Solution of Q9.3.23

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A die is thrown 5 times. Find the probability that an odd number will come up exactly three times.

Solution:

Parameter	Values	Description
n	5	Number of throws
k	3	Number of favourable outcomes
p	0.5	Probability of getting odd number
X	$1 \leq X \leq 5$	X favourable out of 5 total outcomes
Y	$1 \leq Y \leq 5$	gaussian variable
$\mu = np$	2.5	mean
$\sigma = \sqrt{np(1-p)}$	1.118	standard deviation

1) *Binomial Distribution :*
The X is the random variable, the pmf of X is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{1}$$

We require $\Pr(X = 3)$. Since $n = 5$,

$$p_X(3) = 0.3125 \tag{2}$$

2) *Gaussian Distribution*
Let Y be gaussian variable. Using central limit theorem, we can use the gaussian distribution function:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3}$$

$$\tag{4}$$

Using Normal distribution at X=3.

$$p_Y(3) = \frac{1}{\sqrt{2\pi\left(\frac{5}{4}\right)}} e^{-\frac{\left(x-\frac{5}{2}\right)^2}{2\left(\frac{5}{4}\right)}} \tag{5}$$

$$= \frac{1}{\sqrt{2\pi\left(\frac{5}{4}\right)}} e^{-\frac{1}{10}} \tag{6}$$

$$= 0.3228684517 \tag{7}$$

3) *using Q function:*

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{8}$$

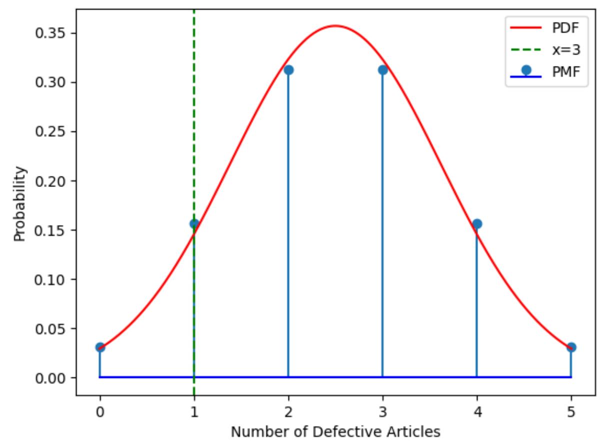

Fig. 3. Binomial-PMF and Gaussian-PDFof X

The CDF of Y:

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu-y}{\sigma}\right), & y < \mu \end{cases} \tag{9}$$

But,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{10}$$

$$\implies F_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) \tag{11}$$

to include correction of 0.5,

$$p_Y(2.5 < Y < 3.5) = F_Y(3.5) - F_Y(2.5) \tag{12}$$

$$= Q\left(\frac{3.5-\mu}{\sigma}\right) - Q\left(\frac{2.5-\mu}{\sigma}\right) \tag{13}$$

$$= Q(0.8944) - Q(0) \tag{14}$$

$$= 0.314446 \tag{15}$$