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A parcel locker network as a solution to the logistics last mile problem

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We consider the problem of designing a parcel locker network as a solution to the Logistics Last Mile Problem: Choosing the optimal number, locations, and sizes of parcel locekers facilities. The objective is to maximize the total profit, consisting of the revenue from customers who use the service, minus the facilities' fixed and operational setup costs, the discounts in the delivery costs for customers who need to travel in order to collect their parcels, and the loss of potential customers who are not willing to travel for service. The problem is expressed as a 0–1 integer linear program. We show that it is equivalent to the well-known Uncapacitated Facility Location Problem. We then solve the modified problem, and apply it to an industrial-sized network.

Keywords: supply chain design; facility location; city logistics; design for service; E-commerce; mixed integer linear programming

1. Introduction

In recent years, the E-commerce market has been expanding at an ever-increasing rate, with products ranging from high-value durable goods to low-value consumer goods. In 2016, E-commerce sales worldwide amounted to 1.86 trillion US dollars, and they are projected to grow to 4.48 trillion US dollars in 2021 (Statista 2017). The expansion and continuous growth of this market lead to a dramatic upsurge in direct-to-consumer deliveries, and therefore, new attention should be drawn to certain issues in the final part of the supply chain.

Many stages in the process of transporting goods to consumers have undergone significant improvements over the years and are now handled in an efficient and cost effective manner (e.g. transporting goods via freight rail networks to a certain station, or via container ships to some port). However, the final stage in the process – that of bringing the goods to the doorstep of the consumer – is often the least efficient, and the most expensive and polluting part of this process, comprising up to 28% of the total cost of delivery (Goodman 2005; Spiegler 2004). Finding ways to improve this final stage is known as the Logistics Last Mile Problem. In general, the Logistics Last Mile Problem does not refer only to the E-commerce and private consumers market. However, in this research we focus only on this market.

As Mr. Alex Walker[1] points out (Goodman 2005): 'It doesn't seem like it would be that difficult. I've got a DC and 20 miles to drive. It shouldn't be that hard. Well, there's a tremendous amount of complexity'.

Why is the 'last mile' so costly and ineffective? The main reasons are: (i) the lack of economies of scale: business-to-consumer deliveries often involve one package per stop, compared with a large number of packages per delivery up to that point, (ii) the difficulty of finding the specific home address of the end consumer, either in large apartment blocks in the city, or in rural areas, where the roads may not have proper signs, and the consumer may live in a remote ranch house or a small community, and (iii) the 'Not-at-Home Problem', especially when the end consumer needs to sign a receipt confirming delivery, which results in a high delivery failure and empty trip rates (Gevaers, Van de Voorde, and Vanelslander 2011).

Further, as consumer preferences have moved more and more to the center of attention, large E-commerce parties identify last-mile services as a key differentiator vs. their competitors (Joerss et al. 2016).

The articles addressing the Logistics Last Mile Problem can be categorized into four main research streams. The first stream was launched more than 60 years ago in the seminal work of Dantzig and Ramser (1959) and has continued ever since. It concerns the optimization of delivery routes, by solving vehicle routing problems, with or without time window constraints, see e.g. Caceres-Cruz et al. (2015), Vidal et al. (2013), Toth and Vigo (2014) and Crainic and Laporte (2012).

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The second stream focuses on invention of new delivery options, such as cargo-bikes, electric vehicles, autonomous vehicles, and drones, see e.g. Anderluh, Hemmelmayr, and Nolz (2017), Klumpp, Witte, and Zelewski (2014), Chong et al. (2011), Perboli, Rosano, and Gobbato (2017) and Lee et al. (2016).

The third stream focuses on the environmental aspects of the problem, either by comparing the pollution levels that result from customers' pick-up strategies with those of home deliveries (Brown and Guiffrida 2014; Edwards, McKinnon, and Cullinane 2010), or by estimation of the environmental impact of using alternative customers' outlets, through case studies. For example, the case study described in Song et al. (2013) shows that using local collection and delivery points for failed first time home shopping deliveries instead of traditional carrier redelivery methods may reduce significantly greenhouse gas emissions.

The fourth stream focuses on designing a connected logistics distribution strategy. Some of it is done by exploiting key concepts of the *Physical Internet* (PI) (Montreuil, Meller, and Ballot 2010). The idea is to design a logistics system that moves parcels in standard-sized, modular containers as efficiently and seamlessly as the internet moves digital information in data packets.

