残差Jacobian的推导

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视觉残差为:

$$r_c = \begin{bmatrix} \frac{x_{cj}}{z_{cj}} - u_{cj} \\ \frac{y_{cj}}{z_{cj}} - v_{cj} \end{bmatrix}$$

对于第i帧中的特征点，它投影到第j帧相机坐标系下的值为:

$$\begin{bmatrix} x_{cj} \\ y_{cj} \\ z_{cj} \\ 1 \end{bmatrix} = T_{bc}^{-1} T_{wb_j}^{-1} T_{wb_i} T_{bc} \begin{bmatrix} \frac{1}{\lambda} u_{ci} \\ \frac{1}{\lambda} v_{ci} \\ \frac{1}{\lambda} \\ 1 \end{bmatrix}$$

换成三维坐标形式为:

$$f_{cj} = \begin{bmatrix} x_{cj} \\ y_{cj} \\ z_{cj} \end{bmatrix} = R_{bc}^T R_{wb_j}^T R_{wb_i} R_{bc} \overbrace{\boxed{\frac{1}{\lambda} \begin{bmatrix} u_{ci} \\ v_{ci} \\ 1 \end{bmatrix}}}^{f_{ci}} + R_{bc}^T \left(R_{wb_j}^T \left(\left(R_{wb_i} p_{bc} + p_{wb_i} \right) - p_{wb_j} \right) - p_{bc} \right)$$

为了简化公式，先定义如下变量:

$$f_{b_i} = R_{bc} f_{ci} + p_{bc}$$
$$f_w = R_{wb_i} f_{b_i} + p_{wb_i}$$
$$f_{b_j} = R_{wb_j}^T (f_w - p_{bc})$$

Jacobian 为视觉误差对两个时刻的状态量、外参、以及逆深度求导.

$$J = \begin{bmatrix} \frac{\partial r_c}{\partial \begin{bmatrix} \delta p_{b_i b_i'} \\ \delta \theta_{b_i b_i'} \end{bmatrix}} & \frac{\partial r_c}{\partial \begin{bmatrix} \delta p_{b_j b_j'} \\ \delta \theta_{b_j b_j'} \end{bmatrix}} & \frac{\partial r_c}{\partial \begin{bmatrix} \delta p_{cc'} \\ \delta \theta_{cc'} \end{bmatrix}} & \frac{\partial r_c}{\partial \delta \lambda} \end{bmatrix}$$

根据链式法则，Jacobian 的计算可以分为两步走，第一步误差对 f_{cj} 求导

$$\frac{\partial r_c}{\partial f_{cj}} = \begin{bmatrix} \frac{1}{z_{cj}} & 0 & -\frac{x_{cj}}{z_{cj}^2} \\ 0 & \frac{1}{z_{cj}} & -\frac{y_{cj}}{z_{cj}^2} \end{bmatrix}$$

i时刻

第二步: ① f_{cj} 对状态量求导

a). 对i时刻的位移求导，可直接写出如下:

$$\frac{\partial f_{cj}}{\partial \delta p_{b_i b_i'}} = R_{bc}^T R_{wb_j}^T$$

b). 对i时刻角度偏差求导.

$$f_{cj} = R_{bc}^T R_{wb_j}^T R_{wb_i} R_{bc} f_{ci} + R_{bc}^T \left(R_{wb_j}^T \left(\left(R_{wb_i} p_{bc} + p_{wb_i} \right) - p_{wb_j} \right) - p_{bc} \right)$$

b> 对 i时刻 角度偏差 求导.

$$f_{c_i} = R_{bc}^T R_{wb_j}^T R_{wb_i} R_{bc} f_{c_i} + R_{bc}^T (R_{wb_j}^T ((R_{wb_i} p_{bc} + p_{wb_i}) - p_{wb_j}) - p_{bc})$$

$$= R_{bc}^T R_{wb_j}^T R_{wb_i} R_{bc} f_{c_i} + R_{bc}^T R_{wb_j}^T R_{wb_i} p_{bc} + (\cdots)$$

$$= R_{bc}^T R_{wb_j}^T R_{wb_i} (R_{bc} f_{c_i} + p_{bc}) + \cdots$$

$$= R_{bc}^T R_{wb_j}^T R_{wb_i} f_{b_i} + \cdots$$

$$\Rightarrow \frac{\partial f_{c_i}}{\partial \delta\theta_{b_i b_i'}} = \frac{\partial R_{bc}^T R_{wb_j}^T R_{wb_i} (I + [\delta\theta_{b_i b_i'}]_\times) f_{b_i}}{\partial \delta\theta_{b_i b_i'}}$$

$$= - R_{bc}^T R_{wb_j}^T R_{wb_i} [f_{b_i}]_\times$$

② 对 j时刻的状态量求导:

a> 对位移求导.

