

Supplementary material: “Solving the Regge-Wheeler and
Teukolsky equations: supervised vs. unsupervised
physics-informed neural networks”

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QNMs for spin sequences $a = 0$, $\ell = \{2, 3\}$, $m = 0$, $n \in \{1, 2, 3, 4, 5\}$.

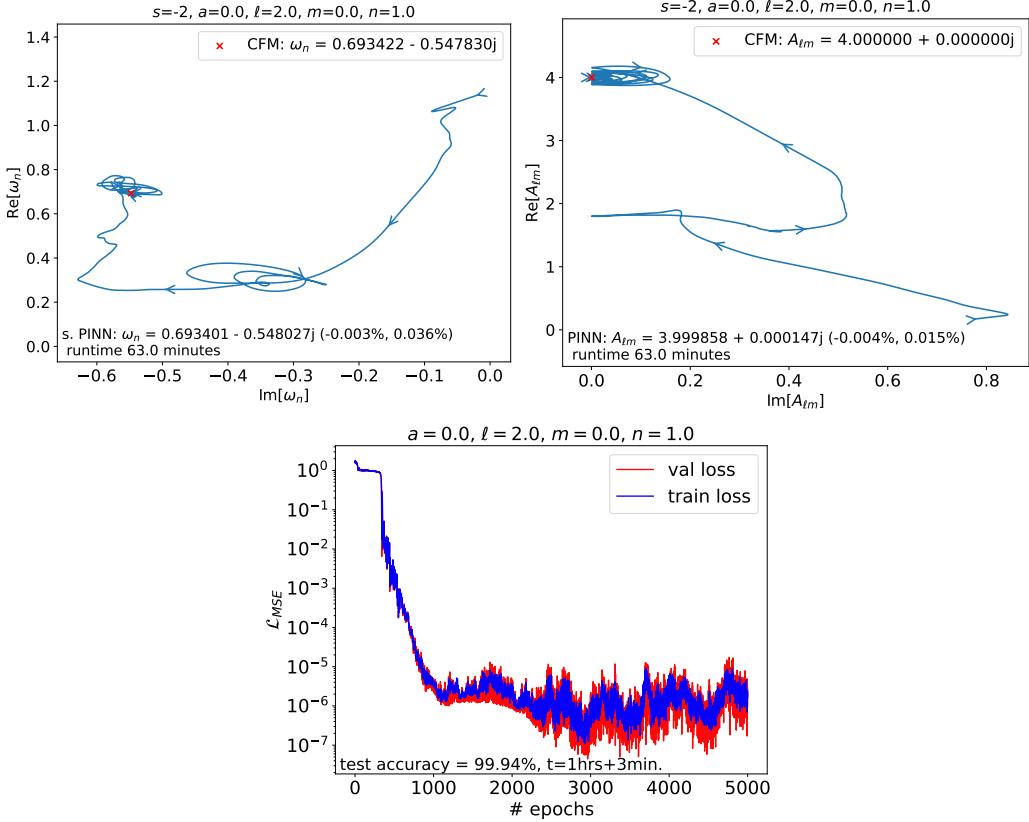


Figure 1: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0$, $\ell = 2$, $m = 0$, $n = 1$.

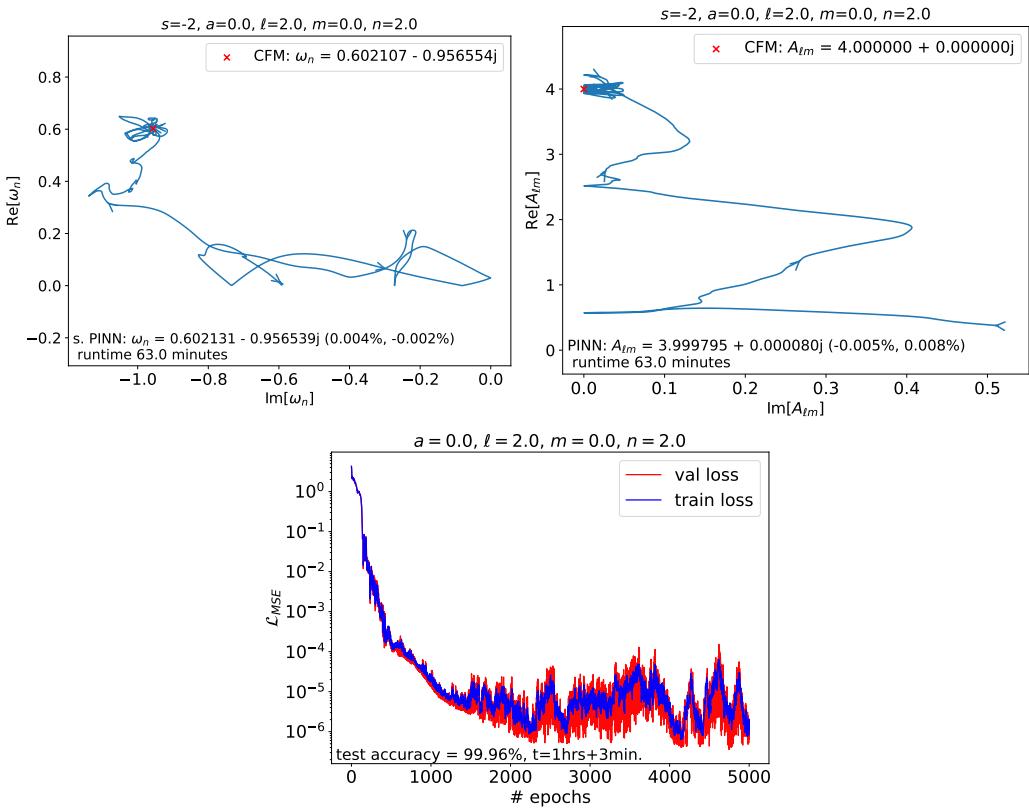


Figure 2: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0$, $\ell = 2$, $m = 0$, $n = 2$.

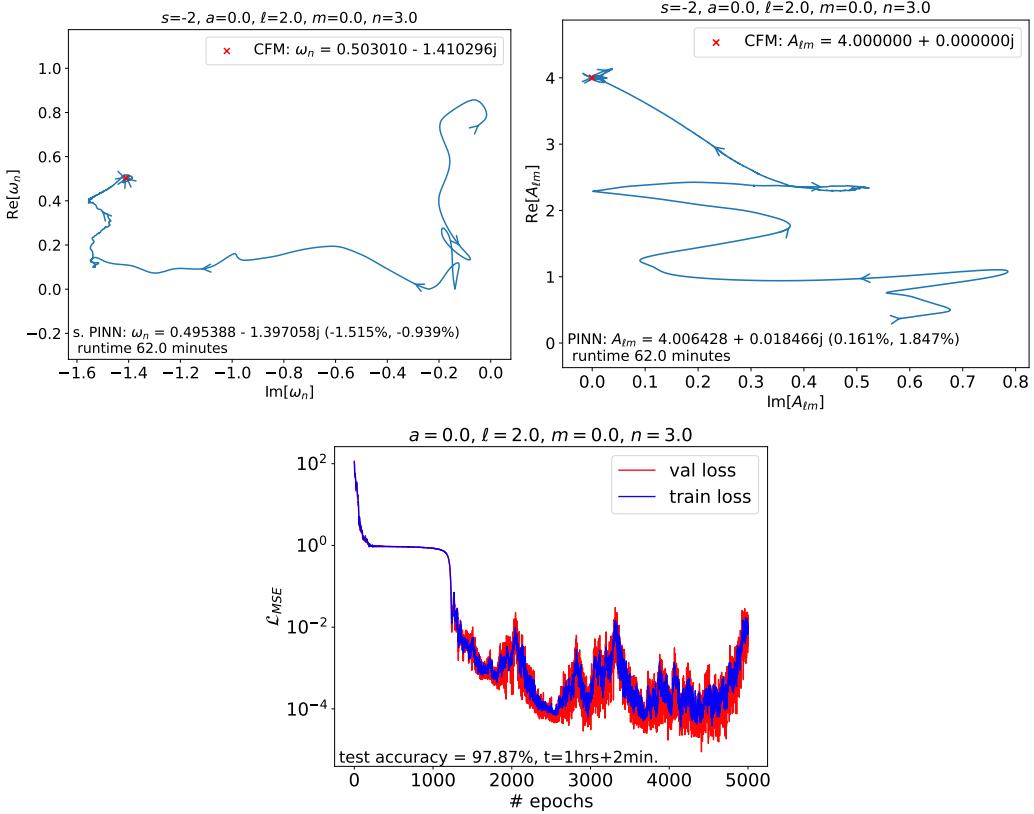


Figure 3: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0, \ell = 2, m = 0, n = 3$.

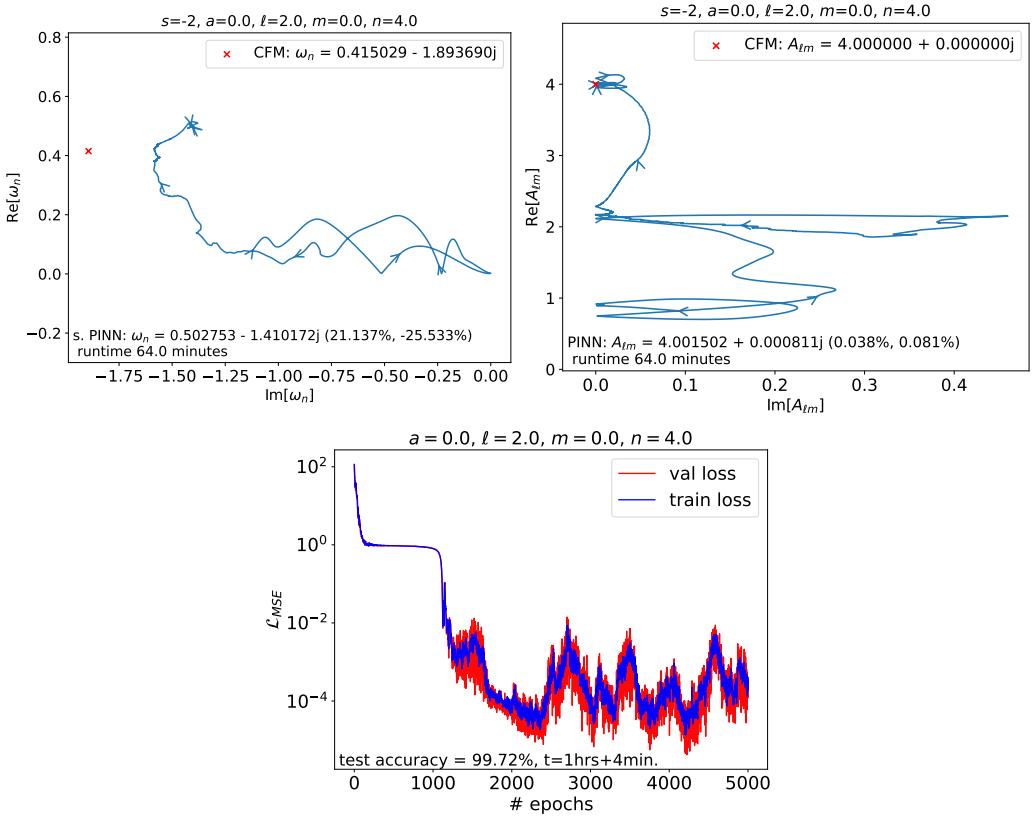


Figure 4: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0, \ell = 2, m = 0, n = 4$.

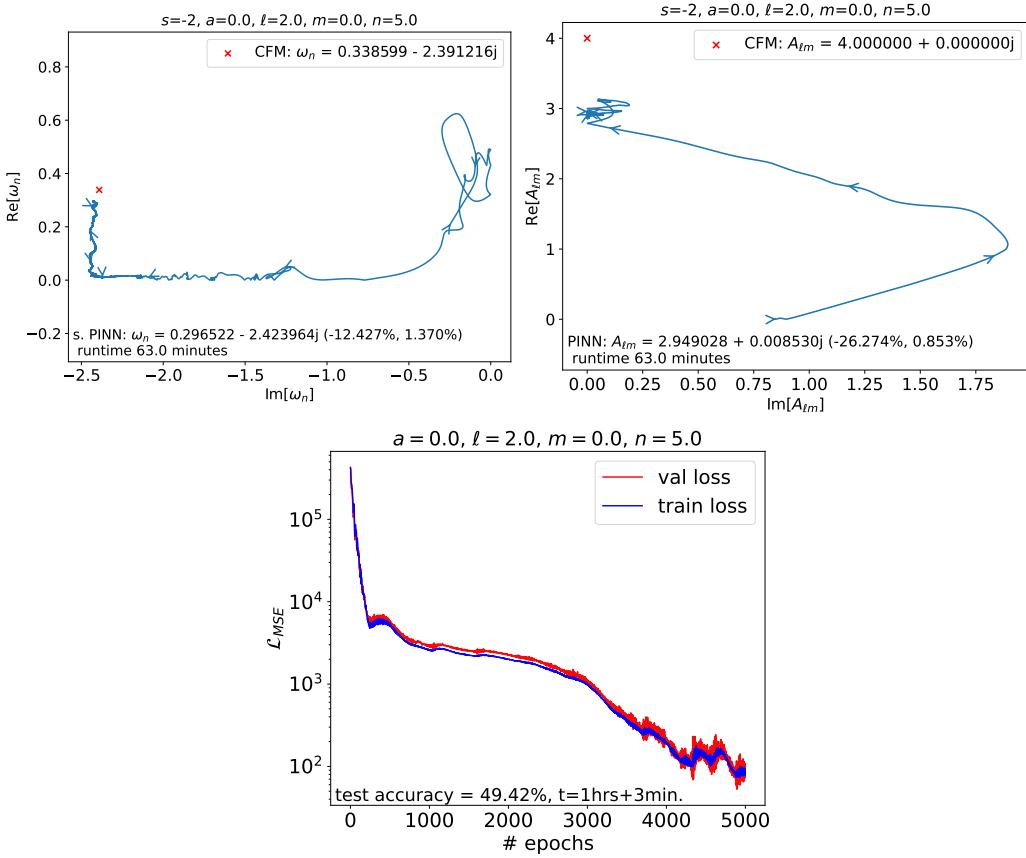


Figure 5: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0$, $\ell = 2$, $m = 0$, $n = 5$.

