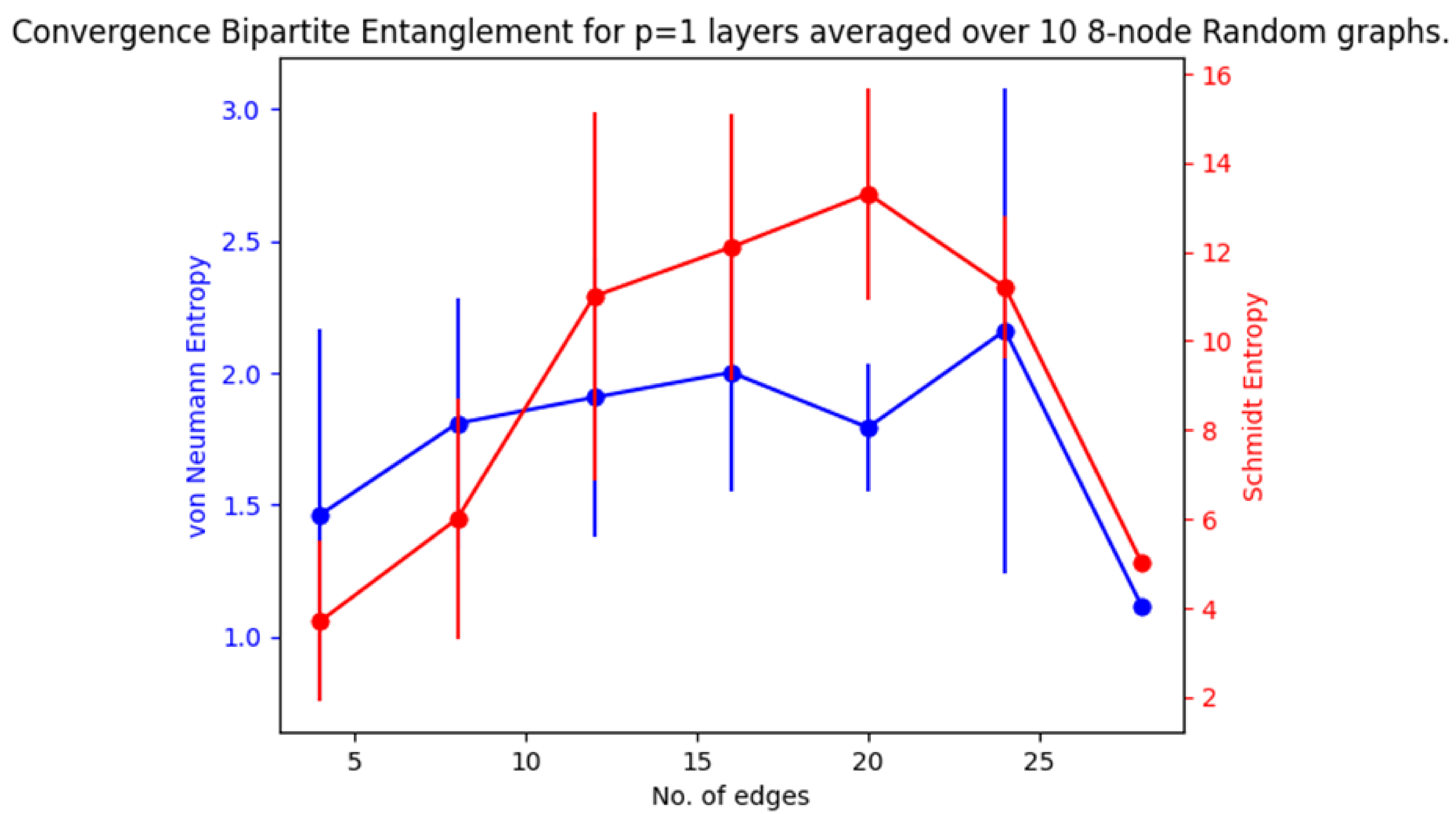
Analysis of 6-node Random Graphs



Analysis of 6-node Random Graphs

