Question 11.16.4.9
If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when:

1. The digits are repeated?
2. The repetition of digits is not allowed?

solution: Let X be a random variable such that:

$$X = \begin{cases} 0 & n \not\equiv 0 \pmod 5 \\ 1 & n \equiv 0 \pmod 5 \end{cases} \tag{1}$$

Let X_1, X_2, X_3, X_4 be the first,second,third and fourth digits ofa 4-digit number N respectively.

Let's solve each part separately.

(i) Repetition of digits
Let number of favourable outcomes be N(A) and total outcomes be N(T).

For N > 5000,

1. X_1 must be 5 or 7.
2. X_2, X_3, X_4 can be 0,1,3,5,7.

We must also exclude the case of 5000. Hence,

$$N(T) = (2 * 5 * 5 * 5) - 1 \tag{2}$$
$$\implies N(T) = 249 \tag{3}$$

For N to be greater than 5000 and also divisble by 5:

1. X_1 must be 5 or 7.
2. X_2, X_3 can be 0,1,3,5,7.
3. X_4 must be 0 or 5.

Here also we must exclude the case of 5000.

$$N(A) = (2 * 5 * 5 * 2) - 1 \tag{4}$$
$$\implies N(A) = 99 \tag{5}$$

With this information we can find the required answer,

$$\Pr(X = 1) = \frac{N(A)}{N(T)} \tag{6}$$
$$\implies \Pr(X = 1) = \frac{33}{83} \tag{7}$$

(ii) No Repetition of Digits
Let number of favourable outcomes be N(B) and total outcomes be N(T).

For N > 5000,

1. X_1 must be 5 or 7.

2. X_2, X_3, X_4 have no constraints.

Hence,

$$N(T) = (2 * 4 * 3 * 2) \tag{8}$$
$$\implies N(T) = 48 \tag{9}$$

For N > 5000 and also divisble by 5:

$$X_4 = \begin{cases} 0 & X_1 = 5 \\ 5, 0 & X_1 = 7 \end{cases} \tag{10}$$

Hence,

$$N(B) = (1 * 3 * 2 * 1) + (1 * 3 * 2 * 2) \tag{11}$$
$$\implies N(B) = 18 \tag{12}$$

With this information we can find the required answer,

$$\Pr(X = 1) = \frac{N(B)}{N(T)} \tag{13}$$
$$\implies \Pr(X = 1) = \frac{3}{8} \tag{14}$$