

Problem 1

EE22BTECH11007 - Anek

1.2.2 Find the equations of AD, BE, CF where D, E, F are midpoints of the triangle ABC. The vertices A, B, C are:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}; \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}; \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}; \quad (3)$$

Given this information and using the midpoint values found in the questions 1.2.1, the midpoints D, E, F are:

$$\mathbf{D} = \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix}; \quad (4)$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}; \quad (5)$$

$$\mathbf{F} = \begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix}; \quad (6)$$

The equation for AD is given by

$$\mathbf{x} = \mathbf{A} + k\mathbf{m} \quad (7)$$

where

$$\mathbf{m} = \mathbf{D} - \mathbf{A} \quad (8)$$

$$= \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (9)$$

$$= \begin{pmatrix} -\frac{9}{2} \\ \frac{3}{2} \end{pmatrix} \quad (10)$$

Hence we get,

$$\mathbf{AD} : \mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + k \begin{pmatrix} -\frac{9}{2} \\ \frac{3}{2} \end{pmatrix} \quad (11)$$

The parametric equation for BE is given by

$$\mathbf{x} = \mathbf{B} + k\mathbf{m} \quad (12)$$

where

$$\mathbf{m} = \mathbf{E} - \mathbf{B} \quad (13)$$

$$= \begin{pmatrix} -1 \\ -3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (14)$$

$$= \begin{pmatrix} 3 \\ -9 \end{pmatrix} \quad (15)$$

Hence we get,

$$\mathbf{BE} : \mathbf{x} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + k \begin{pmatrix} 3 \\ -9 \end{pmatrix} \quad (16)$$

The parametric equation for CF is given by

$$\mathbf{x} = \mathbf{C} + k\mathbf{m} \quad (17)$$

where

$$\mathbf{m} = \mathbf{F} - \mathbf{C} \quad (18)$$

$$= \begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (19)$$

$$= \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \quad (20)$$

Hence we get,

$$\mathbf{CF} : \mathbf{x} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} + k \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \quad (21)$$

So the equations of AD, BE, CF are:

$$\mathbf{AD} : \mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + k \begin{pmatrix} -\frac{9}{2} \\ \frac{3}{2} \end{pmatrix} \quad (22)$$

$$\mathbf{BE} : \mathbf{x} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + k \begin{pmatrix} 3 \\ -9 \end{pmatrix} \quad (23)$$

$$\mathbf{CF} : \mathbf{x} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} + k \begin{pmatrix} \frac{3}{2} \\ \frac{15}{2} \end{pmatrix} \quad (24)$$