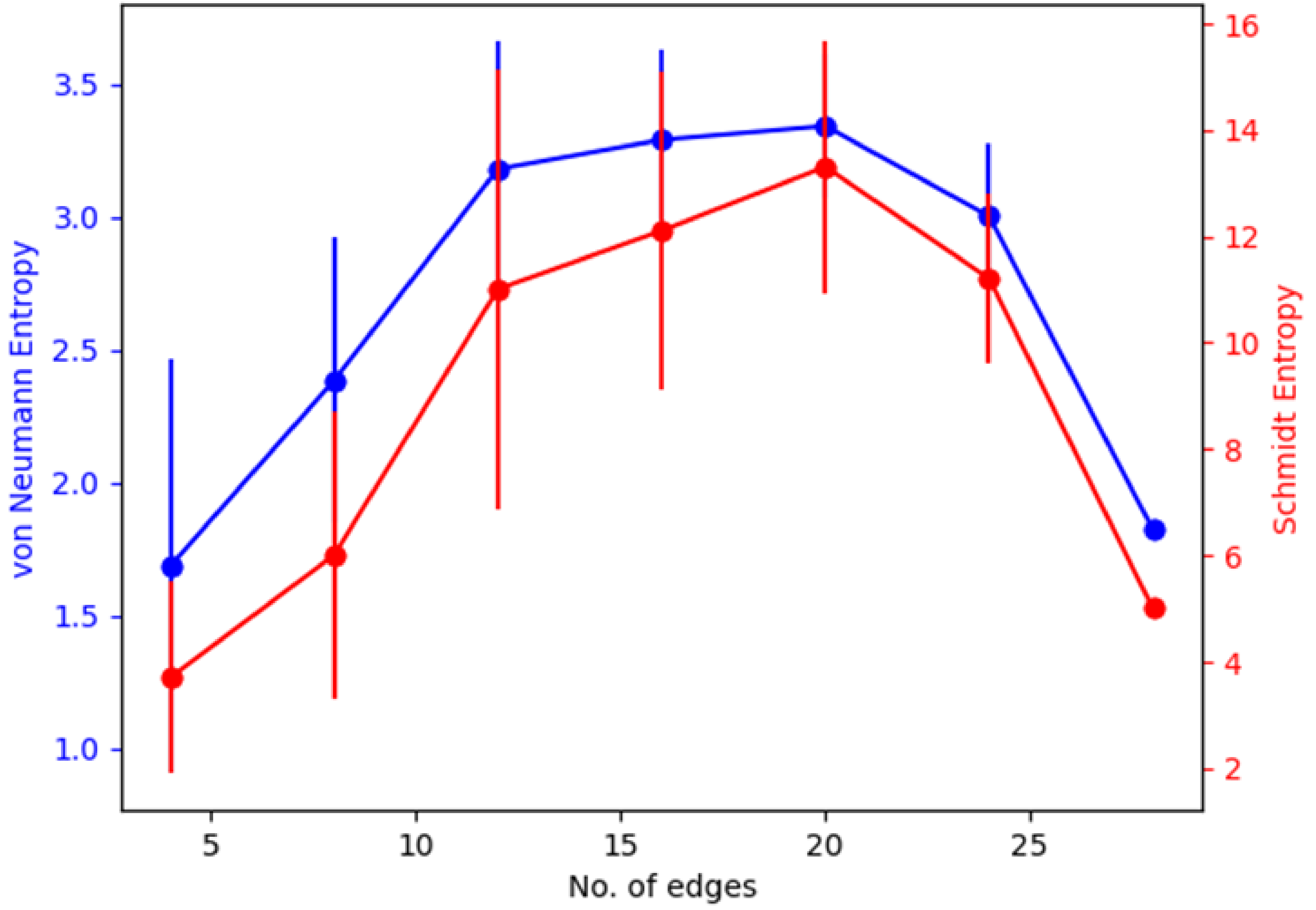


Analysis of 8-node Random Graphs