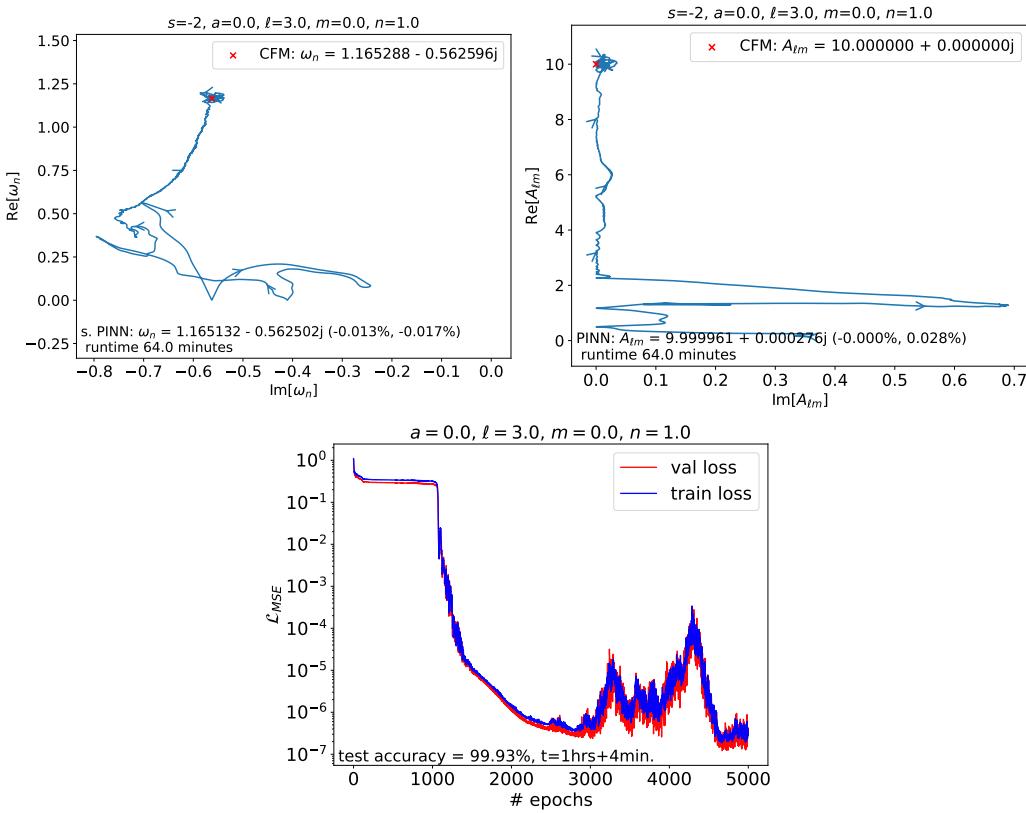


Figure 6: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0$, $\ell = 3$, $m = 0$, $n = 1$.

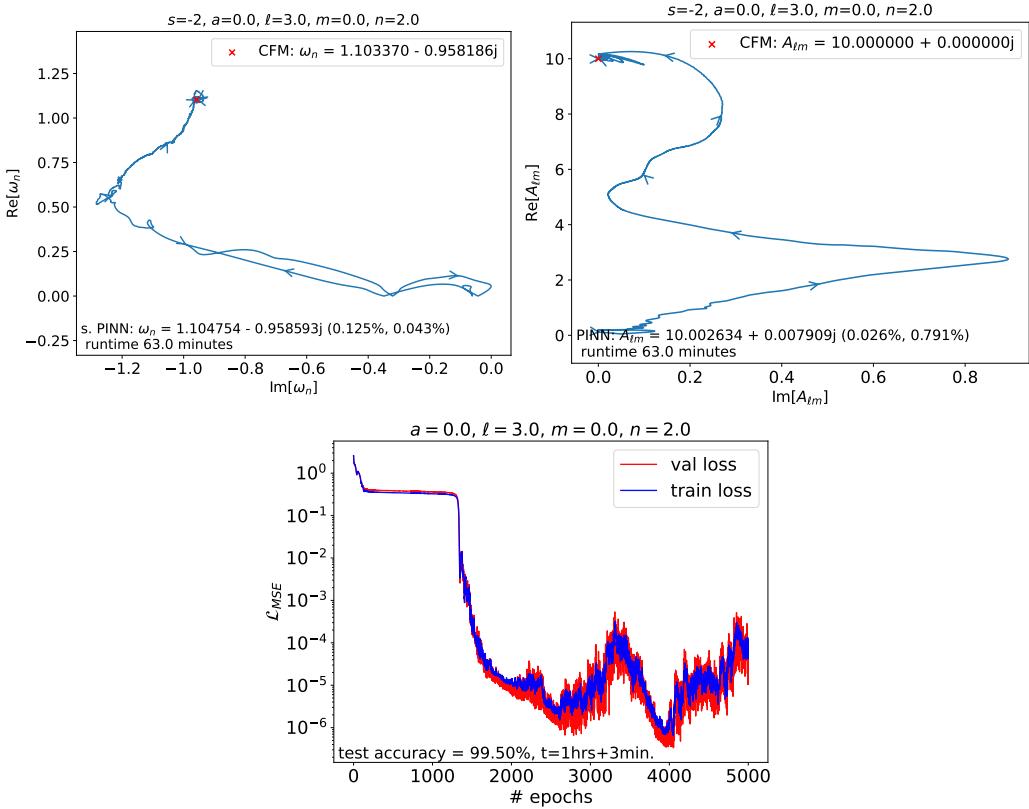


Figure 7: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0, \ell = 3, m = 0, n = 2$.

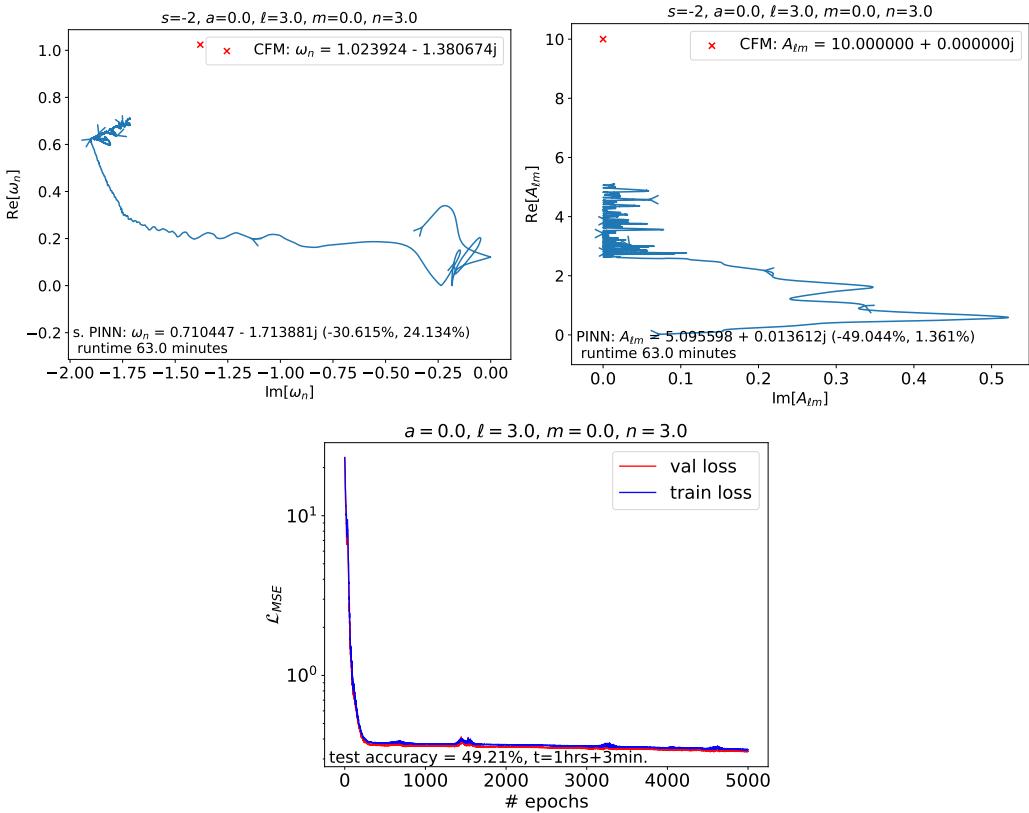


Figure 8: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0, \ell = 3, m = 0, n = 3$.

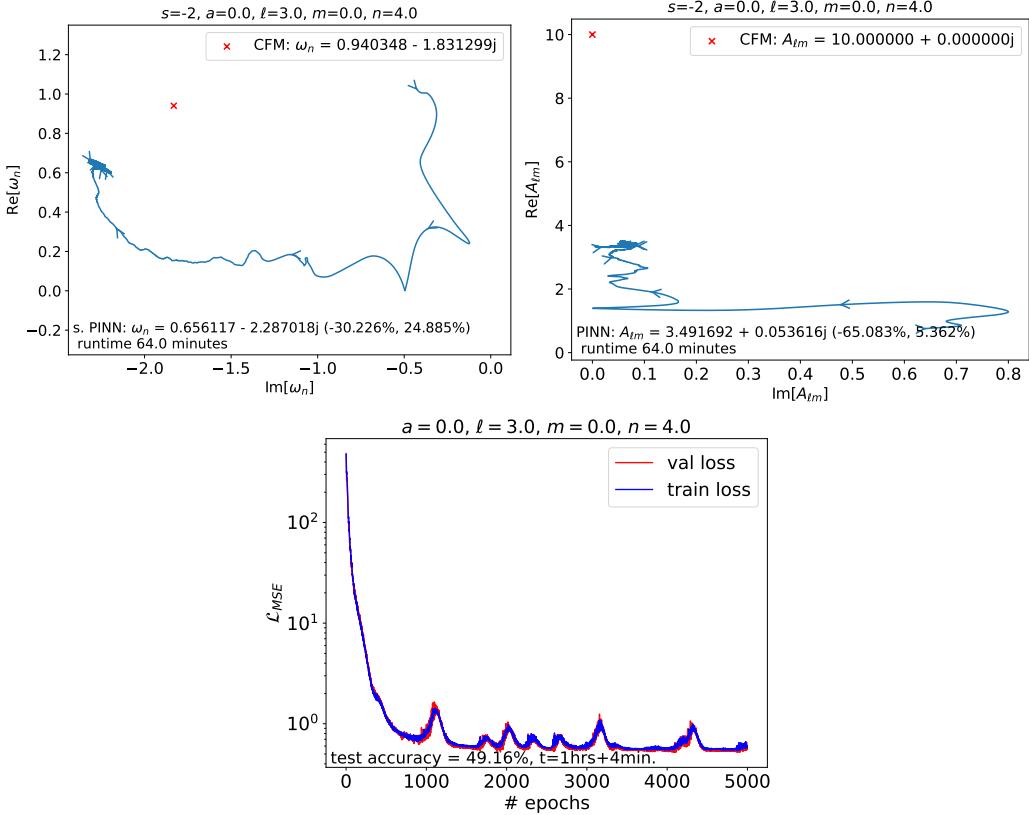


Figure 9: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0$, $\ell = 3$, $m = 0$, $n = 4$.

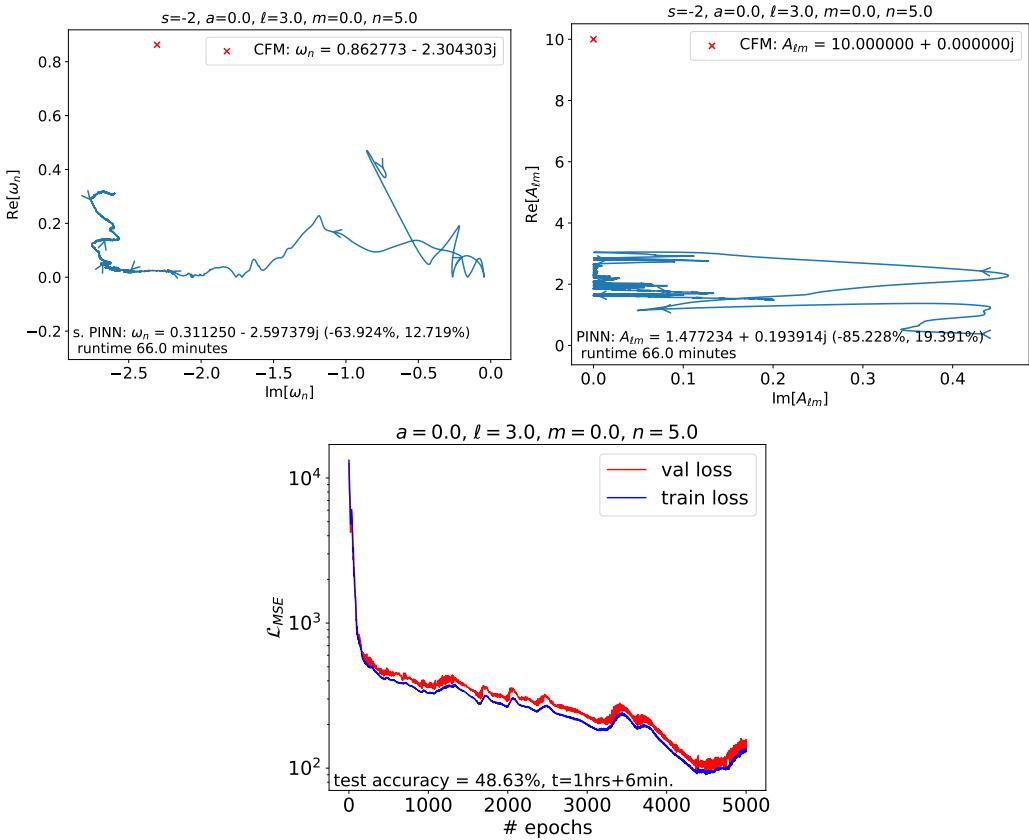


Figure 10: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0$, $\ell = 3$, $m = 0$, $n = 5$.