$$\frac{\partial f_{c_j}}{\partial \delta p_{b_j b_j'}} = -R_{bc}^T R_{wb_j}^T$$

b> 对角度偏差求导,

$$f_{c_j} = R_{bc}^T R_{wb_j}^T (R_{wb_i}(R_{bc} f_{c_i} + p_{bc}) + p_{wb_i} - p_{wb_j}) + \cdots$$

$$= R_{bc}^T R_{wb_j}^T (f_w - p_{wb_j}) + \cdots$$

Jacobian:

$$\frac{\partial f_{c_j}}{\partial \delta\theta_{b_j b_j'}} = \frac{\partial R_{bc}^T (I - [\delta\theta_{b_j b_j'}]_\times) R_{wb_j}^T (f_w - p_{wb_j})}{\partial \delta\theta_{b_j b_j'}}$$

$$= \frac{\partial R_{bc}^T (I - [\delta\theta_{b_j b_j'}]_\times) f_{b_j}}{\partial \delta\theta_{b_j b_j'}}$$

$$= R_{bc}^T [f_{b_j}]_\times$$

③ 对IMU和相机外参之间的求导

a> 对位移求导:

$$\frac{\partial f_{c_j}}{\partial \delta p_{c c'}} = R_{bc}^T (R_{wb_j}^T R_{wb_i} - I_{3\times 3})$$

b> 对角度偏差求导.

$$f_{c_j} = f_{c_j}^1 + f_{c_j}^2$$

$$\Rightarrow \frac{\partial f_{c_j}^1}{\partial \delta\theta_{cc'}} = \frac{\partial (I - [\delta\theta_{cc'}]_\times) R_{bc}^T R_{wb_j}^T R_{wb_i} R_{bc} (I + [\delta\theta_{cc'}]_\times) f_{c_i}}{\partial \delta\theta_{cc'}}$$

$$= \partial R_{bc}^T R_{wb_j}^T R_{wb_i} R_{bc} [\delta\theta_{cc'}]_\times f_{c_i} - [\delta\theta_{cc'}]_\times R_{bc}^T R_{wb_j}^T R_{wb_i} R_{bc} f_{c_i} + o^2 \cdots$$

$$\partial\, \delta\theta_{ci}' \qquad\qquad \partial\, \delta\theta_{ci}'$$

$$= \frac{\partial\, R_{bc}^T R_{wbj}^T R_{wbi} R_{bc} [\delta\theta_{ci}']_\times f_{ci} - [\delta\theta_{ci}']_\times R_{bc}^T R_{wbj}^T R_{wbi} R_{bc} f_{ci} + O^2(\delta\theta_{ci}')}{\partial\, \delta\theta_{ci}'}$$

$$= -R_{bc}^T R_{wbj}^T R_{wbi} R_{bc} [f_{ci}]_\times + [R_{bc}^T R_{wbj}^T R_{wbi} R_{bc} f_{ci}]_\times$$

$$\frac{\partial f_{cj}^2}{\partial\, \delta\theta_{ci}'} = [R_{bc}^T (R_{wbj}^T ((R_{wbi} P_{bc} + P_{wbi}) - P_{wbj}) - P_{bc})]_\times$$

③ 视觉误差对特征逆深度的求导

$$\frac{\partial f_{cj}}{\partial \lambda} = \frac{\partial f_{cj}}{\partial f_{ci}} \cdot \frac{\partial f_{ci}}{\partial \lambda} = -\frac{1}{\lambda} R_{bc}^T R_{wbj}^T R_{wbi} R_{bc} f_{ci}$$