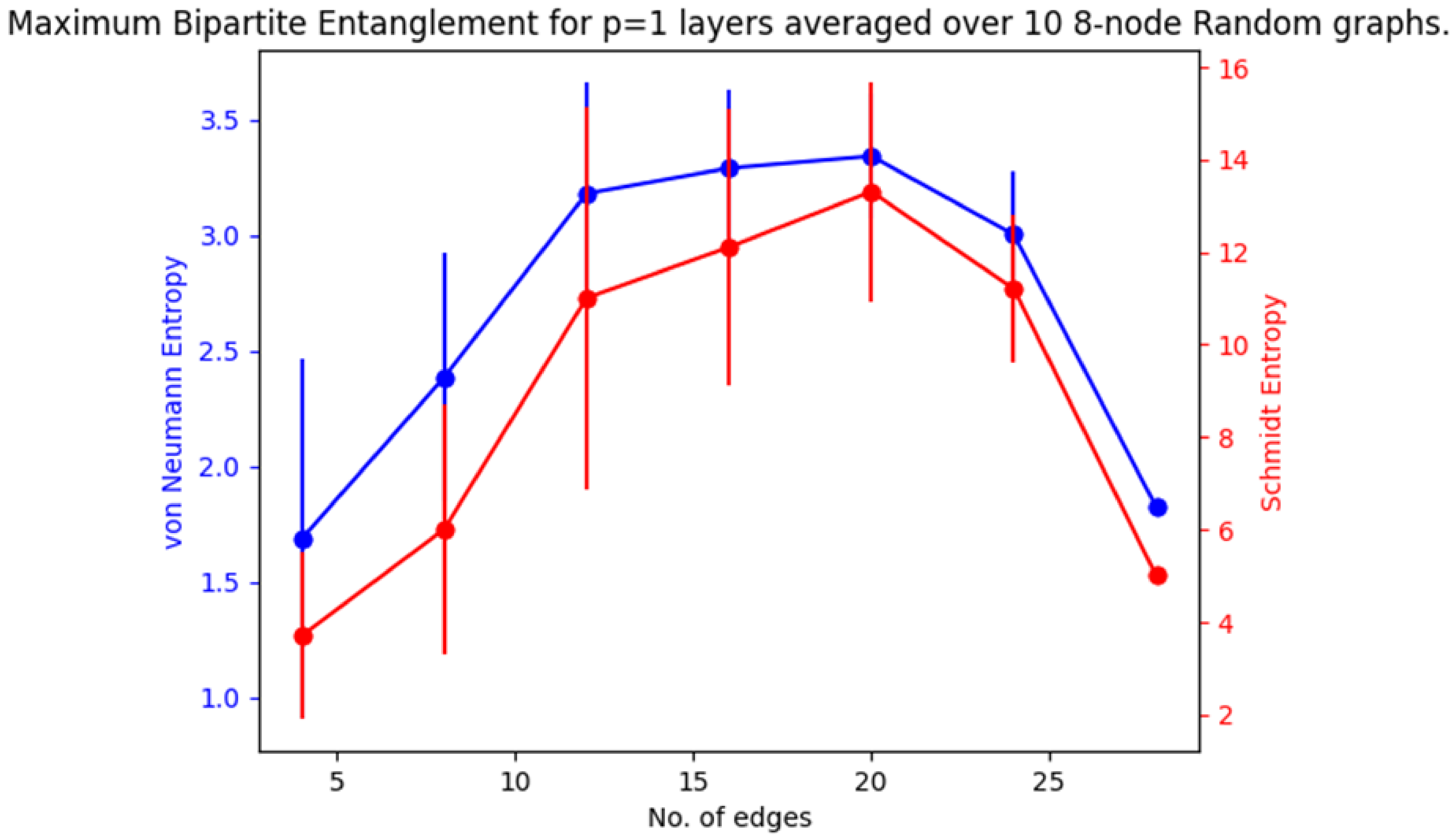


Analysis of 8-node Random Graphs

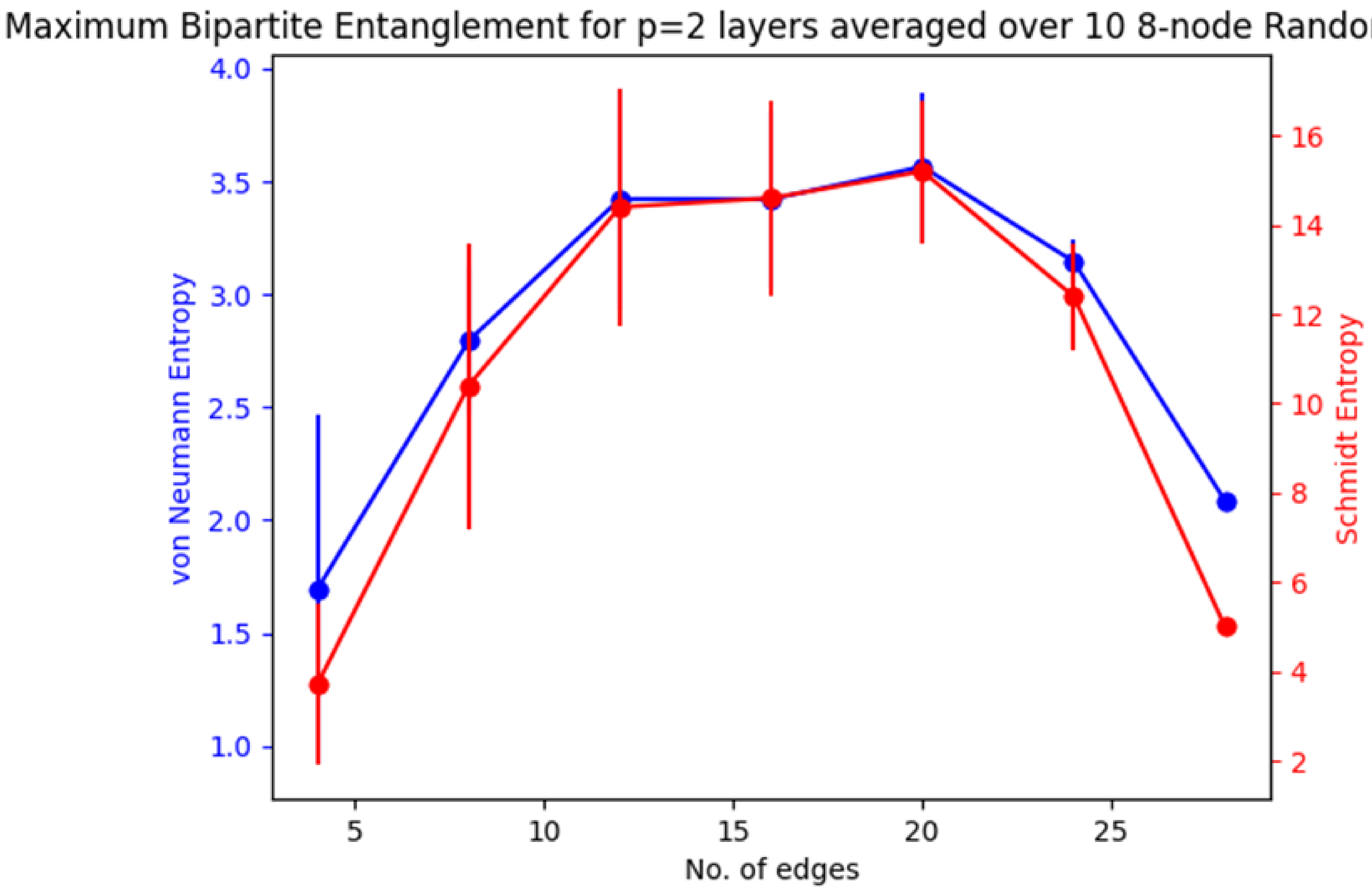
Maximum Bipartite Entanglement for p=1 layers averaged over 10 8-node Random graphs.