QNMs for spin sequences $a \in \{0.1, 0.2, 0.3, 0.4\}$, $\ell = 2$, $m \in \{0, 1\}$, $n = 0$.

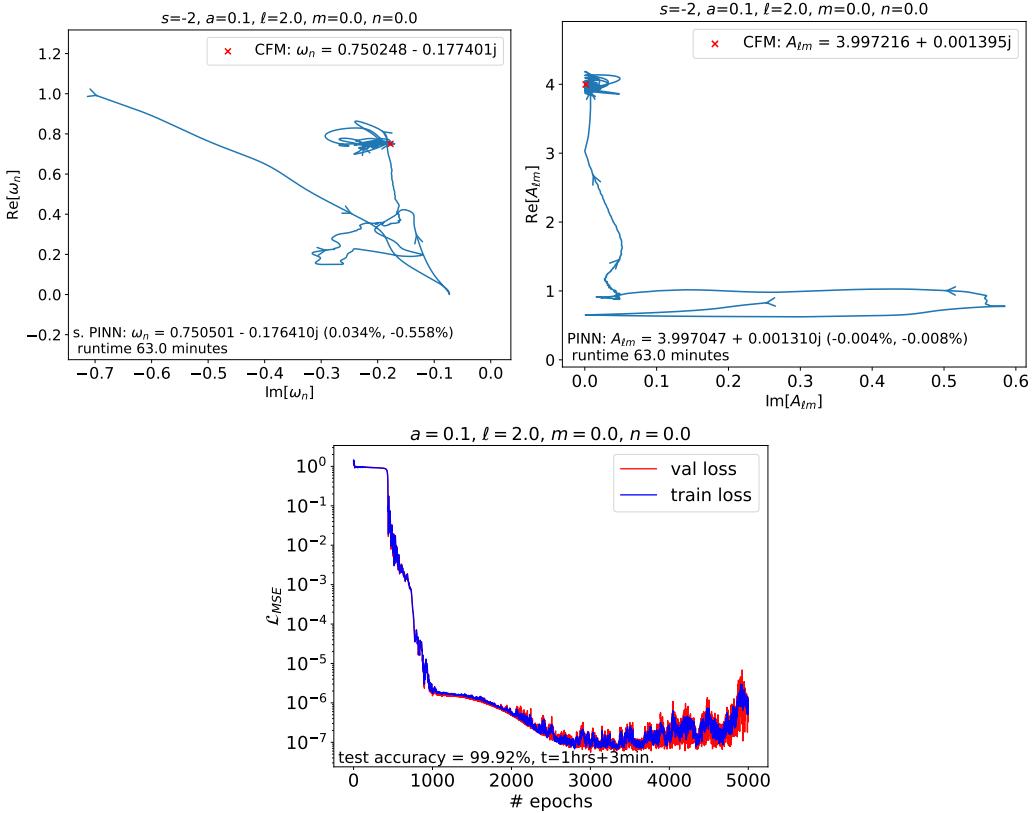


Figure 11: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.1$, $\ell = 2$, $m = 0$, $n = 0$.

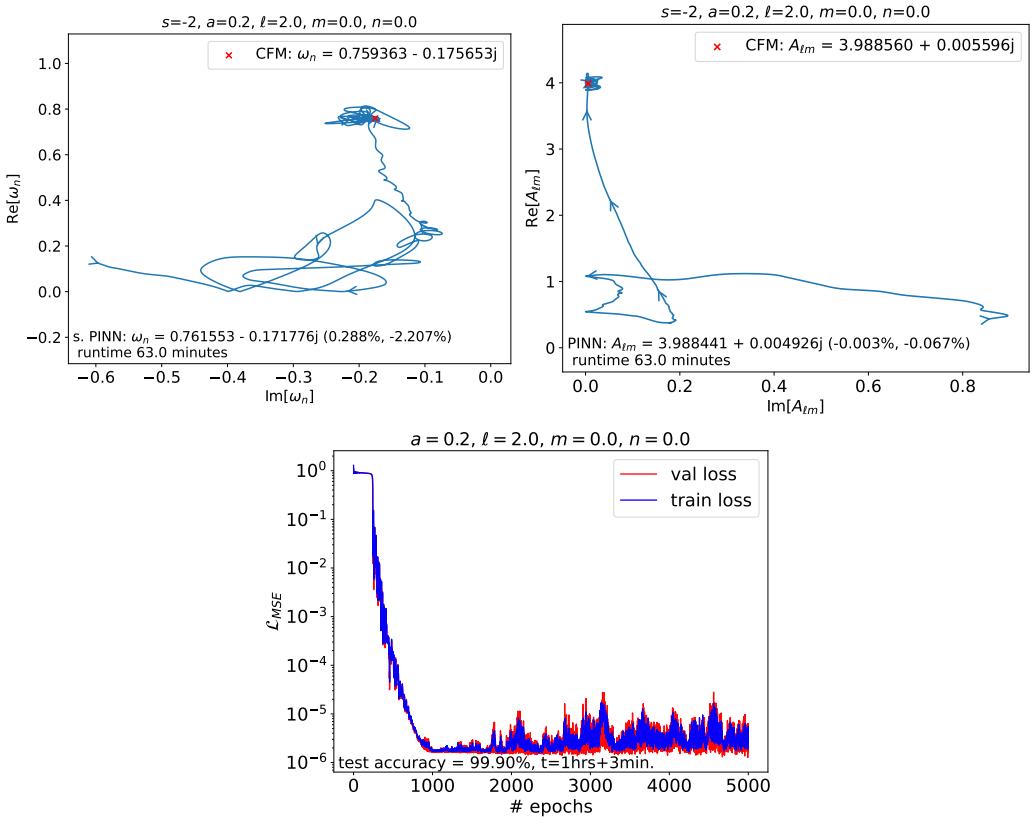


Figure 12: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.2$, $\ell = 2$, $m = 0$, $n = 0$.

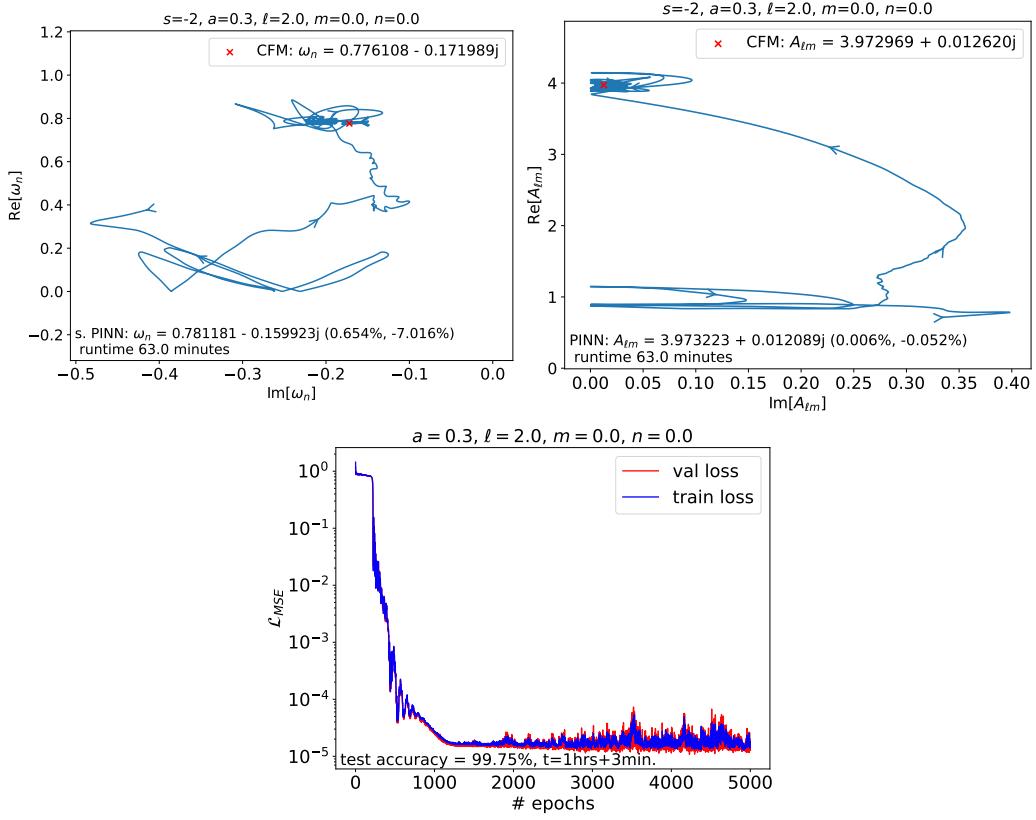


Figure 13: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.3, \ell = 2, m = 0, n = 0$.

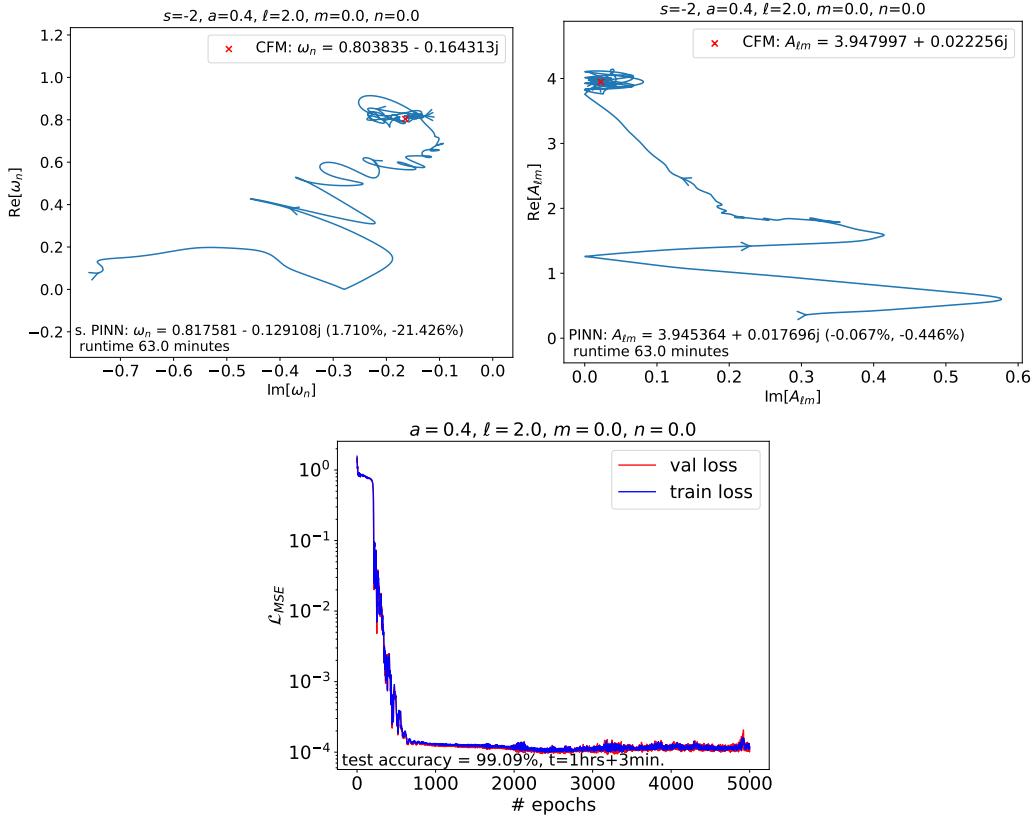


Figure 14: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.4, \ell = 2, m = 0, n = 0$.

