

Question 11.16.4.9

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when:

1. The digits are repeated?
2. The repetition of digits is not allowed?

solution: Let X be a random variable such that

$$X = \begin{cases} 0 & n < 5000 \\ 1 & n > 5000 \end{cases} \quad (1)$$

Let Y be a random variable such that:

$$Y = \begin{cases} 0 & n \not\equiv 0 \pmod{5} \\ 1 & n \equiv 0 \pmod{5} \end{cases} \quad (2)$$

We need to find $\Pr(Y | X)$ in each case.

$$\Pr(Y | X) = \frac{\Pr(XY)}{\Pr(X)} \quad (3)$$

Let's solve each part separately.

(i) Digits are Repeated

For n to be greater than 5000, first digit must be 5 or 7. We must exclude the case '5000'. Hence,

$$\Pr(X = 1) = \frac{249}{3125} \quad (4)$$

For n to be greater than 5000 and also divisible by 5:

1. first digit must be 5 or 7.
2. last digit must be 5 or 0.

Hence,

$$\Pr(XY = 1) = \frac{99}{3125} \quad (5)$$

With this information we can find the required answer,

$$\Pr(Y | X = 1) = \frac{\Pr(XY = 1)}{\Pr(X = 1)} \quad (6)$$

$$\Pr(Y | X = 1) = \frac{99}{249} \quad (7)$$

$$\implies \Pr(Y | X = 1) = \frac{33}{83} \quad (8)$$

(ii) No Repetition of Digits

For n to be greater than 5000, first digit must be 5 or 7. Hence,

$$\Pr(X = 1) = \frac{48}{120} \quad (9)$$

The cases for n to be greater than 5000 and also divisible by 5:

1. first digit must be 5 and last digit must be 0.
2. first digit must be 7 and last digit must be 5 or 0.

Hence,

$$\Pr(XY = 1) = \frac{18}{120} \quad (10)$$

With this information we can find the required answer,

$$\Pr(Y | X = 1) = \frac{\Pr(XY = 1)}{\Pr(X = 1)} \quad (11)$$

$$\Pr(Y | X = 1) = \frac{18}{48} \quad (12)$$

$$\implies \Pr(Y | X = 1) = \frac{3}{8} \quad (13)$$