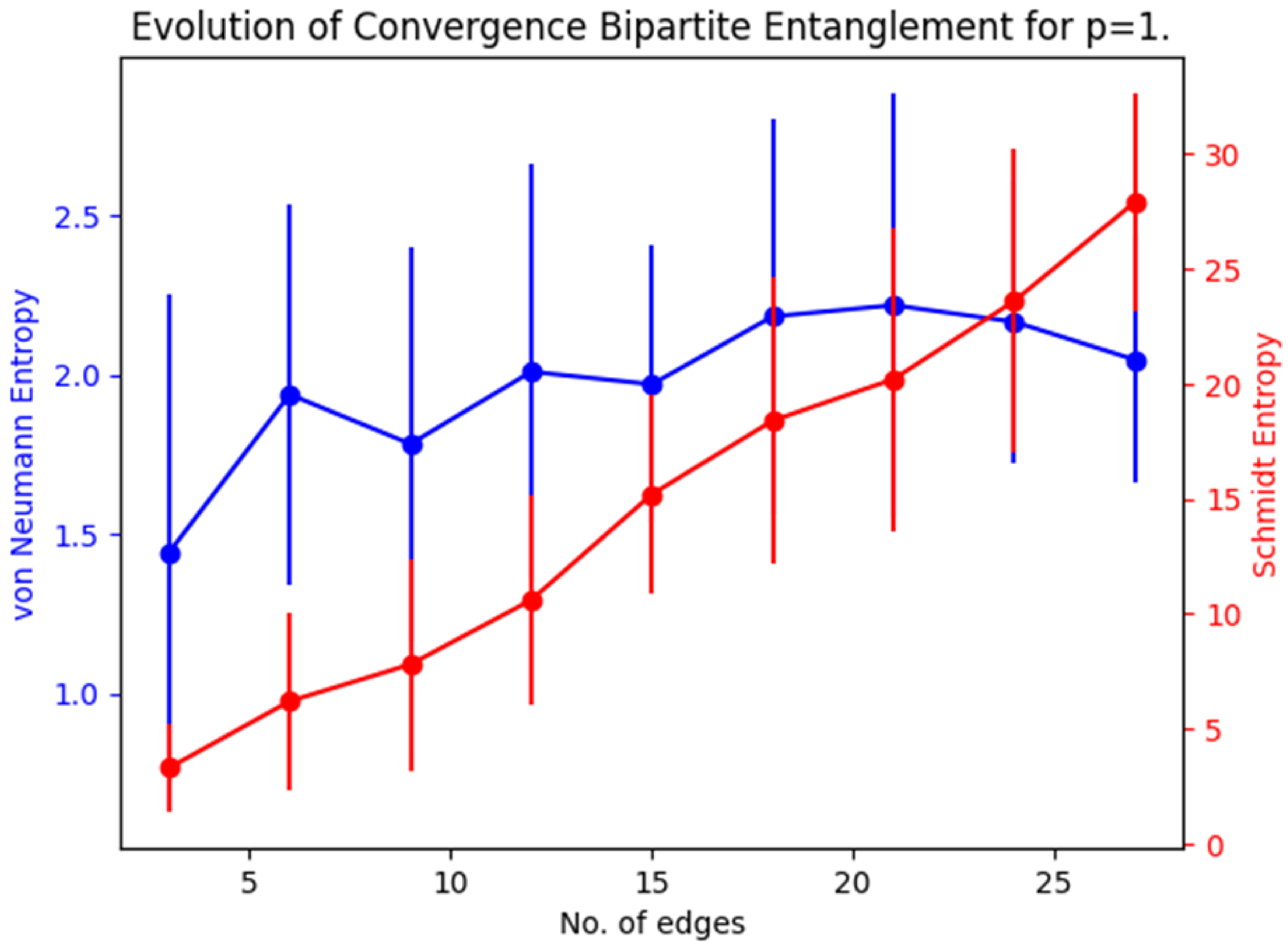
Analysis of 8-node Random Graphs



Analysis of 8-node Random Graphs



Analysis of 10-node Random Graphs

