Question 11.16.4.9

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when:

1. The digits are repeated?
2. The repetition of digits is not allowed?

solution: Let X be a random variable such that:

$$X = \begin{cases} 0 & n \not\equiv 0 \pmod 5 \\ 1 & n \equiv 0 \pmod 5 \end{cases} \tag{1}$$

Let's solve each part separately.

(i) Repetition of digits

Let number of favourable outcomes be N(A) and total outcomes be N(T).

$$N(T) = 249 \tag{2}$$

For n to be greater than 5000 and also divisble by 5:

1. first digit must be 5 or 7.
2. last digit must be 5 or 0.

Hence,

$$N(A) = 99 \tag{3}$$

With this information we can find the required answer,

$$\Pr(X = 1) = \frac{N(A)}{N(T)} \tag{4}$$

$$\implies \Pr(X = 1) = \frac{33}{83} \tag{5}$$

(ii) No Repetition of Digits

Let number of favourable outcomes be N(B) and total outcomes be N(T).

$$N(T) = 48 \tag{6}$$

The cases for n to be greater than 5000 and also divisble by 5:

1. first digit must be 5 and last digit must be 0.
2. first digit must be 7 and last digit must be 5 or 0.

Hence,

$$N(B) = 18 \tag{7}$$

$$\Pr(X = 1) = \frac{N(A)}{N(T)} \tag{8}$$

$$\implies \Pr(X = 1) = \frac{3}{8} \tag{9}$$