Problem 1

EE22BTECH11007 - Anek

1.2.2 Find the equations of $\mathbf{AD}, \mathbf{BE}, \mathbf{CF}$ where $\mathbf{D}, \mathbf{E}, \mathbf{F}$ are midpoints of the triangle $\mathbf{ABC}$. The vertices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}; \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}; \tag{3}$$

Solution:
Given this information and using the midpoint values found in the questions 1.2.1, the midpoints D,E,F are:

$$\mathbf{D} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix}; \tag{4}$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}; \tag{5}$$

$$\mathbf{F} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix}; \tag{6}$$

The normal form of the equation of a line between arbitary points A and B $\mathbf{AB}$ is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{A}\right) = 0 \tag{7}$$

where

$$\mathbf{n}^\top \mathbf{m} = \mathbf{n}^\top \left(\mathbf{B} - \mathbf{A}\right) = 0 \tag{8}$$

or,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{9}$$

1) Using the form mentioned in (7) The normal equation for the median AD is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{A}\right) = 0 \tag{10}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{A} \tag{11}$$

Now we should find $\mathbf{n}$ so that we can find $\mathbf{n}^\top$.

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{12}$$

Where $\mathbf{m} = \mathbf{D} - \mathbf{A}$ for median $\mathbf{AD}$

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-7}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{14}$$

we can use this to obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{-9}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} \frac{3}{2} \\ \frac{9}{2} \end{pmatrix} \tag{15}$$

Hence the normal equation of normal $\mathbf{AD}$ is

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3 \tag{17}$$

2) Using the form mentioned in (7) The normal equation for the normal $\mathbf{BE}$ is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{B}\right) = 0 \tag{18}$$

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{B} \tag{19}$$

Now we should find $\mathbf{n}$ so that we can find $\mathbf{n}^\top$.

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{20}$$

Where $\mathbf{m} = \mathbf{E} - \mathbf{B}$ for median $\mathbf{BE}$

$$\mathbf{m} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{21}$$

$$= \begin{pmatrix} 3 \\ -9 \end{pmatrix} \tag{22}$$

we can use this to obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ -9 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \end{pmatrix} \tag{23}$$

Hence the normal equation of side $\mathbf{BE}$ is

$$\begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -9 & -3 \end{pmatrix} \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{24}$$

$$\implies \begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = 18 \tag{25}$$

3) Using the form mentioned in (7) The normal equation for the side **CF** is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{C}\right) = 0 \tag{26}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{C} \tag{27}$$

Now we should find $\mathbf{n}$ so that we can find $\mathbf{n}^\top$.

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{28}$$

Where $\mathbf{m} = \mathbf{F} - \mathbf{C}$ for median **CF**

$$\implies \mathbf{m} = \begin{pmatrix} \frac{-3}{2} \\ \frac{5}{2} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{29}$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \tag{30}$$

we can use this to obtain vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} = \begin{pmatrix} \frac{15}{2} \\ \frac{-3}{2} \end{pmatrix} \tag{31}$$

Hence the normal equation of side **CF** is

$$\begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{32}$$

$$\implies \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = -15 \tag{33}$$

The equations of the medians $\mathbf{AD}, \mathbf{BE}, \mathbf{CF}$ are:

1) $\mathbf{AD} : \begin{pmatrix} \frac{3}{2} & \frac{9}{2} \end{pmatrix} \mathbf{x} = -3$
2) $\mathbf{BE} : \begin{pmatrix} -9 & -3 \end{pmatrix} \mathbf{x} = 18$
3) $\mathbf{CF} : \begin{pmatrix} \frac{15}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} = -15$

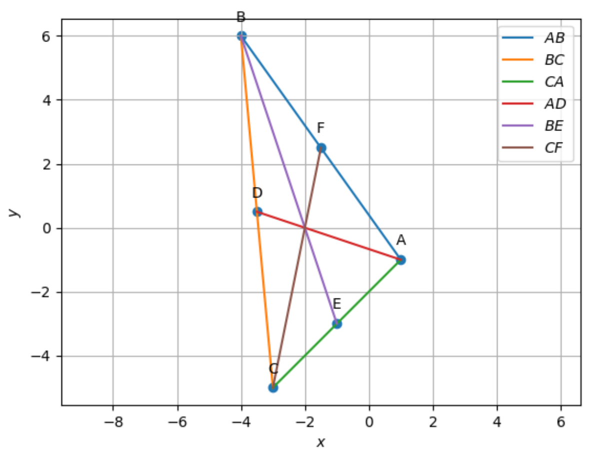

Fig. 3. The triangle ABC and the medians AD,BE,CF plotted using python