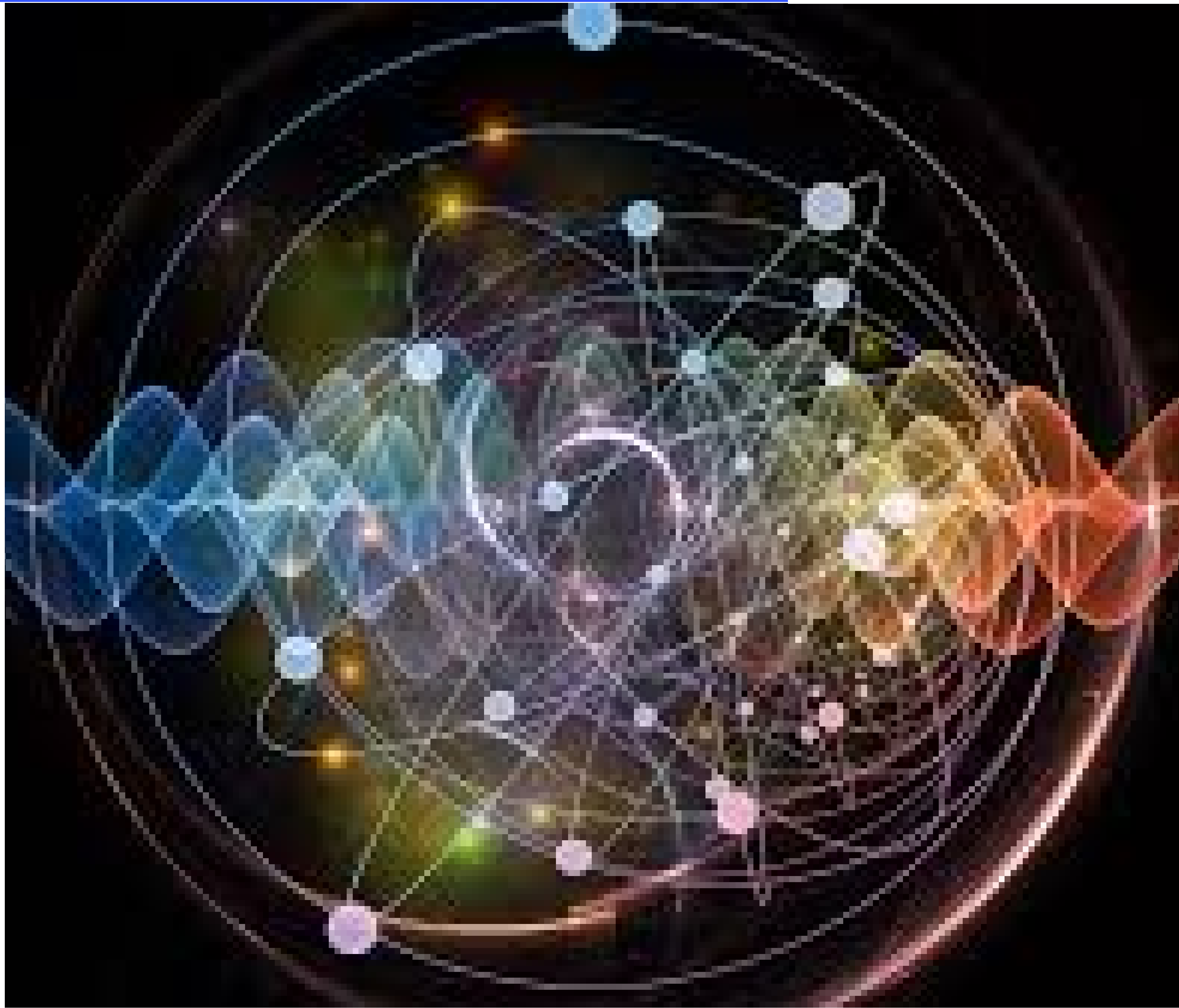


Conclusions

- Bipartite entanglement behaves differently for continuous and discrete measures.

