$$D(a,b) = \left(1-\left(\sum_{m=1}^{M}\beta_m L_m - \sum_{m=1}^{M}\alpha_m U^{(m)}U^{(m)T}\right)\right)_{a,b}.$$

$M=2.$

$$\frac{\partial}{\partial\alpha_1}D(a,b) = \left(U^{(1)}U^{(1)T}\right)_{a,b} = U^{(1)}_{a\cdot}\cdot U^{(1)}_{b\cdot}$$

$$\frac{\partial}{\partial\beta_1}D(a,b) = -L_{1\,a,b}$$

$\frac{\partial}{\partial\alpha_2}$, $\frac{\partial}{\partial\beta_2}$ similar

$$\mathrm{Sil}(C_{1i},C_{2i}) = \frac{\overbrace{\frac{1}{\cancel{|C_{1i}|^2}}\sum_{a,b\in C_{1i}}D(a,b) + \frac{1}{\cancel{|C_{2i}|^2}}\sum_{a,b\in C_{2i}}D(a,b)}^{=f}}{\underbrace{\frac{1}{\cancel{|C_{1i}||C_{2i}|}}\sum_{a\in C_{1i},b\in C_{2i}}D(a,b)}_{=g}}$$

$|C_{1i}|=|C_{2i}|=20.$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial z_1}\frac{\partial z_1}{\partial x}$$

$$\frac{\partial}{\partial\alpha_1}\mathrm{Sil}(C_{1\underset{\text{fixed}}{i}},C_{2i}) = f'g - g'fg^2$$

$$= \left(\sum_{C_{1i}}U^{(1)}_{a\cdot}\cdot U^{(1)}_{b\cdot} + \sum_{C_{2i}}U^{(1)}_{a\cdot}U^{(1)}_{b\cdot}\right)\cdot\sum_{C_{1i},C_{2i}}D(a,b)$$

$$-\sum_{C_{1i},C_{2i}}U^{(1)}_{a\cdot}U^{(1)}_{b\cdot}\cdot\left(\sum_{C_{1i}}D(a,b)+\sum_{C_{2i}}D(a,b)\right)\left(\sum_{C_{1i}C_{2i}}D(a,b)\right)^2$$

$$\frac{\partial}{\partial\beta_1}\mathrm{Sil}(C_{1i},C_{2i}) = -\left(\sum_{C_{1i}}L_{1ab}+\sum_{C_{2i}}L_{1ab}\right)\sum_{C_{1i}C_{2i}}D(a,b)$$

$$+\sum_{C_{1i}C_{2i}}L_{1ab}\cdot\left(\sum_{C_{1i}}D(a,b)+\sum_{C_{2i}}D(a,b)\right)\left(\sum_{C_{1i}C_{2i}}D(a,b)\right)^2$$

$$\rho = \frac{\overbrace{E\left[\left(\mathrm{Sil}(C_{1i},C_{2i})-E[\mathrm{Sil}(C_1,C_2)]\right)\left(\frac{1}{p_i}-E(\frac{1}{p})\right)\right]}^{=f}}{\underbrace{\sigma_{\mathrm{Sil}(C_1,C_2)}\,\sigma_{\frac{1}{p}}}_{=g}}$$

$\leftarrow p_i$ const.

$n=20$

$$\frac{\partial f}{\partial S_i} = \left\{\frac{1}{n}\sum_{k=1}^{n}\left[\left(S_k-\frac{1}{n}\sum S_k\right)\left(\frac{1}{p_k}-E(\frac{1}{p})\right)\right]\right\}' = \frac{n-1}{n^2}\left(\frac{1}{p_i}-E(\frac{1}{p})\right)$$

$S_i = \mathrm{Sil}(C_{1i},C_{2i})$

$$\frac{\partial}{\partial\alpha_1}f\left(\mathrm{Sil}(C_{11},C_{21}),\cdots\mathrm{Sil}(C_{1n},C_{2n})\right) = \sum_{i=1}^{n}\frac{\partial f}{\partial S_i}\cdot\frac{\partial S_i}{\partial\alpha_1} = \cdots\cdots$$

$$\frac{\partial g}{\partial S_i} = \left\{\sqrt{E(S_i^2)-E(S_i)^2}\cdot\sigma_{\frac{1}{p}}\right\}' = \frac{\sigma_{\frac{1}{p}}}{\cancel{2}\sigma_{\mathrm{Sil}(C_1,C_2)}}\cdot\left(\frac{1}{n}\cdot\cancel{2}S_i-\cancel{2}E(S_i)\cdot\frac{1}{n}\right)$$

$$\frac{\partial g}{\partial\alpha_i} = \cdots\cdots \qquad = \frac{\sigma_{\frac{1}{p}}(S_i-E(S_i))}{n\,\sigma_{\mathrm{Sil}(C_1,C_2)}}$$