Question 11.16.4.9

If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when:

1. The digits are repeated?

2. The repetition of digits is not allowed?

solution:

Let's solve each part separately:

(i) Digits are Repeated

Let number of favourable outcomes be N(A) and total outcomes be N(T).
For the number to be greater than 5000, the 1000s digit must be 5 or 7 and there is no constraint on the other digits. And we must exclude the case '5000'. Hence,

$$N(T) = (2 \times 5 \times 5 \times 5) - 1 = 249 \tag{1}$$

For the number to be divisible by 5, the last digit must be either 0 or 5. Here also we must exclude the case '5000'
Hence,

$$N(A) = (2 \times 5 \times 5 \times 2) - 1 = 99 \tag{2}$$

The probability of forming a number divisible by 5 when digits are repeated is:

$$P(\text{divisible by } 5) = \frac{\text{N(A)}}{\text{N(T)}} = \frac{99}{249} = \frac{33}{83}. \tag{3}$$

(ii) No Repetition of Digits

Let number of favourable outcomes be N(B) and total outcomes be N(T).
Hence,

$$N(T) = 2 \times 4 \times 3 \times 2 = 48 \tag{4}$$

For the number to be divisible by 5, the last digit must be either 0 or 5.
The favourable cases are:

1. If the 4-digit number starts with 5, the last digit must be 0.

$$\text{N}_1 = 1 \times 3 \times 2 \times 1 = 6 \tag{5}$$

2. if the 4-digit number starts with 7, the last digit can be 0 or 5

$$N_2 = 1 \times 3 \times 2 \times 2 = 12 \tag{6}$$

The total number of favourable cases N(A) will be:

$$N(B) = N_1 + N_2 = 18 \tag{7}$$

The probability of forming a number divisible by 5 when digits are not repeated is:

$$P(\text{divisible by 5}) = \frac{\text{N(A)}}{\text{N(T)}} = \frac{18}{48} = \frac{3}{8}. \tag{8}$$