

Solution of Q9.3.21

SUJAL GUPTA - EE22BTECH11052

It is known that 10% of certain articles manufactured are defective. What is probability that a random sample space of 12 such articles, 9 are defective?

Solution:

Parameter	Values	Description
n	5	Number of throws
k	3	Number of favourable outcomes
p	0.5	Probability of getting odd number
X	$1 \leq X \leq 5$	X favourable out of 5 total outcomes
Y	$1 \leq Y \leq 5$	gaussian variable
$\mu = np$	2.5	mean
$\sigma = \sqrt{np(1-p)}$	1.118	standard deviation

TABLE 0

TABLE 1

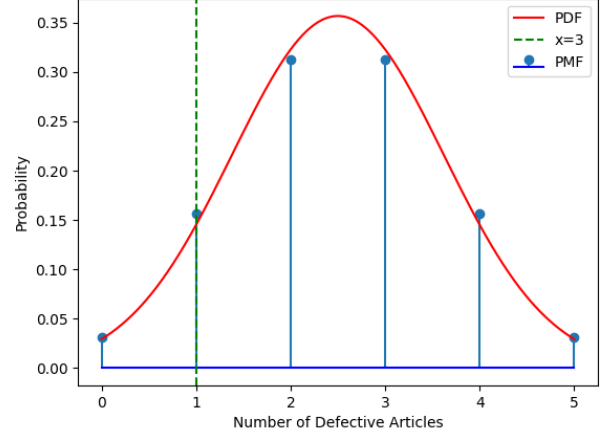


Fig. 3. Binomial-PMF and Gaussian-PDF of X

1) Binomial Distribution :

The X is the random variable, the pmf of X is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \quad (1)$$

We require $\Pr(X = 3)$. Since $n = 5$,

$$p_X(3) = 0.3125 \quad (2)$$

2) Gaussian Distribution

Let Y be gaussian variable. Using central limit theorem, we can use the gaussian distribution function:

$$p_Y(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (3)$$

(4)

Using Normal distribution at $X=3$.

$$p_Y(9) = \frac{1}{\sqrt{2\pi}\left(\frac{5}{4}\right)} e^{-\frac{\left(x-\frac{5}{2}\right)^2}{2\left(\frac{5}{4}\right)}} \quad (5)$$

$$= \frac{1}{\sqrt{2\pi}\left(\frac{5}{4}\right)} e^{-\frac{1}{10}} \quad (6)$$

$$= 7.1094368 \quad (7)$$

3) using Q function:

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (8)$$

The CDF of Y :

$$F_Y(y) = \begin{cases} 1 - Q\left(\frac{y-\mu}{\sigma}\right), & y > \mu \\ Q\left(\frac{\mu-y}{\sigma}\right), & y < \mu \end{cases} \quad (9)$$

But,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (10)$$

$$\Rightarrow F_Y(y) = 1 - Q\left(\frac{y-\mu}{\sigma}\right) \quad (11)$$

to include correction of 0.5,

$$p_Y(2.5 < Y < 3.5) = F_Y(3.5) - F_Y(2.5) \quad (12)$$

$$= Q\left(\frac{3.5-\mu}{\sigma}\right) - Q\left(\frac{2.5-\mu}{\sigma}\right) \quad (13)$$

$$= Q(0.8944) - Q(0) \quad (14)$$

$$= 0.314446 \quad (15)$$