

Problem 1

EE22BTECH11007 - Anek

Question: Find the equations of **AD, BE, CF** where **D, E, F** are midpoints of the triangle **ABC**. The vertices **A, B, C** are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}; \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}; \quad (3)$$

Solution:

Given this information and using the midpoint values found in the questions 1.2.1, the midpoints **D, E, F** are:

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix}; \quad (4)$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}; \quad (5)$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{3}{2} \end{pmatrix}; \quad (6)$$

The normal form of the equation of a line between arbitrary points **A** and **B** i.e., **AB** is

$$\mathbf{n}^T (\mathbf{x} - \mathbf{A}) = 0 \quad (7)$$

where

$$\mathbf{n}^T \mathbf{m} = \mathbf{n}^T (\mathbf{B} - \mathbf{A}) = 0 \quad (8)$$

or,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \quad (9)$$

1) Using the form mentioned in (7) The normal equation for the median **AD** is

$$\mathbf{n}^T (\mathbf{x} - \mathbf{A}) = 0 \quad (10)$$

$$\Rightarrow \mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} \quad (11)$$

Using the equation (8) for the median **AD** $\mathbf{m} = \mathbf{D} - \mathbf{A}$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (12)$$

$$= \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \quad (13)$$

Using the formula (9) We can obtain vector **n**

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{9}{2} \end{pmatrix} \quad (14)$$

Hence the normal equation of median **AD** is

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (15)$$

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3 \quad (16)$$

2) Using the form mentioned in (7) The normal equation for the median **BE** is

$$\mathbf{n}^T (\mathbf{x} - \mathbf{B}) = 0 \quad (17)$$

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{B} \quad (18)$$

Using the equation (8) for the median **BE** $\mathbf{m} = \mathbf{E} - \mathbf{B}$

$$\mathbf{m} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (19)$$

$$= \begin{pmatrix} 3 \\ -9 \end{pmatrix} \quad (20)$$

Using the formula (9), we can obtain vector **n**

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -9 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \end{pmatrix} \quad (21)$$

Hence the normal equation of median **BE** is

$$\begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 & -3 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (22)$$

$$\Rightarrow \begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = 18 \quad (23)$$

3) Using the form mentioned in (7) The normal equation for the median **CF** is

$$\mathbf{n}^T (\mathbf{x} - \mathbf{C}) = 0 \quad (24)$$

$$\Rightarrow \mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{C} \quad (25)$$

Using the equation (8) for the median **CF** $\mathbf{m} = \mathbf{F} - \mathbf{C}$

$$\Rightarrow \mathbf{m} = \begin{pmatrix} \frac{-3}{2} \\ \frac{3}{2} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (26)$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \quad (27)$$

Using the formula (9), we can obtain vector **n**

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} = \begin{pmatrix} \frac{15}{2} \\ \frac{-3}{2} \end{pmatrix} \quad (28)$$

Hence the normal equation of median **CF** is

$$\begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (29)$$

$$\Rightarrow \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = -15 \quad (30)$$

The equations of the medians **AD**, **BE**, **CF** are:

- 1) **AD** : $\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3$
- 2) **BE** : $\begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = 18$
- 3) **CF** : $\begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = -15$

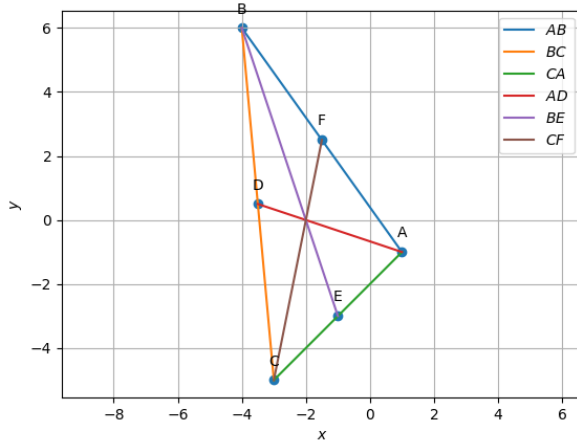


Fig. 3. The triangle ABC and the medians AD, BE, CF plotted using python