Zhang et al. (2016) emphasize the combined characteristics of logistics of physical and non-physical objects and thus, argue for a product service system view on logistics. Logistics services are encapsulated and accessible via XML based description and interface. Fazili et al. (2017) compare the performance of PI, conventional, and hybrid logistic systems in order to quantify the advantages and disadvantages of each one of them. The results show that PI reduces driving distance, greenhouse gas emissions and social costs of truck driving, but increases the number of container transfers within the logistics centres. Zhang et al. (2017) consider a carrier collaboration network with multiple less-than-truckload carriers and multiple vehicle type during a time horizon, with the objective of minimizing the total transportation cost without reducing the individual profit of the carriers. Faugere and Montreuil (2017) list the advantages and disadvantages of four different alternatives for parcel lockers design, from the current solution of a fixed-configuration lockers to innovative solution of smart mobile modular lockers. Cheng, Liao, and Hua (2017) propose a new pickup policy for courier routing, which combines the nearest-neighbour policy with centrality measures, and enables to dispatch an idle courier to the more central request locations.

In the current work we focus on the usage of *parcel lockers* (or *shared reception boxes*) as a solution to the Logistics Last Mile Problem. To clarify the terminology, we next distinguish between the following terms: a reception box, a delivery box, a collection point, and a parcel locker. A *reception box* is a fixed locker installed outside a consumer's home, that can be accessed using a key or an electronic code. A *delivery box* is a locker owned by the delivery company. It is filled with the parcels at a distribution depot, and then temporarily attached to the consumer's home via a fixed locking device. A *collection point* is a place near the consumer's home, to which parcels are delivered. The consumer can collect its parcels only on certain opening hours. A *parcel locker* is a group of lockers, sited in apartment blocks, work places, railway stations, etc. The lockers have electronic locks with variable opening codes, and so they can be used by different consumers, whenever it is convenient to them.

To the best of our knowledge, even though parcel lockers are currently being used in more than 20 countries, including the UK, Europe, US, and Canada, our work is the first attempt to develop a quantitative approach to determine their optimal number, locations, and sizes, aimed at mitigating the Logistics Last Mile Problem.

The few articles that investigate issues related to our work are either qualitative in nature (e.g. Song et al. 2013), focus on other problems (e.g. Faugere and Montreuil 2017), or focus on particular case studies. For example, Kämäräinen (2001) uses delivery data from the suburban area of Helsinki to compare the costs of regular home delivery with that of delivery to reception boxes. The results show a cost reduction of 42% when using reception boxes. Lemke, Iwan, and Korczak (2016) use data provided by a polish postal service company to study the assessment of parcel lockers usage in Poland, focusing on their current locations. The results show that 15% of consumers will use the lockers more often if their locations are improved, and that most consumers prefer to use lockers near their homes. Punakivi (2001) uses sales data from a large retail company in Finland, and compares the operational costs of regular home delivery, with that of delivery to reception boxes, delivery boxes, and parcel lockers. The results show that transportation costs when delivering to parcel lockers are 55–66% lower than those when using an attended reception with a two-hour delivery time window.

As aforementioned, in this work we focus on parcel lockers facilities as a solution to the Logistics Last Mile Problem. We wish to determine, given a certain area topography and consumers' demands, how many locker sites to open, where to locate them, and how many standard lockers to install in each site, so as to maximize the total resulted profit. The total profit consists of the revenue from customers who use the service, minus the facilities' fixed and operational setup costs, the discounts in the delivery costs for customers who need to travel in order to collect their parcels, and the loss of potential customers who are not willing to travel for service. We note that the majority of the papers that deal with the Logistic Last Mile Problem assume that all the demand need to be satisfied, and thus the revenue is fixed, and only the costs need to be considered. Our work is among those few papers that take into account the possibility of lost demand, and consider the profits and costs of the problem (see e.g. Feillet, Dejax, and Gendreau 2005).

The original formulation of our problem results in a 0–1 integer linear program. We show that the problem can be transformed into the well-known Uncapacitated Facility Location Problem (UFLP) (Balinski 1965), and thus can be solved by any one of the many mathematical solutions algorithms that were developed for that purpose, e.g. Beasley (1993), Cornuéjols, Nemhauser, and Wolsey (1983), Eiselt and Marianov (2011), Erlenkotter (1978), Korte et al. (2012), Krarup and Pruzan (1983) and Owen and Daskin (1998). Since our work does not focus on developing new solutions techniques to the UFLP, and since we demonstrate our work using a realistic, yet limited-sized network (∼100 nodes), we implement the mixed-integer linear programming solver of MATLAB, *intlinprog* (Mathworks.com 2017).