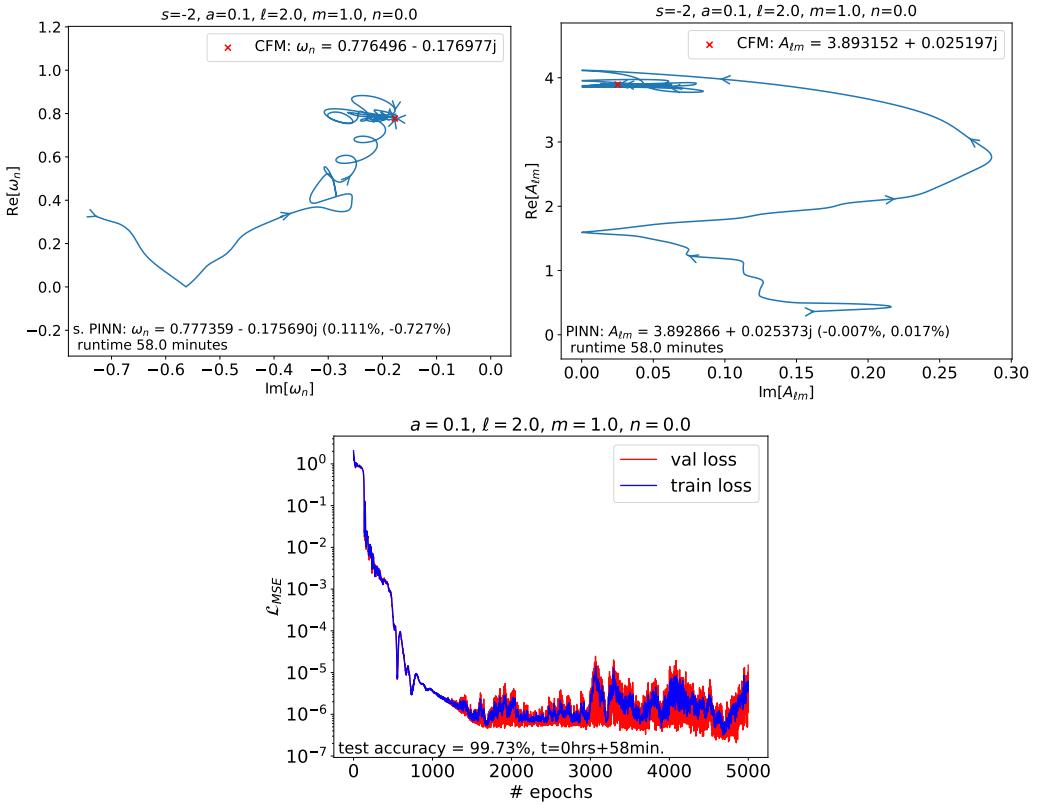


Figure 15: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.1, \ell = 2, m = 1, n = 0$.

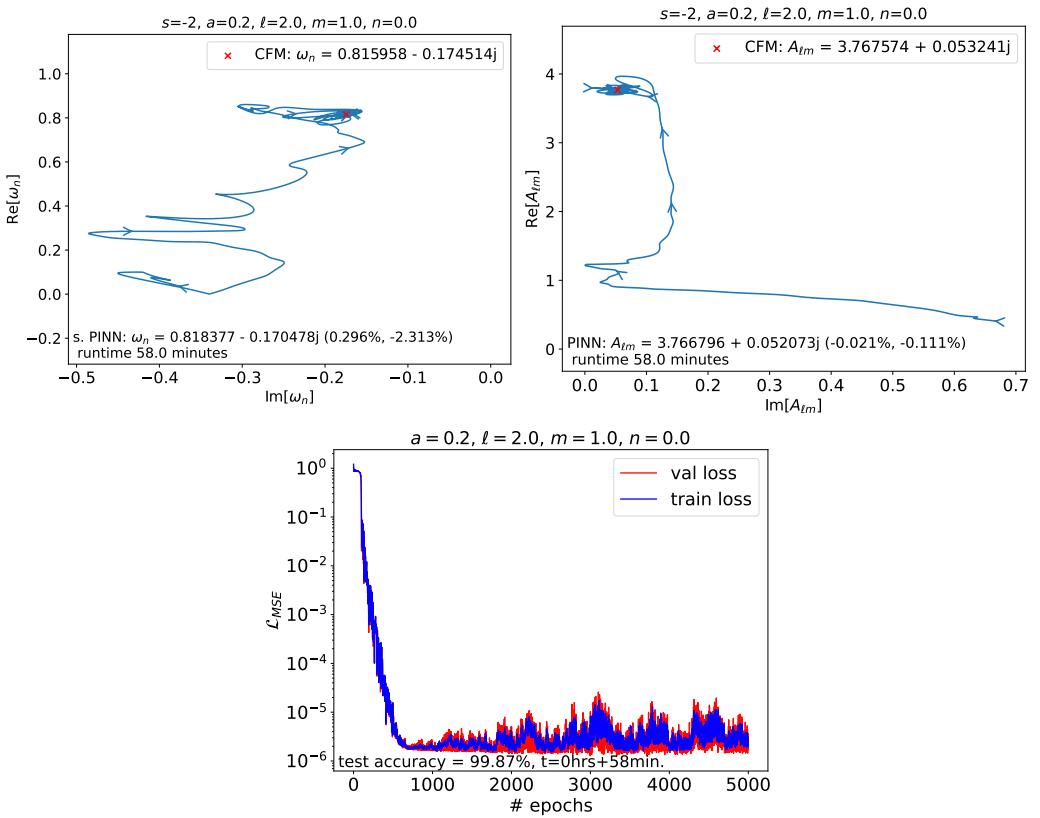


Figure 16: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.2, \ell = 2, m = 1, n = 0$.

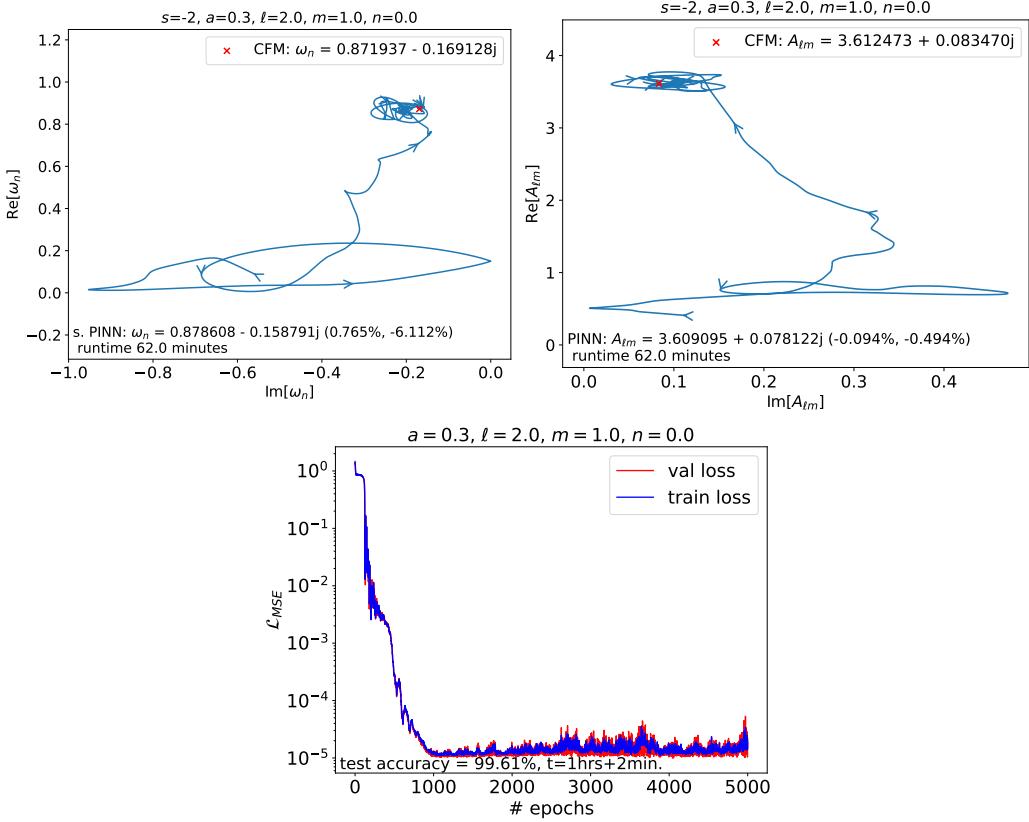


Figure 17: $\omega_n, A_{\ell m}, \mathcal{L}_{MSE}$ for spin indices $a = 0.3, \ell = 2, m = 1, n = 0$.

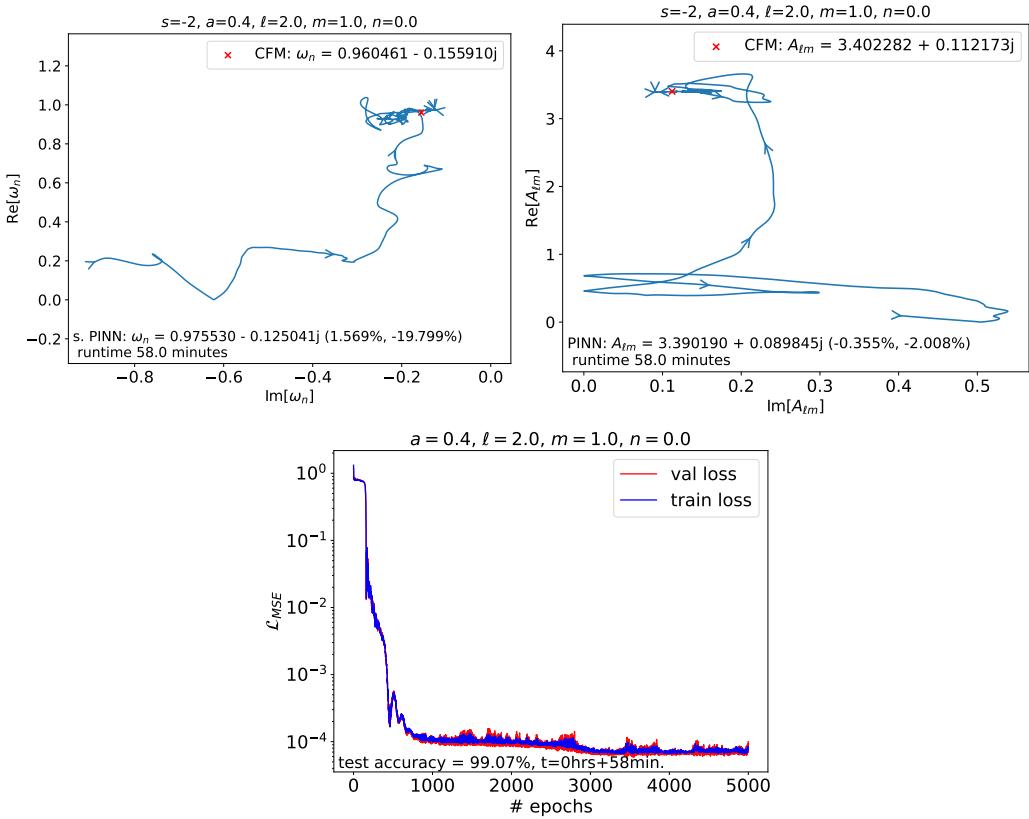


Figure 18: $\omega_n, A_{\ell m}, \mathcal{L}_{MSE}$ for spin indices $a = 0.4, \ell = 2, m = 1, n = 0$.

QNMs for spin sequences $a \in \{0.1, 0.2, 0.3, 0.4\}$, $\ell = m = 2$, $n \in \{1, 2, 3, 4, 5\}$.

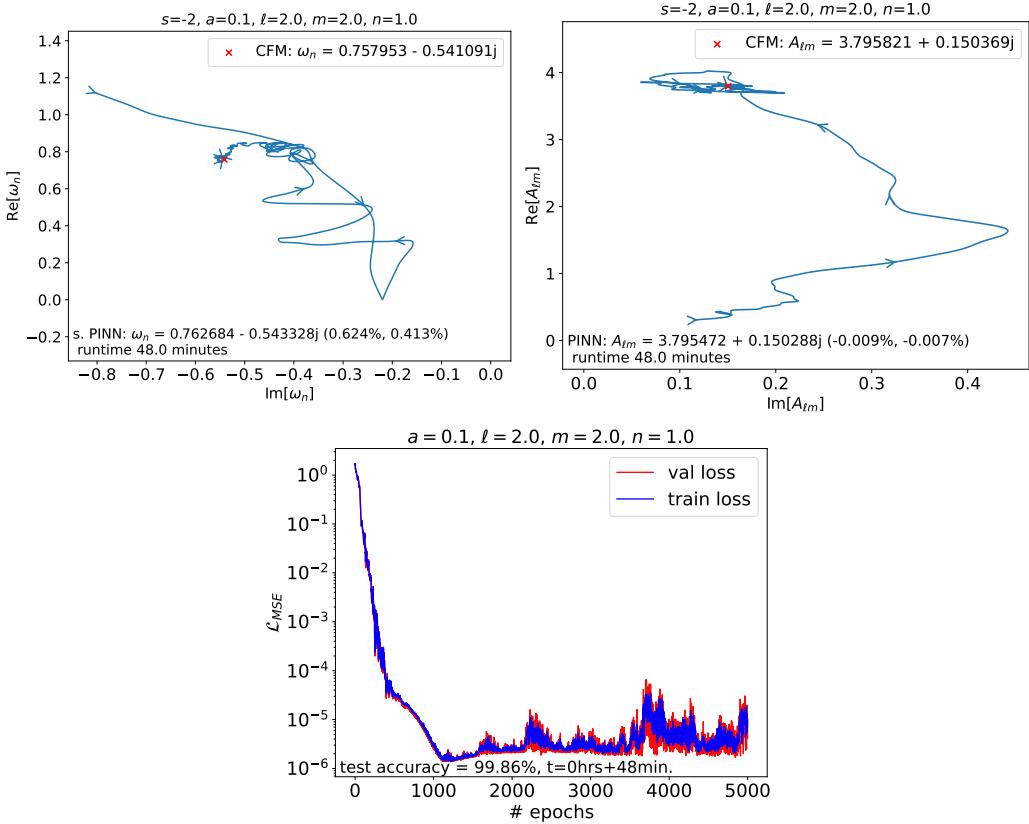


Figure 19: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.1$, $\ell = m = 2$, $n = 1$.

