Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{1}$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} -2 \\ -5 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} -1 \\ 8 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} 3 \\ -3 \end{pmatrix}$	CA
$\|\mathbf{A}-\mathbf{B}\|$	5.38	length of AB
$\|\mathbf{B}-\mathbf{C}\|$	8.06	length of BC
$\|\mathbf{C}-\mathbf{A}\|$	4.24	length of CA
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} -5 \\ 2 \end{pmatrix}$	AB
c_1	-10	
$\mathbf{n_2}$	$\begin{pmatrix} 8 \\ 1 \end{pmatrix}$	BC
c_2	-5	
$\mathbf{n_3}$	$\begin{pmatrix} -3 \\ -3 \end{pmatrix}$	CA
c_3	-6	
Area	10.5	Area of Triangle
$\angle A$	113.19°	Angles
$\angle B$	28.92°	
$\angle C$	37.874°	

TABLE 1: Vectors.

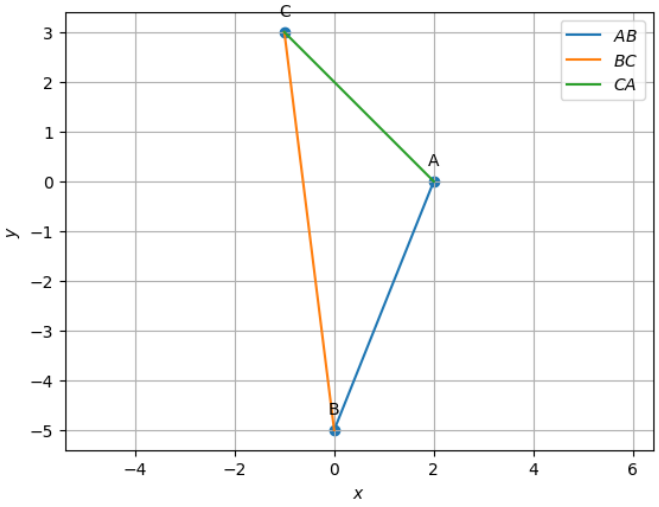

Fig. 1: triangle plotted using python

2 Median

parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} -0.5 \\ -1 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} 0.5 \\ 1.5 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} 1 \\ -2.5 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} -2.5 \\ -1 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} -1 \\ 2.5 \end{pmatrix}$	
c_4	-2	
$\mathbf{m_5}$	$\begin{pmatrix} 0.5 \\ 6.5 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} 6.5 \\ -0.5 \end{pmatrix}$	
c_5	2.5	
$\mathbf{m_6}$	$\begin{pmatrix} 2 \\ -5.5 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} -5.5 \\ -2 \end{pmatrix}$	
c_6	-0.5	
$\mathbf{G}$	$\begin{pmatrix} 0.33 \\ -0.66 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