The remainder of this paper is organized as follows: In Section 2 we formulate the problem. In Section 3 we solve it. In Section 4 we demonstrate our solution with a simple numerical example, and with an industrial-sized network example. In Section 5 we summarize the paper.

2. The parcel locker facilities location problem

We consider the problem of designing a network of parcel locker facilities as a solution to the Logistics Last Mile Problem, where we assume that the lockers can be shared and used by several E-commerce companies. Specifically, we want to decide how many locker sites to open, where to locate them, and how many standard lockers to install in each site, so as to maximize the total resulted profit.

2.1 *Problem formulation*

Let $G = (I, E)$ be an undirected, connected and simple network, with a node set I (of size $n = ||I||$) and an edge set E. The nodes and edges of the network represent areas and distances between the areas, respectively (e.g. neighborhoods of a city, and the distances among them).

Each node $i \in I$ has a demand $P_i > 0$, which represents the population that resides and works at the corresponding area, and an online ordering frequency $F_i > 0$. Let $Q_i \equiv P_i \cdot F_i (> 0)$ denote the total online ordering of node $i \in I$ per unit of time, and let $Q \equiv \sum_{i \in I} Q_i$. It is assumed that the ordering processes are independent of each other. For each $i \in I$, node i is the *home site* of Q_i.

Let $d_{i,j} (\geq 0)$ be the distance between nodes $i, j \in I$, and let $D_{\max} \equiv \max_{i,j \in I} d_{i,j}$.

For $u = 0, 1, \dots, m$, with $m \leq n$, let D_u be a fixed distance, with

$$0 \equiv D_0 < D_1 < \dots < D_m \equiv D_{\max}. \tag{1}$$

For each node $i \in I$, and for each $u = 1, \dots, m$, let $S_i^u \equiv \{j \in I : d_{i,j} \in (D_{u-1}, D_u]\}$, i.e. S_i^u is the set of nodes at the uth layer from node i. Further, for each node $i \in I$, let $S_i^0 \equiv \{i\}$ and $S_i \equiv (S_i^0, \dots, S_i^m)$.

For each node $i \in I$, and for each $u = 0, 1, \dots, m$, let l_i^u indicate whether at least one facility is opened at node i's uth layer. That is,

$$l_i^u = \begin{cases} 1 & \text{if at least one facility is opened at } S_i^u, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The consignment company wants to open several parcel locker facilities at the network so as to maximize the total resulted profit. The company's profit from operating the network consists of the revenue from customers who agree to use the service, minus the facilities' fixed and operational setup costs, the loss of potential customers who do not agree to travel for service, and the discounts in the delivery costs for those who agree.

It is assumed that customers always prefer their nearest facility. Customers that have a facility at their home site would always use the service. However, the percentage of customers who agree to travel and collect their parcels from sites other than their home site decreases with the distance. In particular, the percentage of customers who agree to travel a distance which is not larger than D_1 is higher than the percentage of customers who agree to travel a distance which is larger than D_1 but is not larger than D_2, and so on. That is,

$$1 \equiv \rho^0 > \rho^1 \geq \dots \geq \rho^m (\geq 0). \tag{3}$$

If customers are not willing to travel a distance larger than D_u, $u \geq 1$, then: $\rho^{u+1} = \rho^{u+2} = \dots = \rho^m = 0$. Note that we can also assume that these percentages depend on the nodes. The complexity of the problem will be the same.

Let $R > 0$ denote the consignment company's revenue per ordering. Since not all customers are willing to travel and collect their parcels from sites other than their home site, those that agree are compensated by the company, by receiving a discount in the delivery costs. For $u = 0, 1, \ldots, m$, let $C_d^u \geq 0$ be the discount in the delivery costs for customers who travel to a facility at their uth layer, with:

$$0 \equiv C_d^0 \leq C_d^1 \leq C_d^2 \leq \ldots \leq C_d^m (< R) \tag{4}$$

(note that if $C_d^m \geq R$, the consignment company may not operate the system).

Let $f_i > 0$ be the setup cost of a facility at node $i \in I$. The setup cost consists of the fixed cost of constructing a facility, and a varying cost, such as maintaining the facility, rental, electricity, and more (all costs are normalized per period using the same periods for which the demand is expressed above).