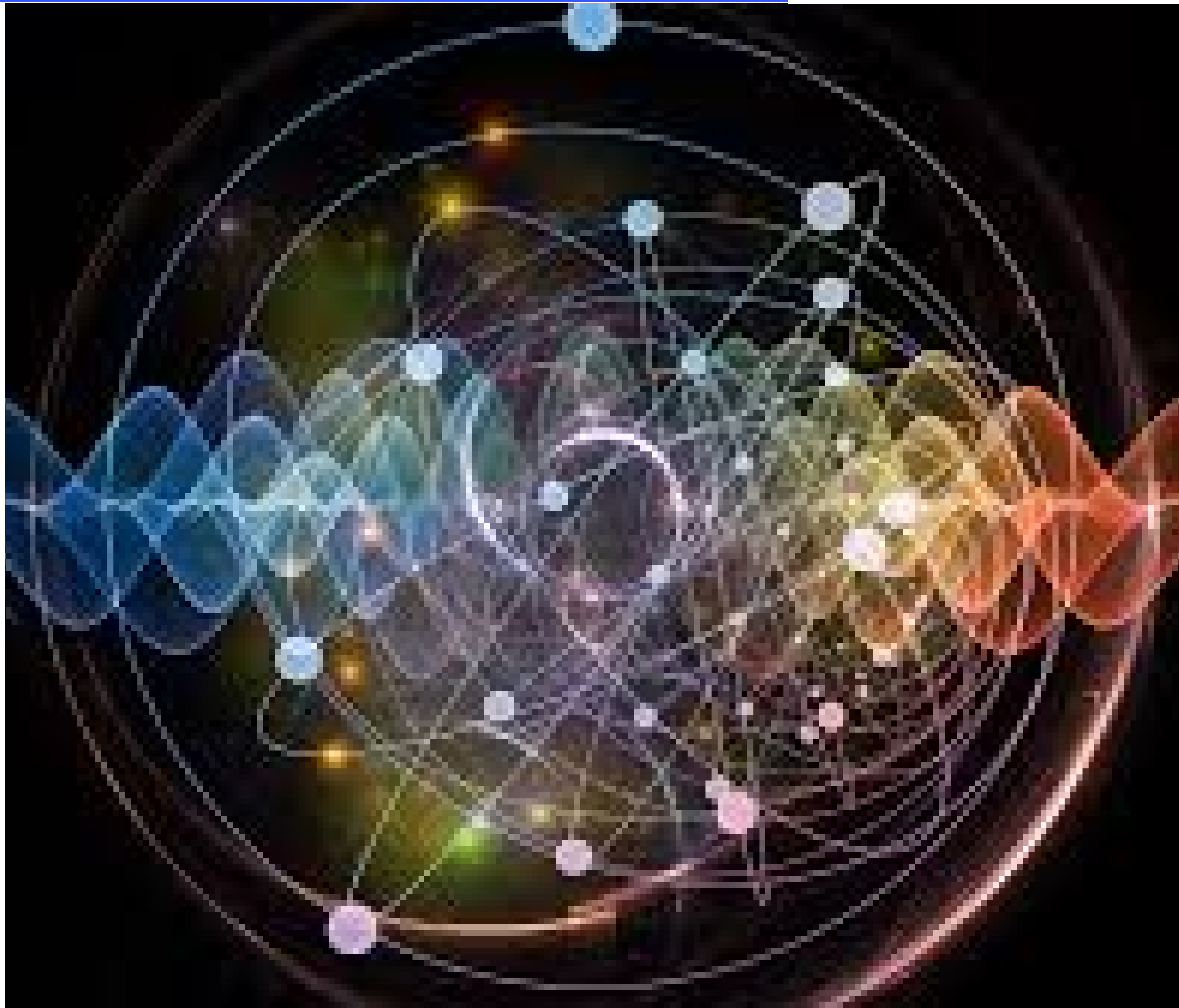
- Low-depth QAOA is sufficient to solve MaxCut on well-connected graphs.

- For Random graphs, a distinct peak in entanglement is observed near phase transition.



Further Research

- Random initialization of parameters
- Conduct a scaling analysis for larger graphs
- Averaging over multiple bipartitions
- Categorizing graph structure for hard problems
- Comparing with classical MaxCut solvers



Discussion