

Problem 1

EE22BTECH11007 - Anek

1.2.2 Find the equations of AD,BE,CF where D,E,F are midpoints of the triangle ABC. The vertices A,B,C are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}; \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}; \quad (3)$$

Solution:

Given this information and using the midpoint values found in the questions 1.2.1, the midpoints D,E,F are:

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix}; \quad (4)$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}; \quad (5)$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{3}{2} \end{pmatrix}; \quad (6)$$

The normal equation for the side AD is

$$\mathbf{n}^T (\mathbf{x} - \mathbf{A}) = 0 \quad (7)$$

$$\Rightarrow \mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} \quad (8)$$

Now we should find \mathbf{n} so that we can find \mathbf{n}^T .

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \quad (9)$$

Where $\mathbf{m} = \mathbf{D} - \mathbf{A}$ for median AD

$$\Rightarrow \mathbf{m} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (10)$$

$$= \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \quad (11)$$

we can use this to obtain vector \mathbf{n}

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{9}{2} \end{pmatrix} \quad (12)$$

The transpose of \mathbf{n} is

$$\mathbf{n}^T = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \quad (13)$$

Hence the normal equation of side AD is

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (14)$$

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3 \quad (15)$$

The normal equation for the side BE is

$$\mathbf{n}^T (\mathbf{x} - \mathbf{B}) = 0 \quad (16)$$

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{B} \quad (17)$$

Now we should find \mathbf{n} so that we can find \mathbf{n}^T .

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \quad (18)$$

Where $\mathbf{m} = \mathbf{E} - \mathbf{B}$ for median BE

$$\mathbf{m} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (19)$$

$$= \begin{pmatrix} 3 \\ -9 \end{pmatrix} \quad (20)$$

we can use this to obtain vector \mathbf{n}

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -9 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \end{pmatrix} \quad (21)$$

The transpose of \mathbf{n} is

$$\mathbf{n}^T = \begin{pmatrix} -9 & -3 \end{pmatrix} \quad (22)$$

Hence the normal equation of side BE is

$$\begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 & -3 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (23)$$

$$\Rightarrow \begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = 18 \quad (24)$$

The normal equation for the side CF is

$$\mathbf{n}^T (\mathbf{x} - \mathbf{C}) = 0 \quad (25)$$

$$\Rightarrow \mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{C} \quad (26)$$

Now we should find \mathbf{n} so that we can find \mathbf{n}^T .

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \quad (27)$$

Where $\mathbf{m} = \mathbf{F} - \mathbf{C}$ for median CF

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (28)$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \quad (29)$$

we can use this to obtain vector \mathbf{n}

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} = \begin{pmatrix} \frac{15}{2} \\ \frac{-3}{2} \end{pmatrix} \quad (30)$$

The transpose of \mathbf{n} is

$$\mathbf{n}^\top = \left(\frac{15}{2} \quad \frac{-3}{2} \right) \quad (31)$$

Hence the normal equation of side AD is

$$\left(\frac{15}{2} \quad \frac{-3}{2} \right) \mathbf{x} = \left(\frac{15}{2} \quad \frac{-3}{2} \right) \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (32)$$

$$\implies \left(\frac{15}{2} \quad \frac{-3}{2} \right) \mathbf{x} = -15 \quad (33)$$

The equations of the medians AD,BE,CF are:

$$1) \text{ AD } \left(\frac{3}{2} \quad \frac{9}{2} \right) \mathbf{x} = -3$$

$$2) \text{ BE } \left(-9 \quad -3 \right) \mathbf{x} = 18$$

$$3) \text{ CF } \left(\frac{15}{2} \quad \frac{-3}{2} \right) \mathbf{x} = -15$$

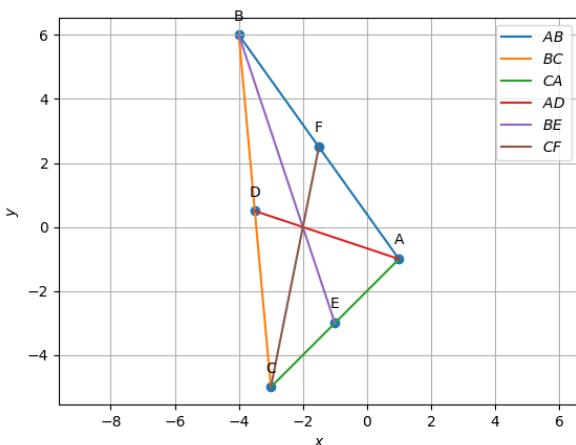


Fig. 3. The triangle ABC and the medians AD,BE,CF plotted using python