We assume throughout that the consignment company can open facilities only at the nodes of the network.

Let X be the facilities' location vector, i.e. for each $i \in I$:

$$x_i = \begin{cases} 1 & \text{if a facility is opened at node } i, \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Let Y be the customers' layer allocation matrix, i.e. for each $i \in I$, $u = 0, 1, \ldots, m$:

$$y_{i,u} \in \begin{cases} \{1\} & \text{if all customers at node } i \text{ travel to a facility at their } u\text{th layer,} \\ (0,1) & \text{if a fraction of the customers at node } i \text{ travel to a facility at their } u\text{th layer,} \\ \{0\} & \text{otherwise.} \end{cases} \tag{6}$$

With this notation, the problem formulation, which we denote by Problem (7), is:

$$\min_{X,Y,L} \sum_{i=1}^{n} \left[f_i x_i + Q_i \sum_{u=0}^{m} (C_d^u - R) \rho^u y_{i,u} \right] \tag{7a}$$

$$\text{s.t.:} \quad y_{i,u} \leq l_i^u \ \forall\, i \in I, u = 0, 1, \ldots, m, \tag{7b}$$

$$\sum_{u=0}^{m} y_{i,u} = 1 \ \forall\, i \in I, \tag{7c}$$

$$l_i^u \leq \sum_{j \in S_i^u} x_j \ \forall\, i \in I, u = 0, 1, \ldots, m, \tag{7d}$$

$$l_i^u \geq x_j \ \forall\, j \in S_i^u, u = 0, 1, \ldots, m, \ i \in I, \tag{7e}$$

$$x_i \in \{0,1\} \forall\, i \in I, \tag{7f}$$

$$l_i^u \in \{0,1\} \ \forall\, i \in I, u = 0, 1, \ldots, m. \tag{7g}$$

$$y_{i,u} \geq 0 \ \forall\, i \in I, u = 0, 1, \ldots, m. \tag{7h}$$

The first term in the objective function (7a) is the facilities' setup cost. The second term is the net discounts in the delivery costs for customers who travel in order to collect their parcels. Since $\rho^0 = 1$ and $C_d^0 = 0$ by (3) and (4), respectively, if a facility is located at customers' home site, the customers use it and do not get any discount. Constraints (7b) guarantee that customers in each given node only travel to relevant layers, where a facility is located. Constraints (7c) ensure that the demand in each node can be fully serviced through facilities established in the network. Constraints (7d), (7e), and (7g) determine the values of the layers' indicators. Specifically, Constraints (7d) along with Constraints (7g) ensure that the value of node i's uth layer indicator is smaller than the number of facilities opened at node i's uth layer, so that if no facility is opened there, the indicator's value will be 0. Constraints (7e) along with Constraints (7g) ensure that if at least a single facility is opened at node i's uth layer, the indicator's value will be 1. Note that we do not need to impose close assignment constraints, since the ordering of the discounts (4) guarantees that each customer will travel to its nearest layer u in which at least one facility is located. Constraints (7f) and (7g) are the binary restrictions of the x_i and l_i^u variables, respectively. Constraints (7h) state that the assignment variables must be non-negative. Note that we do not require the $y_{i,u}$ variables to be binary. Since the unit cost from a demand node to the nearest layer with an open facility is strictly less than the unit cost between that node and any other layer with an open facility, then the corresponding assignment variables for that demand node will naturally be binary. That is, all of the demand at that node will be assigned to the nearest layer with an open facility.

Finally, note that we implicitly assume that the parcel locker facilities have an unlimited capacity, and that their sizes can be determined without any restrictions.

In the next example we demonstrate the construction of S_i^u, for $i \in I$ and $u = 0, 1, \ldots, m$.

Example 1 Demonstrating the Construction of S_i^u for $i \in I$ and $u = 0, 1, \ldots, m$:

Consider a path graph on 3 vertices, with $V = \{1, 2, 3\}$, $E = \{[1, 2], [2, 3]\}$, $d_{1,2} = 1$, $d_{2,3} = 1$, $D_1 = 1$, and $D_2 = 2$.