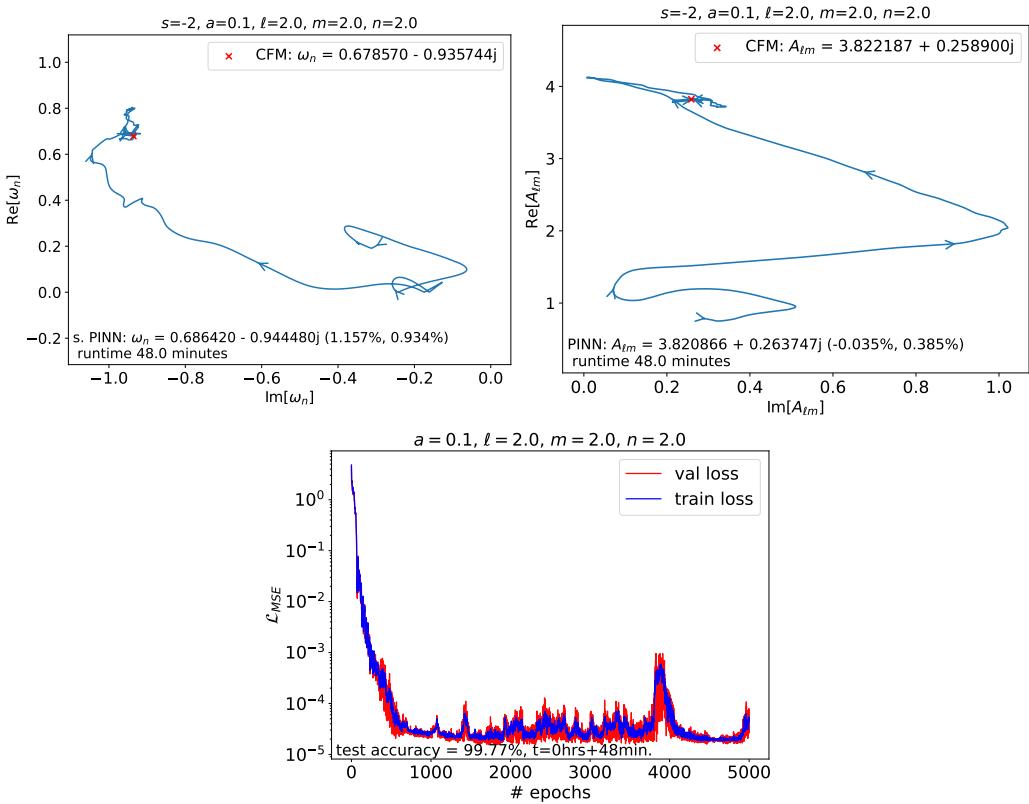


Figure 20: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.1$, $\ell = m = 2$, $n = 2$.

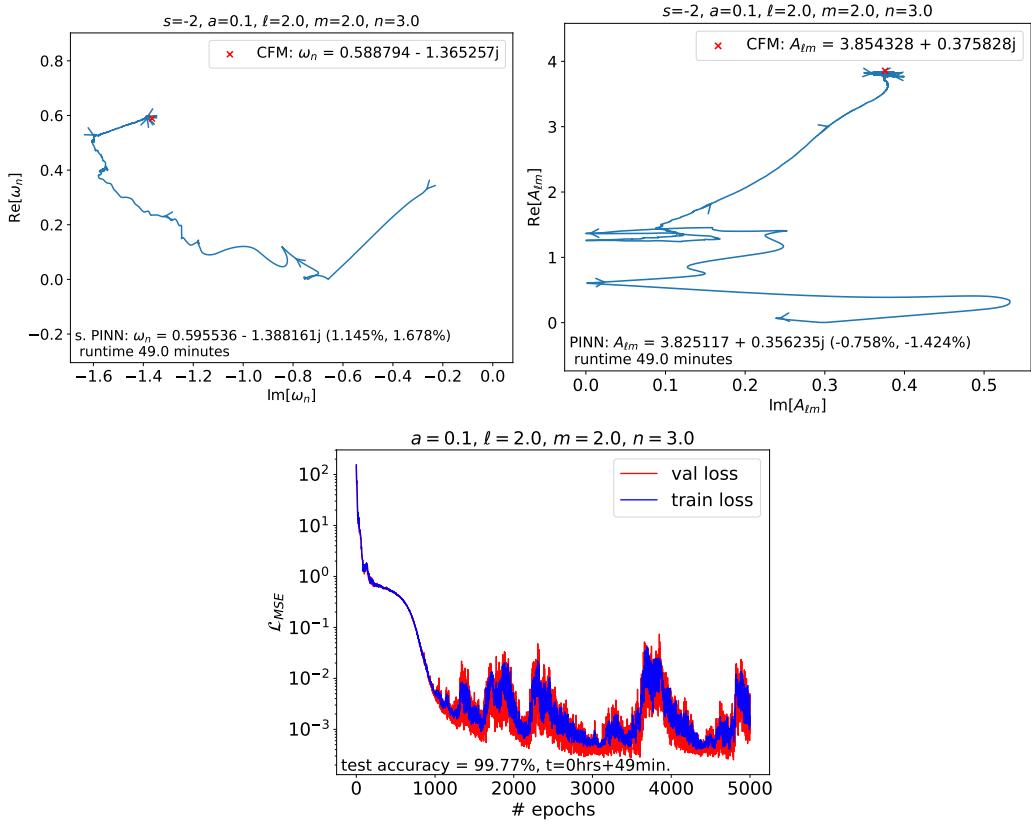


Figure 21: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.1$, $\ell = m = 2$, $n = 3$.

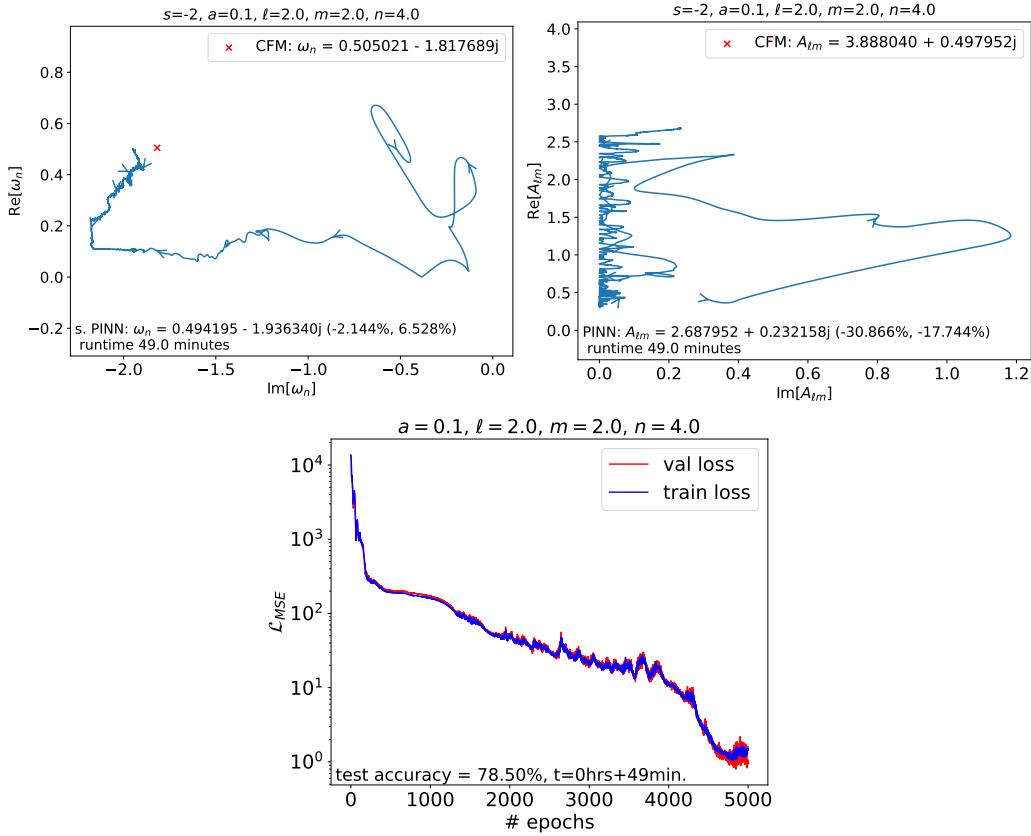


Figure 22: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.1$, $\ell = m = 2$, $n = 4$.

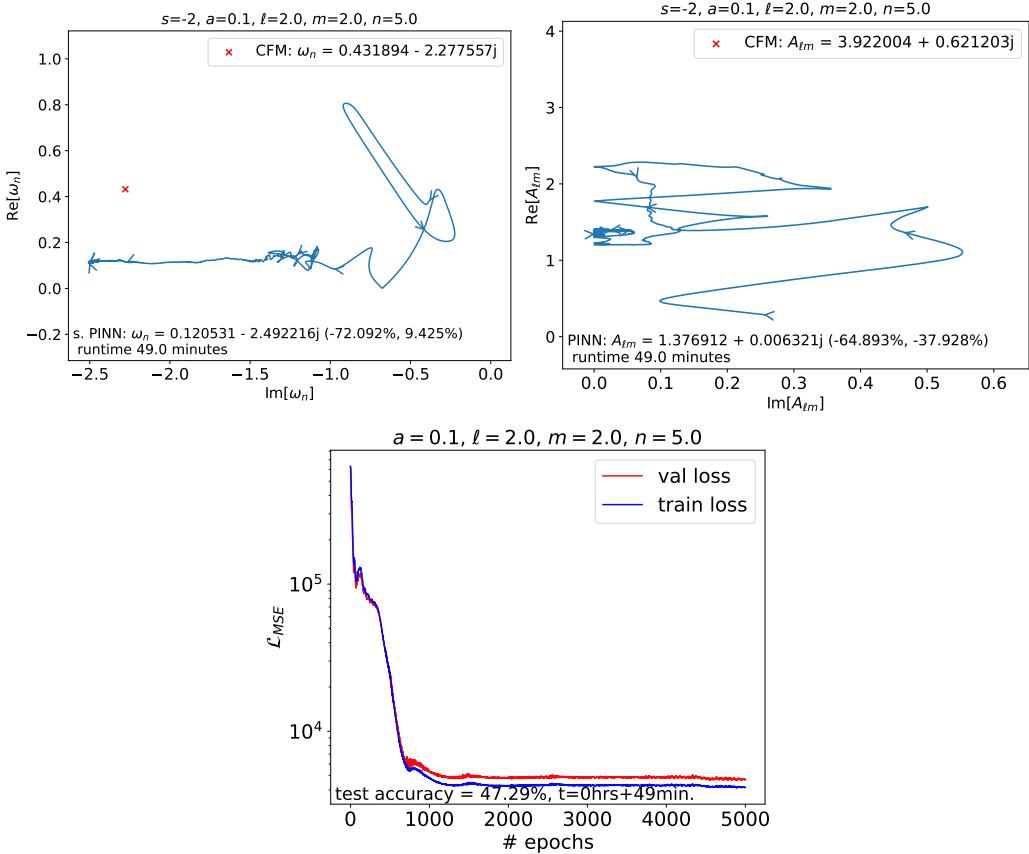


Figure 23: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.1, \ell = m = 2, n = 5$.

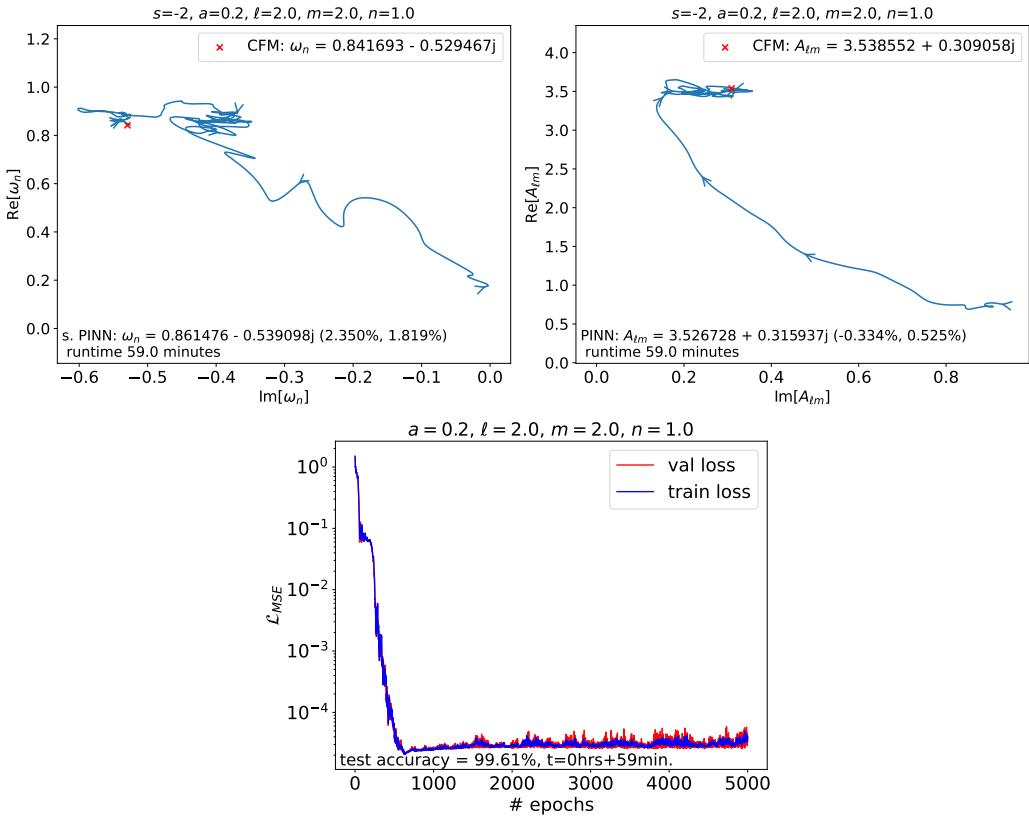


Figure 24: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.2, \ell = m = 2, n = 1$.

