Problem 1

EE22BTECH11007 - Anek

Question: Find the equations of **AD**, **BE**, **CF** where **D**, **E**, **F** are midpoints of the triangle **ABC**. The vertices **A**, **B**, **C** are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}; \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}; \tag{3}$$

Solution:

Given this information and using the midpoint values found in the questions 1.2.1, the midpoints D,E,F are:

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix}; \tag{4}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}; \tag{5}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix}; \tag{6}$$

The normal form of the equation of a line between arbitary points A and B i.e, **AB** is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{A} \right) = 0 \tag{7}$$

where

$$\mathbf{n}^\top \mathbf{m} = \mathbf{n}^\top \left(\mathbf{B} - \mathbf{A} \right) = 0 \tag{8}$$

or,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{9}$$

1) Using the form mentioned in (7) The normal equation for the median AD is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{A} \right) = 0 \tag{10}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{A} \tag{11}$$

Using the equation (8) for the median AD $\mathbf{m} = \mathbf{D} - \mathbf{A}$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{13}$$

Using the formula (9) We can obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{9}{2} \end{pmatrix} \tag{14}$$

Hence the normal equation of median AD is

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{15}$$

$$\implies \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3 \tag{16}$$

2) Using the form mentioned in (7) The normal equation for the median BE is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{B} \right) = 0 \tag{17}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{B} \tag{18}$$

Using the equation (8) for the median BE $\mathbf{m} = \mathbf{E} - \mathbf{B}$

$$\implies \mathbf{m} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{19}$$

$$= \begin{pmatrix} 3 \\ -9 \end{pmatrix} \tag{20}$$

Using the formula (9),we can obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -9 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \end{pmatrix} \tag{21}$$

Hence the normal equation of median BE is

$$\begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 & -3 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{22}$$

$$\implies \begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = 18 \tag{23}$$

3) Using the form mentioned in (7) The normal equation for the median CF is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{C} \right) = 0 \tag{24}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{C} \tag{25}$$

Using the equation (8) for the median CF $\mathbf{m} = \mathbf{F} - \mathbf{C}$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{26}$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \tag{27}$$

Using the formula (9), we can obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} = \begin{pmatrix} \frac{15}{2} \\ \frac{-3}{2} \end{pmatrix} \tag{28}$$

Hence the normal equation of median CF is

$$\begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{29}$$

$$\implies \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = -15 \tag{30}$$

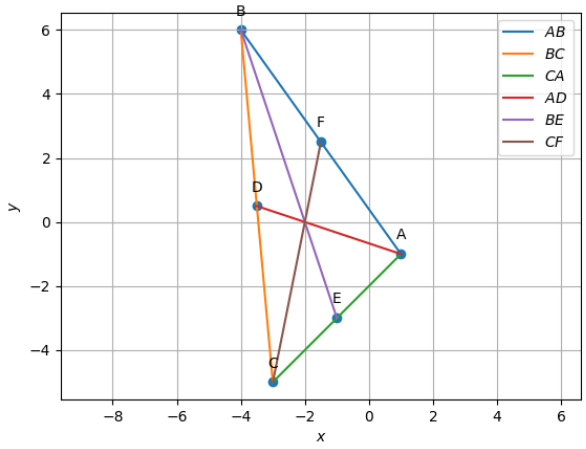

Fig. 3. The triangle ABC and the medians AD,BE,CF plotted using python