Question 11.16.4.9
If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when:

1. The digits are repeated?
2. The repetition of digits is not allowed?

solution: Let X be a random variable such that

$$X = \begin{cases} 0 & n < 5000 \\ 1 & n > 5000 \end{cases} \tag{1}$$

Let Y be a random variable such that:

$$Y = \begin{cases} 0 & n \not\equiv 0 \pmod 5 \\ 1 & n \equiv 0 \pmod 5 \end{cases} \tag{2}$$

We need to find $\Pr(Y \mid X)$ in each case.

$$\Pr(Y \mid X) = \frac{\Pr(XY)}{\Pr(X)} \tag{3}$$

Let's solve each part separately.

(i) Digits are Repeated
For n to be greater than 5000, first digit must be 5 or 7. Hence,

$$\Pr(X = 1) = \frac{249}{3125} \tag{4}$$

For n to be greater than 5000 and also divisble by 5:

1. first digit must be 5 or 7.
2. last digit must be 5 or 0.

Hence,

$$\Pr(XY = 1) = \frac{99}{3125} \tag{5}$$

With this information we can find the required answer,

$$\Pr(Y \mid X = 1) = \frac{\Pr(XY = 1)}{\Pr(X = 1)} \tag{6}$$

$$\Pr(Y \mid X = 1) = \frac{99}{249} \tag{7}$$

$$\implies \Pr(Y \mid X = 1) = \frac{33}{83} \tag{8}$$

(ii) No Repetition of Digits

For n to be greater than 5000, first digit must be 5 or 7. Hence,

$$\Pr\left(X=1\right)=\frac{48}{120} \tag{9}$$

The cases for n to be greater than 5000 and also divisble by 5:

1. first digit must be 5 and last digit must be 0.
2. first digit must be 7 and last digit must be 5 or 0.

Hence,

$$\Pr\left(XY=1\right)=\frac{18}{120} \tag{10}$$

With this information we can find the required answer,

$$\Pr\left(Y \mid X=1\right)=\frac{\Pr\left(XY=1\right)}{\Pr\left(X=1\right)} \tag{11}$$

$$\Pr\left(Y \mid X=1\right)=\frac{18}{48} \tag{12}$$

$$\implies \Pr\left(Y \mid X=1\right)=\frac{3}{8} \tag{13}$$