

# Solution of Q9.3.23

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A die is thrown 5 times. Find the probability that an odd number will come up exactly three times.

**Solution:**

Parameter	Values	Description
$n$	5	Number of throws
$k$	3	Number of favourable outcomes
$p$	0.5	Probability of getting odd number
$X$	$1 \leq X \leq 5$	X favourable out of 5 total outcomes
$Y$	$1 \leq Y \leq 5$	gaussian variable
$\mu = np$	2.5	mean
$\sigma = \sqrt{np(1-p)}$	1.118	standard deviation

## 1) Binomial Distribution :

The  $X$  is the random variable, the pmf of  $X$  is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \quad (1)$$

We require  $\Pr(X = 3)$ . Since  $n = 5$ ,

$$p_X(3) = 0.3125 \quad (2)$$

## 2) Gaussian Distribution

Let  $Y$  be gaussian variable. Using central limit theorem, we can use the gaussian distribution function:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (3)$$

(4)

Using Normal distribution at  $X=3$ .

$$p_Y(3) = \frac{1}{\sqrt{2\pi\left(\frac{5}{4}\right)}} e^{-\frac{\left(3-\frac{5}{2}\right)^2}{2\left(\frac{5}{4}\right)}} \quad (5)$$

$$= \frac{1}{\sqrt{2\pi\left(\frac{5}{4}\right)}} e^{-\frac{1}{10}} \quad (6)$$

$$= 0.3228684517 \quad (7)$$

## 3) using $Q$ function:

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (8)$$

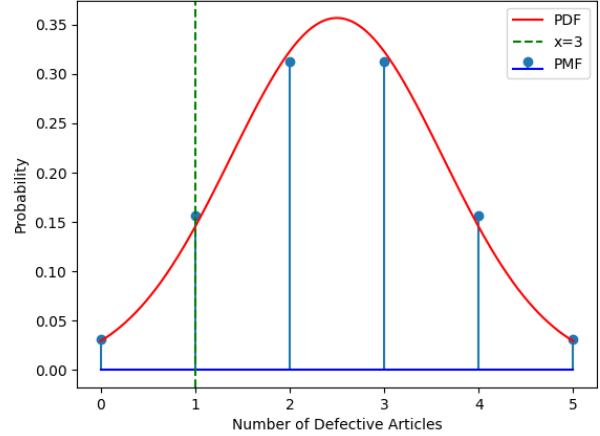


Fig. 3. Binomial-PMF and Gaussian-PDF of  $X$

The CDF of  $Y$ :

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu-y}{\sigma}\right), & y < \mu \end{cases} \quad (9)$$

But,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (10)$$

$$\Rightarrow F_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) \quad (11)$$

to include correction of 0.5,

$$p_Y(2.5 < Y < 3.5) = F_Y(2.5) - F_Y(3.5) \quad (12)$$

$$= Q\left(\frac{2.5-\mu}{\sigma}\right) - Q\left(\frac{3.5-\mu}{\sigma}\right) \quad (13)$$

$$= Q(0) - Q(0.8944) \quad (14)$$

$$= 0.314446 \quad (15)$$