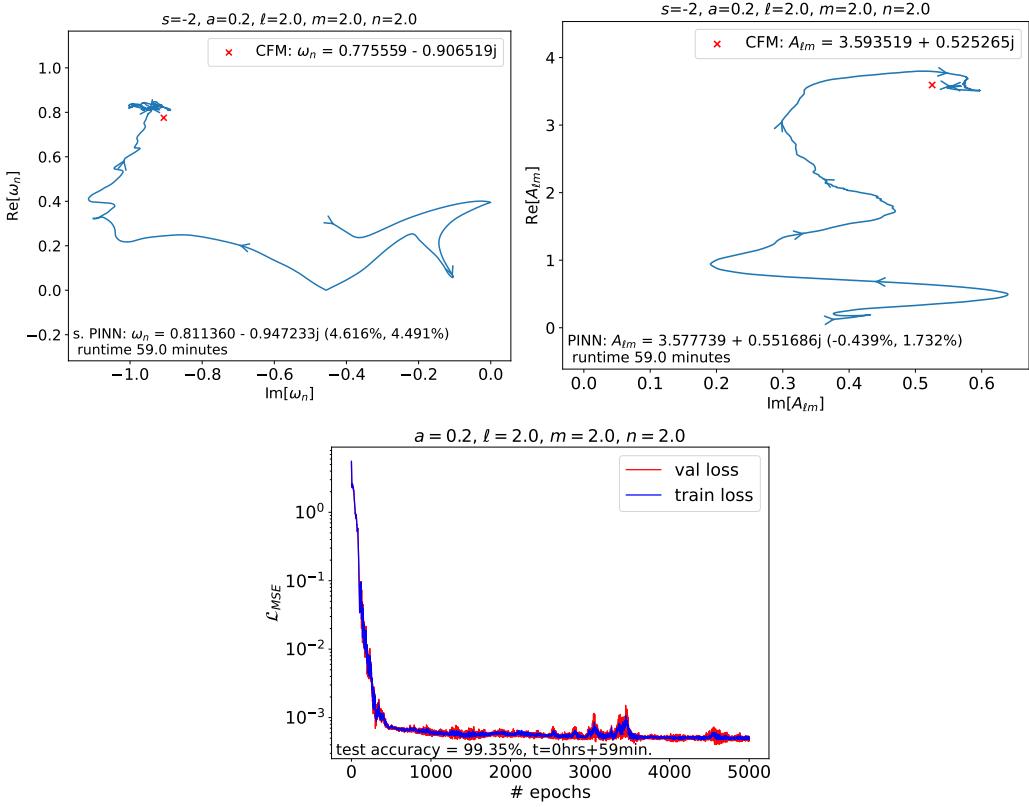


Figure 25: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.2, \ell = m = 2, n = 2$.

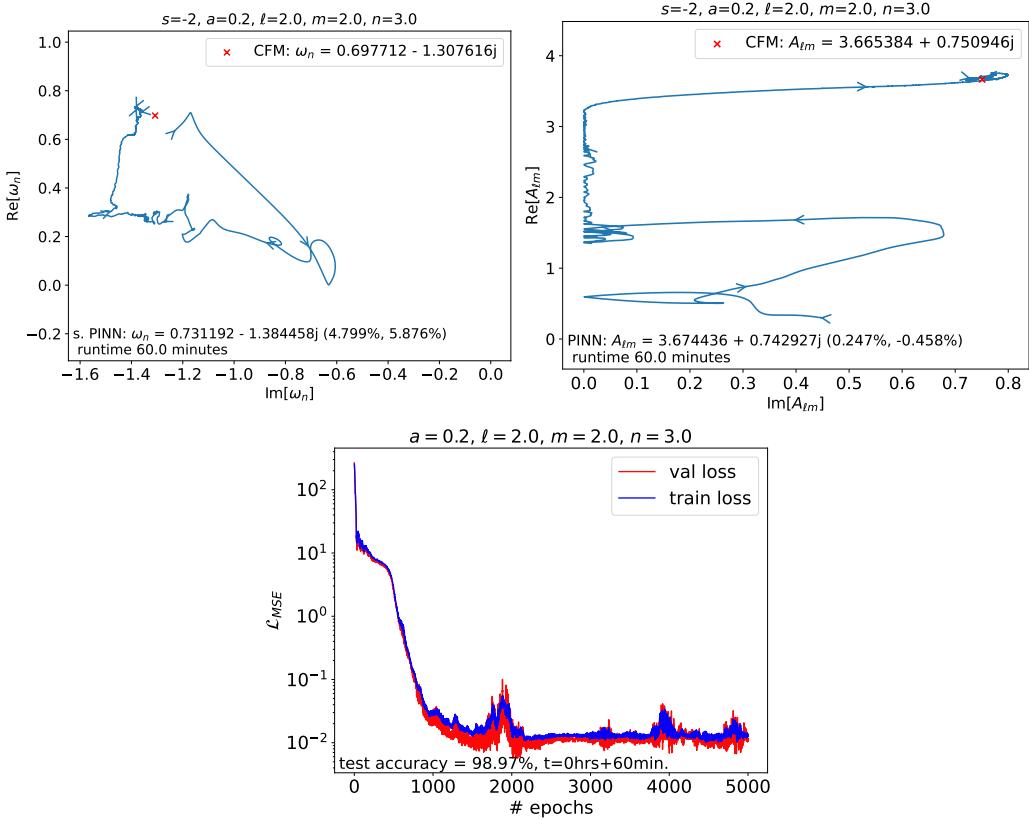


Figure 26: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.2, \ell = m = 2, n = 3$.

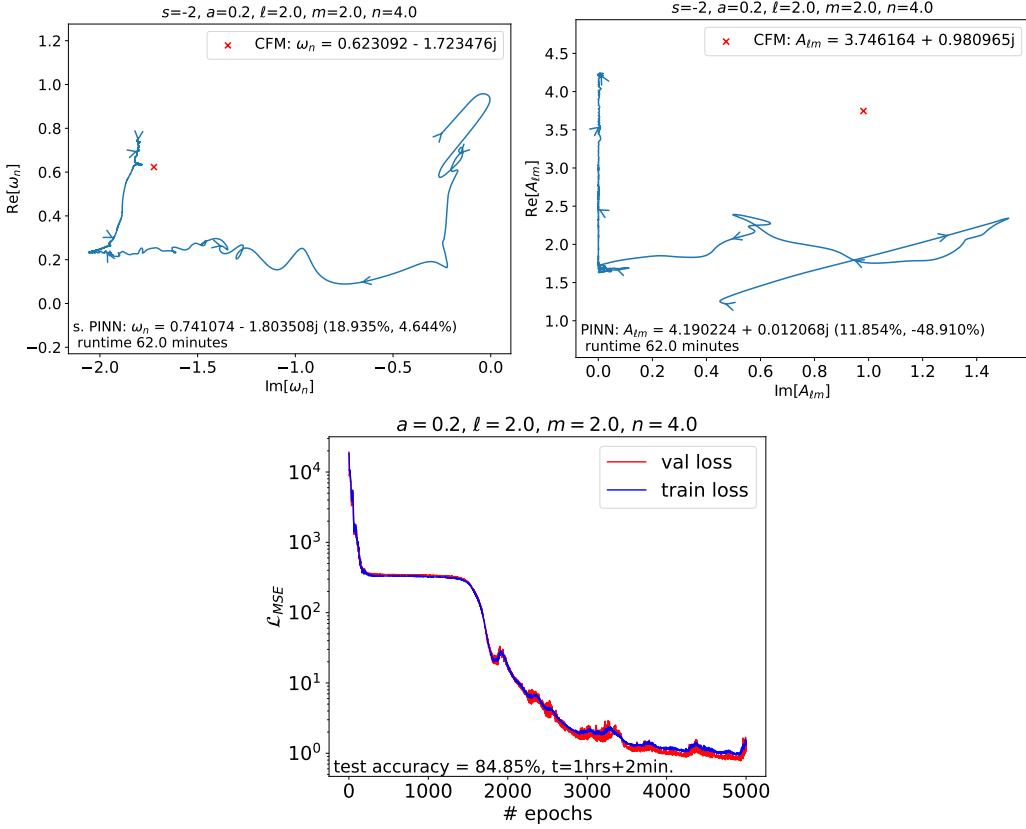


Figure 27: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.2$, $\ell = m = 2$, $n = 4$.

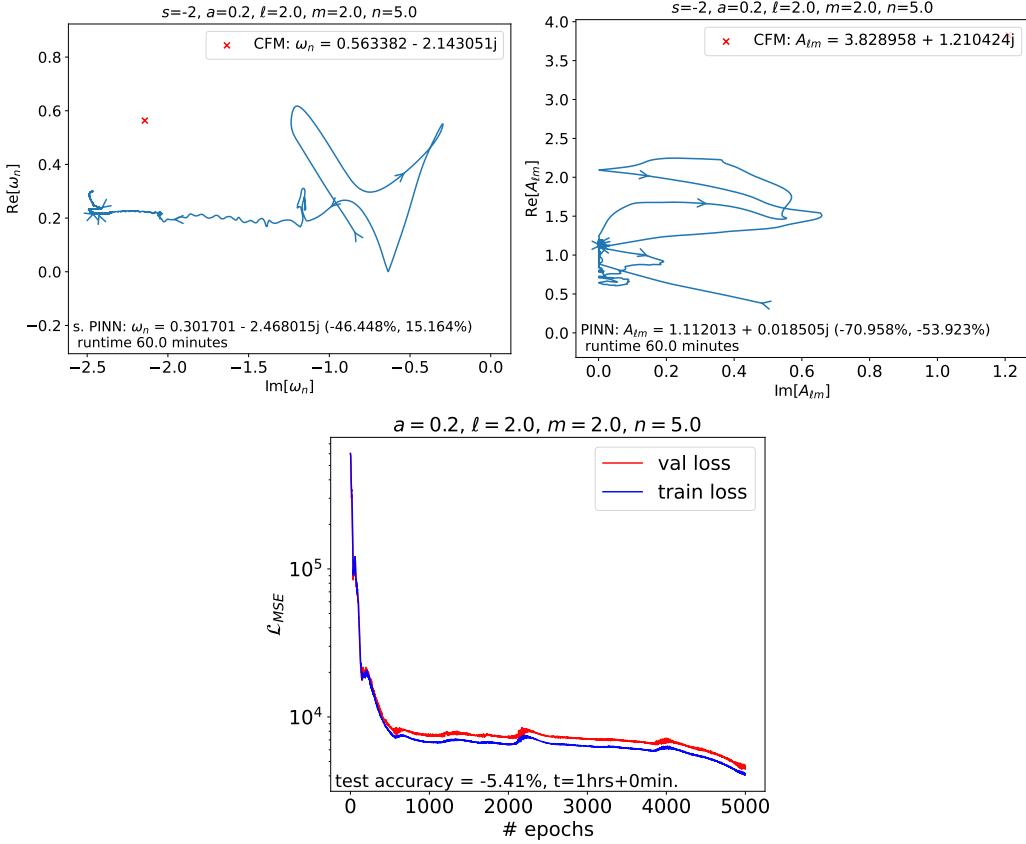


Figure 28: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.2$, $\ell = m = 2$, $n = 5$.

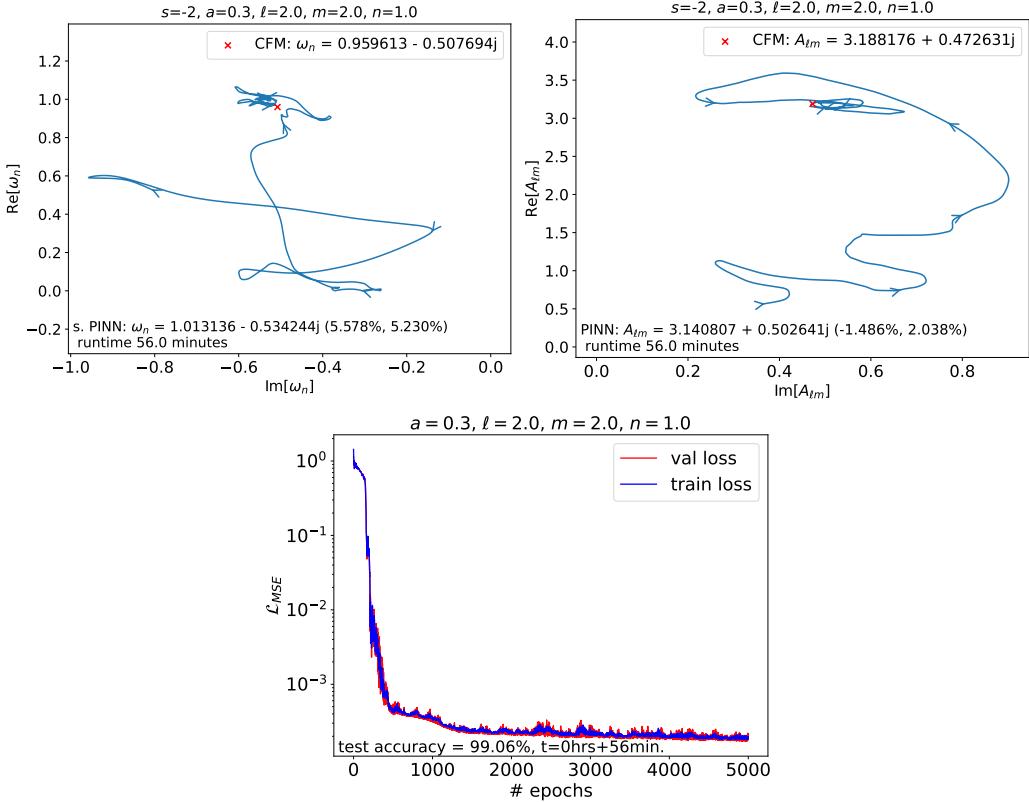


Figure 29: ω_n , $A_{\ell m}$, \mathcal{L}_{MSE} for spin indices $a = 0.3$, $\ell = m = 2$, $n = 1$.

