Problem 1

EE22BTECH11007 - Anek

1.2.2 Find the equations of AD,BE,CF where D,E,F are midpoints of the triangle ABC. The vertices A,B,C are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}; \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}; \tag{3}$$

Solution:
Given this information and using the midpoint values found in the questions 1.2.1, the midpoints D,E,F are:

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix}; \tag{4}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}; \tag{5}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix}; \tag{6}$$

The normal equation for the side AD is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{A}\right) = 0 \tag{7}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{A} \tag{8}$$

Now we should find $\mathbf{n}$ so that we can find $\mathbf{n}^\top$.

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{9}$$

Where $\mathbf{m} = \mathbf{D} - \mathbf{A}$ for median $\mathbf{AD}$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{11}$$

we can use this to obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{9}{2} \end{pmatrix} \tag{12}$$

The transpose of $\mathbf{n}$ is

$$\mathbf{n}^\top = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \tag{13}$$

Hence the normal equation of side AD is

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3 \tag{15}$$

The normal equation for the side BE is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{B}\right) = 0 \tag{16}$$

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{B} \tag{17}$$

Now we should find $\mathbf{n}$ so that we can find $\mathbf{n}^\top$.

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{18}$$

Where $\mathbf{m} = \mathbf{E} - \mathbf{B}$ for median $\mathbf{BE}$

$$\mathbf{m} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{19}$$

$$= \begin{pmatrix} 3 \\ -9 \end{pmatrix} \tag{20}$$

we can use this to obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -9 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \end{pmatrix} \tag{21}$$

The transpose of $\mathbf{n}$ is

$$\mathbf{n}^\top = \begin{pmatrix} -9 & -3 \end{pmatrix} \tag{22}$$

Hence the normal equation of side BE is

$$\begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 & -3 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{23}$$

$$\implies \begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = 18 \tag{24}$$

The normal equation for the side CF is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{C}\right) = 0 \tag{25}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{C} \tag{26}$$

Now we should find $\mathbf{n}$ so that we can find $\mathbf{n}^\top$.

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{27}$$

Where $\mathbf{m} = \mathbf{F} - \mathbf{C}$ for median $\mathbf{CF}$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{28}$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \tag{29}$$

we can use this to obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} = \begin{pmatrix} \frac{15}{2} \\ \frac{-3}{2} \end{pmatrix} \tag{30}$$

The transpose of $\mathbf{n}$ is

$$\mathbf{n}^\top = \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \tag{31}$$

Hence the normal equation of side CF is

$$\begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{32}$$

$$\implies \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = -15 \tag{33}$$

The equations of the medians AD,BE,CF are:

1) AD: $\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix}$x=-3
2) BE: $\begin{pmatrix} -9 & -3 \end{pmatrix}$x=18
3) CF: $\begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix}$x=-15

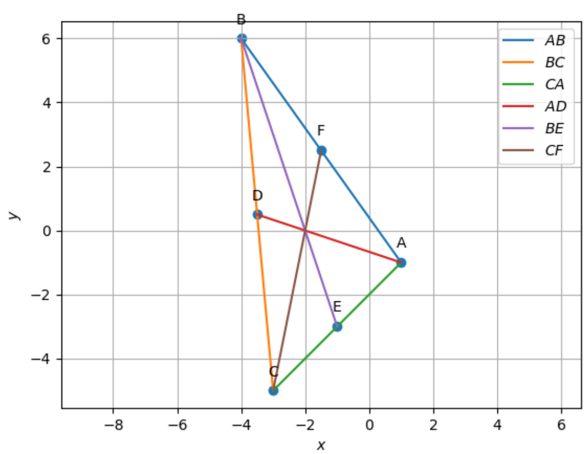

Fig. 3. The triangle ABC and the medians AD,BE,CF plotted using python