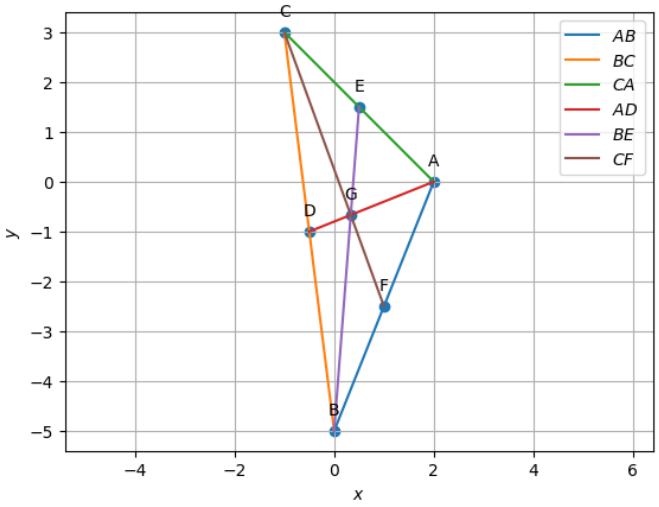

Fig. 2: medians plotted using python

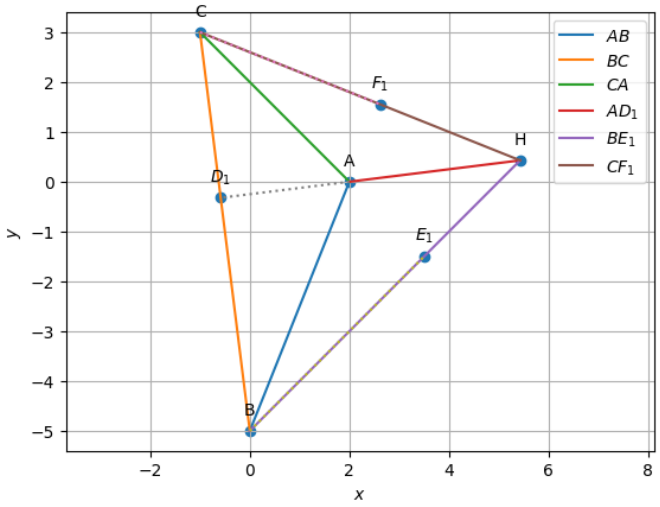

Fig. 3: altitudes plotted using python

3 Altitude

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix}-0.58\\-0.32\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$\begin{pmatrix}3.5\\-1.5\end{pmatrix}$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}2.62\\1.55\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}-2.58\\-0.32\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}-0.32\\2.58\end{pmatrix}$	
c_7	-0.64	
$\mathbf{m_8}$	$\begin{pmatrix}3.5\\3.5\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}3.5\\-3.5\end{pmatrix}$	
c_8	17.5	
$\mathbf{m_9}$	$\begin{pmatrix}3.62\\-1.44\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}-1.44\\-3.62\end{pmatrix}$	
c_9	-9.41	
H	$\begin{pmatrix}5.42\\0.42\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 Perpendicular Bisector

parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix}-2\\1\end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix}-1\\-2\end{pmatrix}$	
c_{10}	-7.5	
$\mathbf{m_{11}}$	$\begin{pmatrix}3\\3\end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix}-3\\3\end{pmatrix}$	
c_{11}	3	
$\mathbf{m_{12}}$	$\begin{pmatrix}-5\\-2\end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix}2\\-5\end{pmatrix}$	
c_{12}	-10.5	
O	$\begin{pmatrix}1.83\\2.83\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	2.83	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	46.39°	$\angle BOC = 2\angle BAC$
$\angle BAC$	23.19°	
$\angle AOC$	96.7°	$\angle AOC = 2\angle ABC$
$\angle ABC$	48.36°	
$\angle AOB$	216.86°	$\angle AOB = 2\angle BCA$
$\angle BCA$	108.43°	

TABLE 4: Perpendicular Bisector.

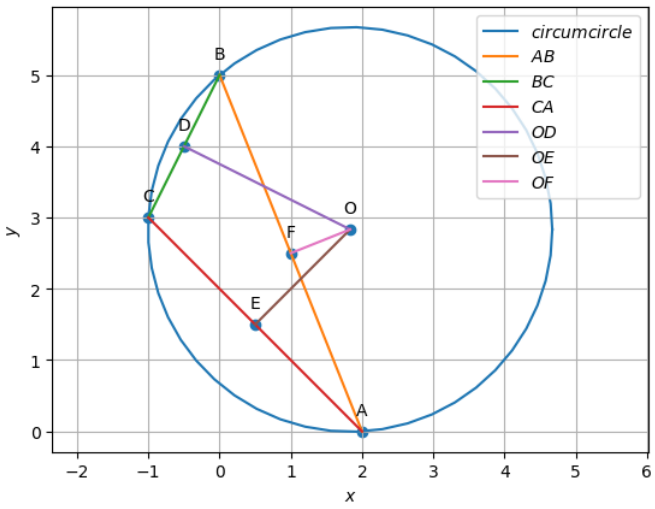

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix}1.07\\-0.22\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}0.22\\-1.07\end{pmatrix}$	
c_{13}	0.44	
$\mathbf{m_{14}}$	$\begin{pmatrix}0.24\\1.92\end{pmatrix}$	BI
$\mathbf{n_{14}}$	$\begin{pmatrix}-1.92\\0.24\end{pmatrix}$	
c_{14}	-1.23	
$\mathbf{m_{15}}$	$\begin{pmatrix}-0.831\\1.69\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}-1.69\\-0.83\end{pmatrix}$	
c_{15}	-0.79	
$\mathbf{I}$	$\begin{pmatrix}0.60\\-0.28\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}-0.57\\-0.43\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}1.44\\0.55\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}1.70\\-0.72\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	0.43	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	56.59°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	14.46°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	18.93°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.

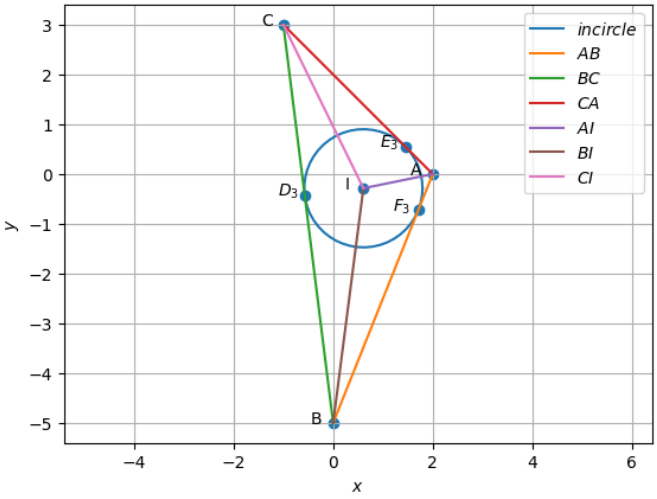

Fig. 5: Angle bisectors plotted using python