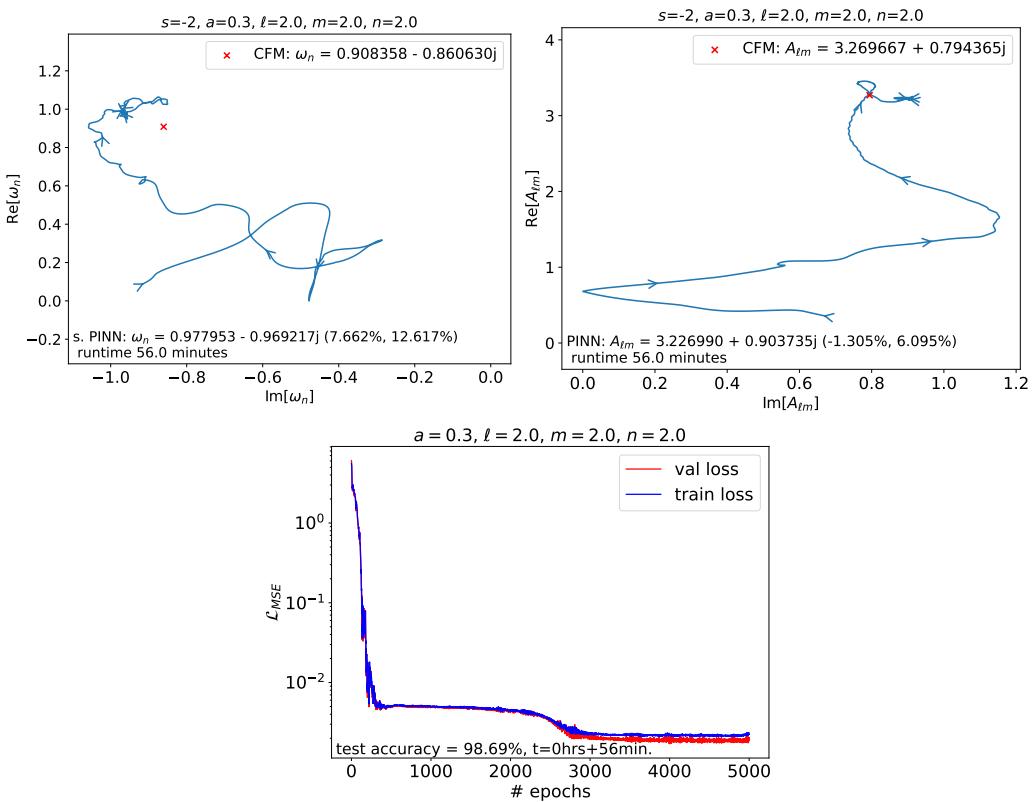


Figure 30: ω_n , $A_{\ell m}$, \mathcal{L}_{MSE} for spin indices $a = 0.3$, $\ell = m = 2$, $n = 2$.

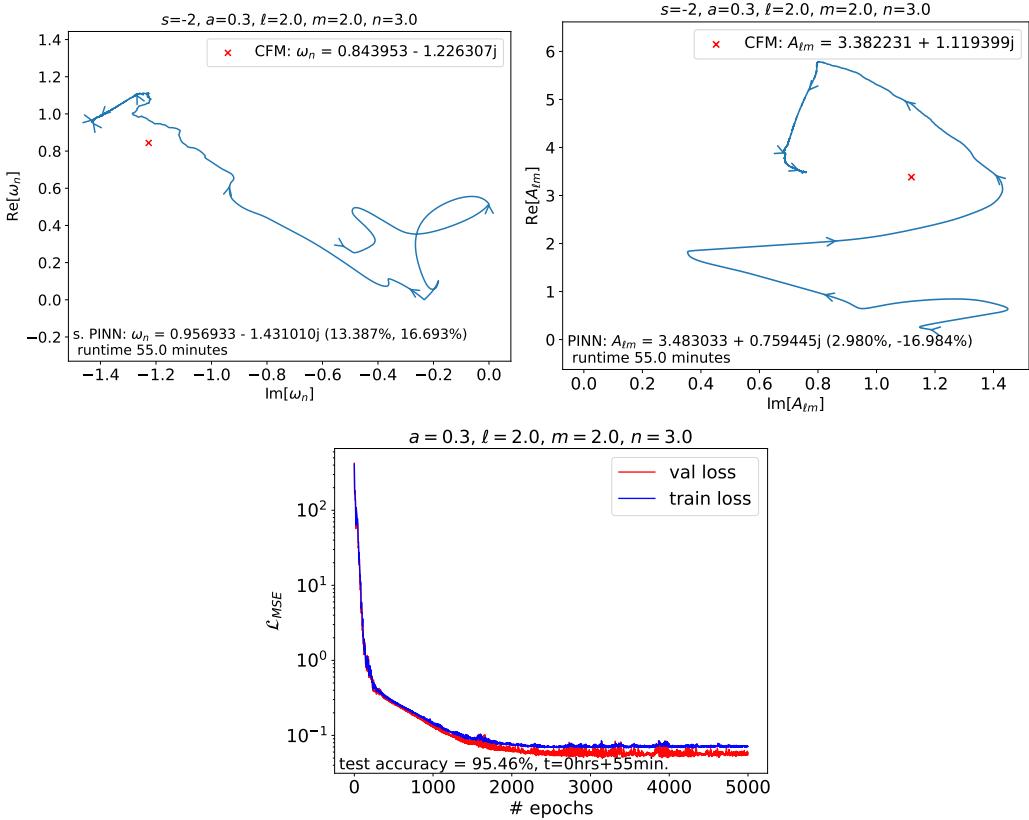


Figure 31: $\omega_n, A_{\ell m}, \mathcal{L}_{MSE}$ for spin indices $a = 0.3, \ell = m = 2, n = 3$.

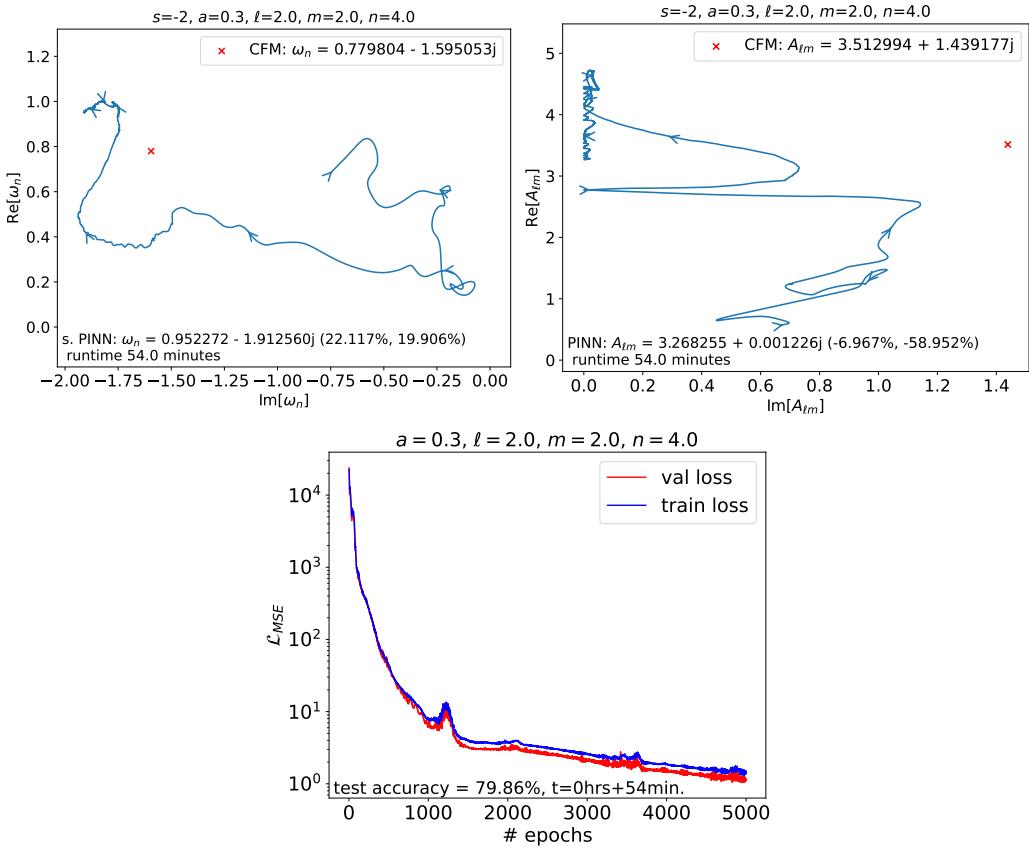


Figure 32: $\omega_n, A_{\ell m}, \mathcal{L}_{MSE}$ for spin indices $a = 0.3, \ell = m = 2, n = 4$.

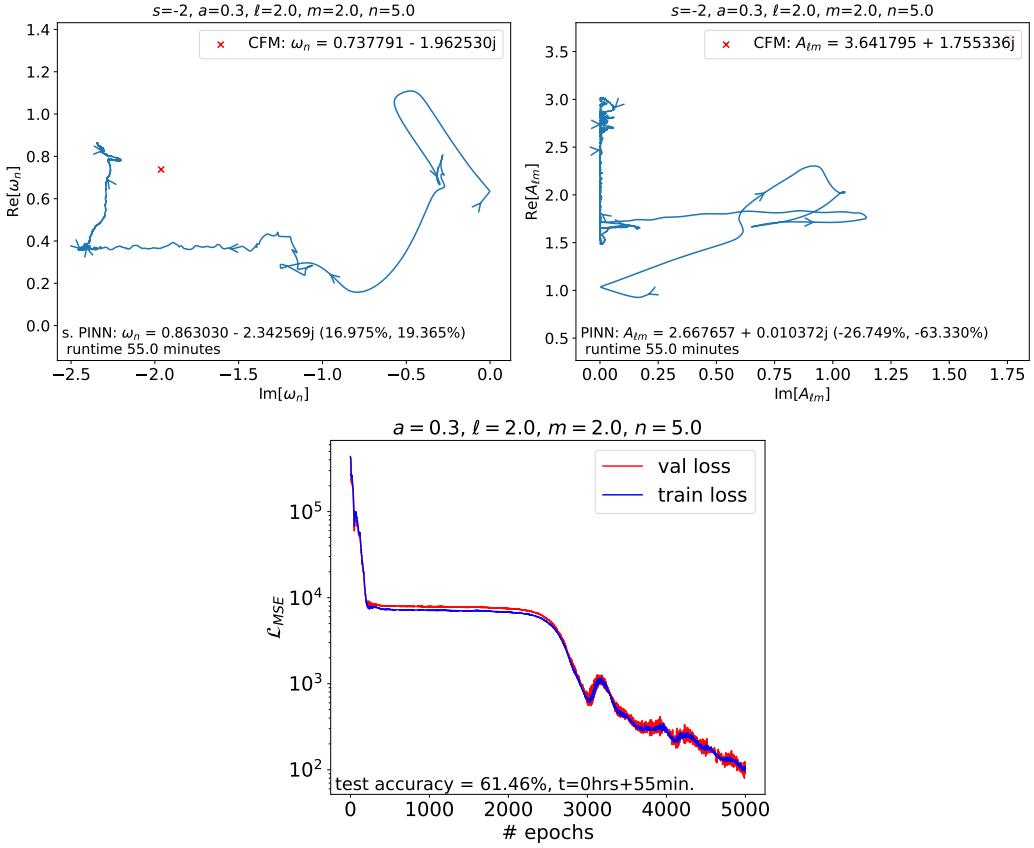


Figure 33: $\omega_n, A_{\ell m}, \mathcal{L}_{MSE}$ for spin indices $a = 0.3, \ell = m = 2, n = 5$.

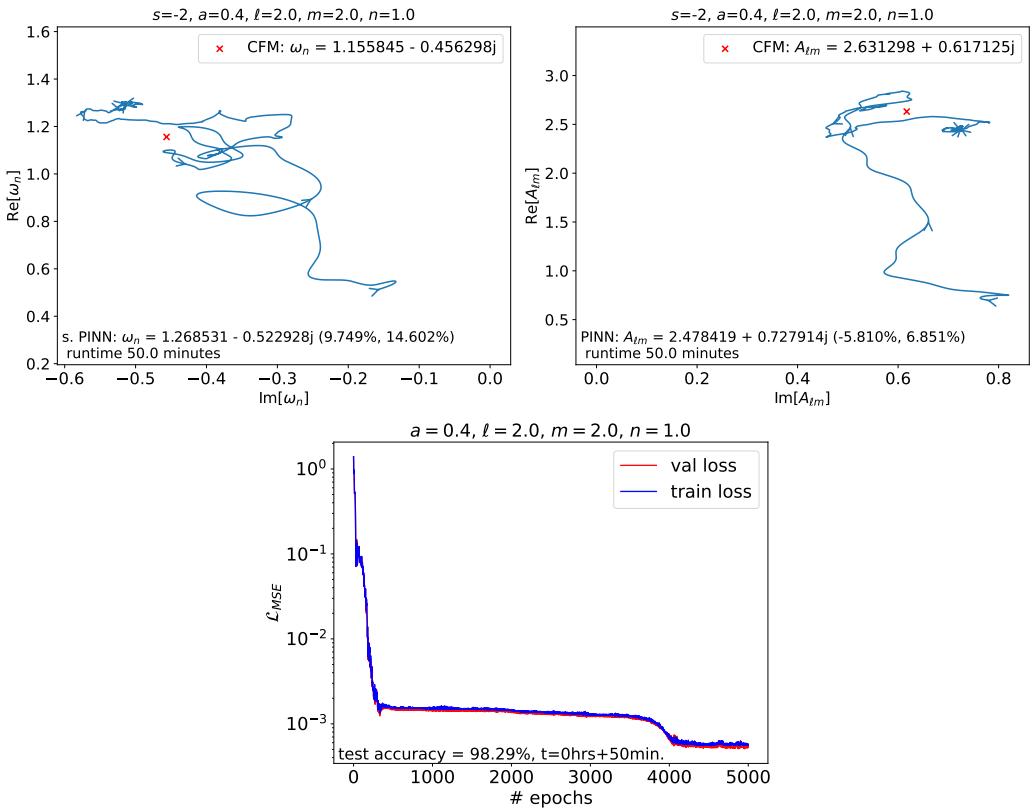


Figure 34: $\omega_n, A_{\ell m}, \mathcal{L}_{MSE}$ for spin indices $a = 0.4, \ell = m = 2, n = 1$.

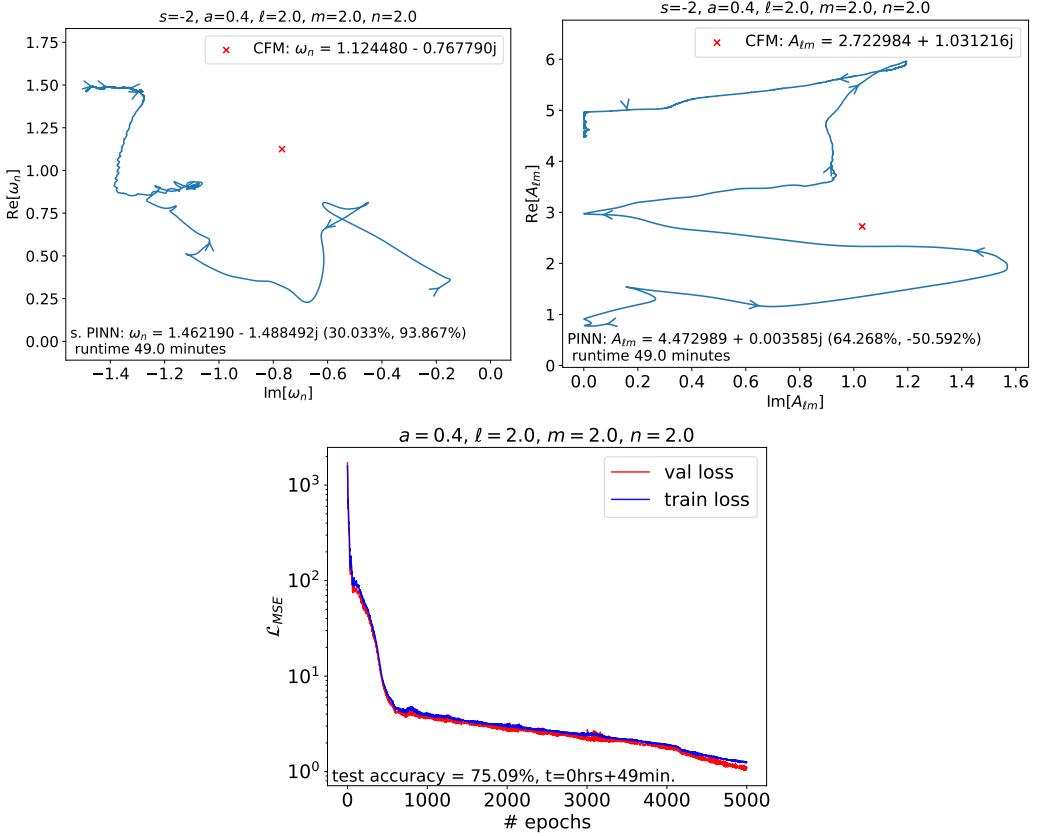


Figure 35: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.4$, $\ell = m = 2$, $n = 2$

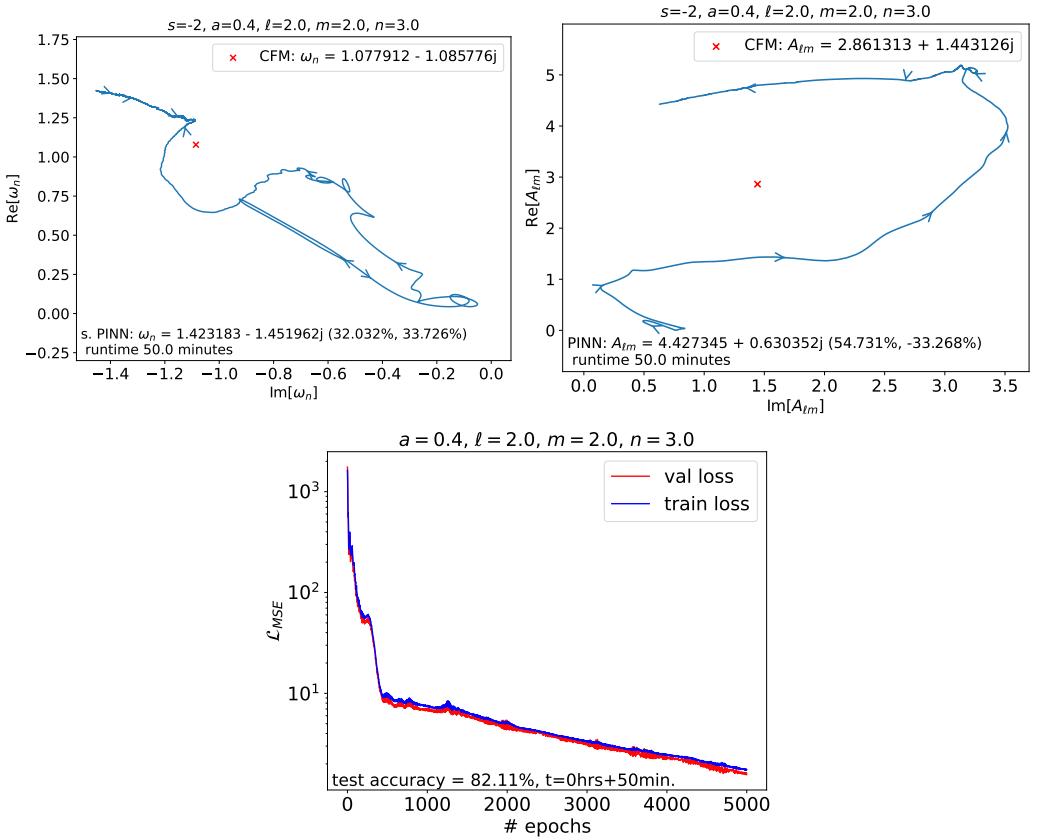


Figure 36: ω_n , A_{lm} , \mathcal{L}_{MSE} for spin indices $a = 0.4$, $\ell = m = 2$, $n = 3$

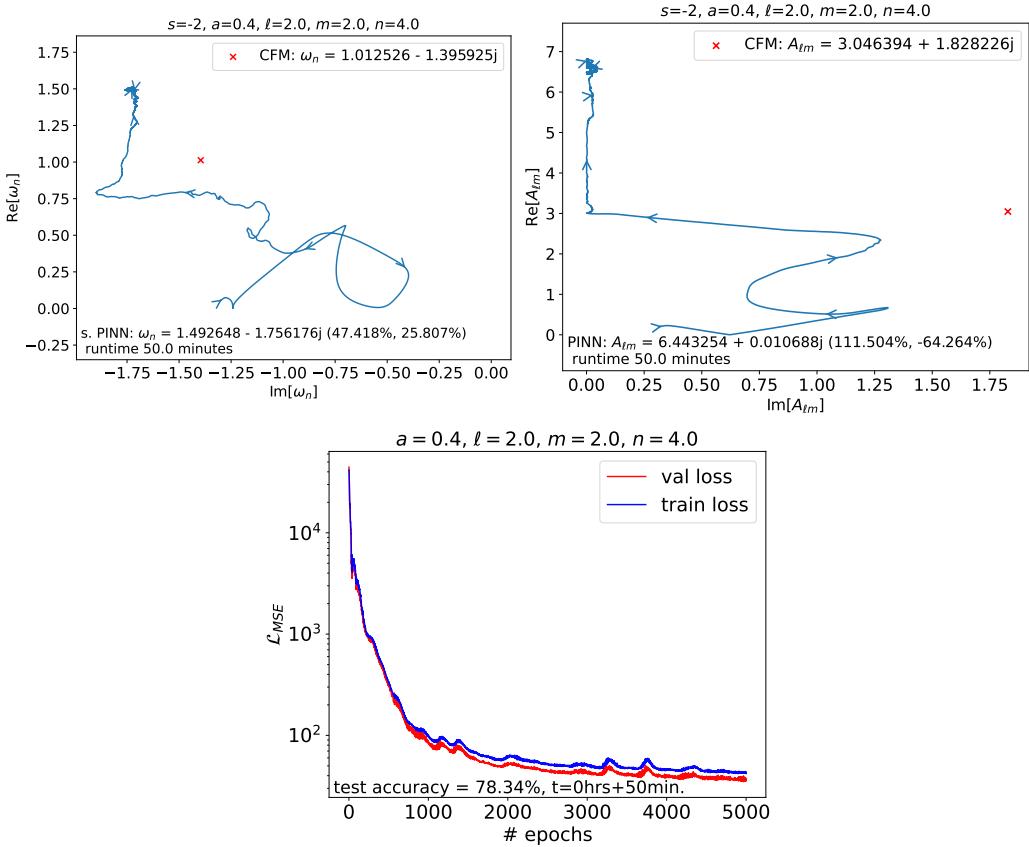


Figure 37: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.4, \ell = m = 2, n = 4$

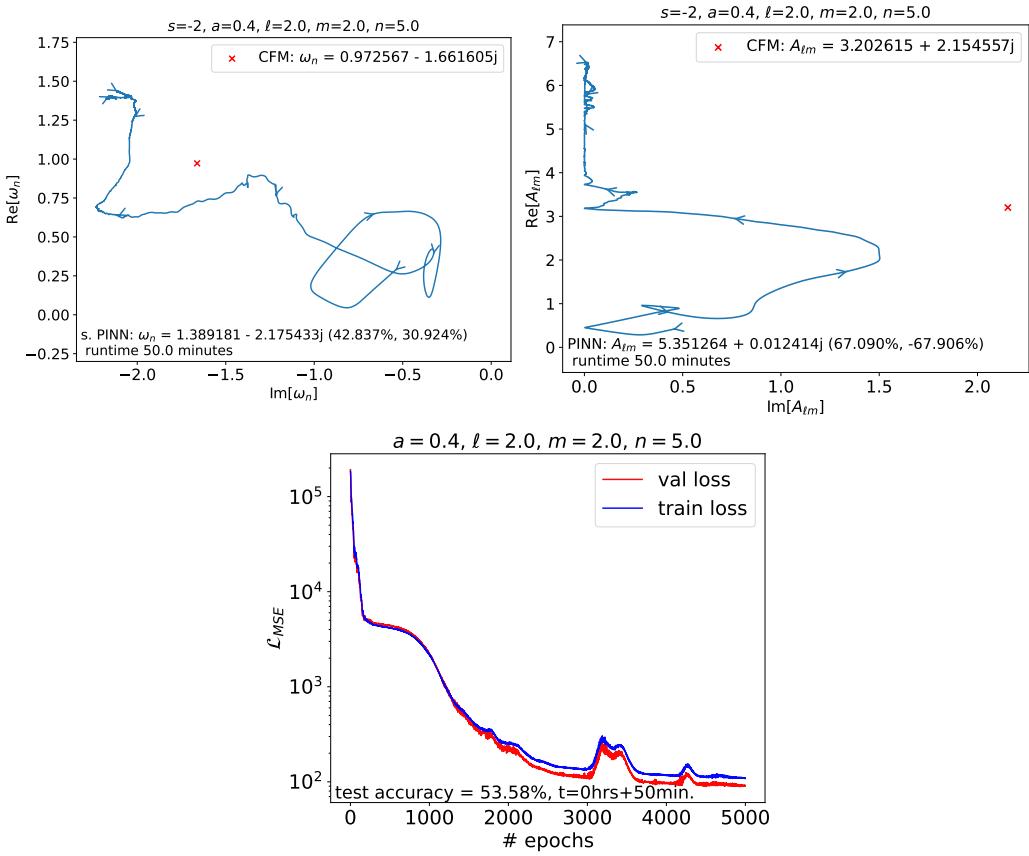


Figure 38: $\omega_n, A_{lm}, \mathcal{L}_{MSE}$ for spin indices $a = 0.4, \ell = m = 2, n = 5$