Then, for node 1: $S_1^0 = \{1\}$, $S_1^1 = \{j \in I : d_{1,j} \in (0,1]\} = \{1\}$, and $S_1^2 = \{j \in I : d_{1,j} \in (1,2]\} = \{2\}$. For node 2: $S_2^0 = \{2\}$, $S_2^1 = \{j \in I : d_{2,j} \in (0,1]\} = \{1,3\}$, and $S_2^2 = \{j \in I : d_{2,j} \in (1,2]\} = \emptyset$. Finally, for node 3: $S_3^0 = \{3\}$, $S_3^1 = \{j \in I : d_{3,j} \in (0,1]\} = \{2\}$, and $S_3^2 = \{j \in I : d_{3,j} \in (1,2]\} = \{1\}$.

3. Solution

For each $u = 0, 1, \dots, m$, let $\gamma_u \equiv R(1-\rho^u) + C_d^u \rho^u$, and for each $i \in I$, $u = 0, 1, \dots, m$, and $j \in S_i^u$, let $C_{i,j} \equiv Q_i \gamma_u$. Since $R(1-\rho^u)$ is the loss of potential profit from customers who do not agree to travel to their uth layer, and $C_d^u \rho^u$ is the discount given to customers who agree to travel to their uth layer, γ_u is basically the consignment company's loss from not opening any facility at a nearer layer, namely, layer $0, 1, \dots, u-1$.

LEMMA 1 *For each $i \in I$, $u = 0, 1, \dots, m$, and $j \in S_i^u$, $C_{i,j} \geq 0$.*

Proof. For each $i \in I$, $u = 0, 1, \dots, m$, and $j \in S_i^u$, $C_{i,j} = Q_i \gamma_u = Q_i(R(1-\rho^u) + C_d^u \rho^u) \geq 0$ as $R > 0$, $Q_i > 0$ for each $i \in I$, and $1 - \rho^u \geq 0$, $C_d^u \geq 0$, $\rho^u \geq 0$ for each $u = 0, 1, \dots, m$. □

Let Z be the customers' node allocation matrix, i.e. for each $i, j \in I$:

$$z_{i,j} \in \begin{cases} \{1\} & \text{if all customers at node } i \text{ travel to a facility at node } j, \\ (0,1) & \text{if a fraction of the customers at node } i \text{ travel to a facility at node } j, \\ \{0\} & \text{otherwise.} \end{cases} \tag{8}$$

Then, Problem (7) is equivalent to the following problem, which we denote by Problem (9):

$$\min_{X,Z} \sum_{i\in I} f_i x_i + \sum_{i\in I}\sum_{j\in I} C_{i,j} z_{i,j} \tag{9a}$$

$$\text{s.t.:} \quad \sum_{j\in I} z_{i,j} = 1 \quad \forall\, i \in I, \tag{9b}$$

$$z_{i,j} \leq x_j \quad \forall\, i, j \in I, \tag{9c}$$

$$x_i \in \{0,1\} \quad \forall\, i \in I, \tag{9d}$$

$$z_{i,j} \geq 0 \quad \forall\, i, j \in I. \tag{9e}$$

The first term in the objective function (9a) is the facilities' setup cost, and the second term is the service cost. Constraints (9b) ensure that the demand in each node can be fully serviced through facilities established in the network. Constraints (9c) guarantee that customers in each given node only travel to relevant nodes. Constraints (9d) are the binary restrictions of the x_i variables. Constraints (9e) state that the assignment variables must be non-negative.

Problem (9) is the classic UFLP, which means that we can use any one of the solutions algorithms that were developed to solve it, e.g. Beasley (1993), Cornuéjols, Nemhauser, and Wolsey (1983), Eiselt and Marianov (2011), Erlenkotter (1978), Korte et al. (2012), Krarup and Pruzan (1983) and Owen and Daskin (1998).

Let $Cost^J$ and $\tilde{Cost}^J$ denote the total cost that results from opening facilities at $J \subseteq I$, $J \neq \emptyset$ under (7) and (9), respectively. The next lemma proves that $Cost^J$ and $\tilde{Cost}^J$ are equivalent up to a certain constant, and therefore, under both formulations, the optimal solution is the same.

LEMMA 2 *Consider the subset $J \subseteq I$, $J \neq \emptyset$. Then,*

$$Cost^J = \tilde{Cost}^J - RQ. \tag{10}$$

Proof. According to (7a):

$$\begin{aligned} Cost^J &= \sum_{j\in J} f_i + \sum_{j\in I} Q_j \sum_{u=\min\{\{0,1,\dots,m\}\mid \emptyset\neq S_j^u\subseteq J\}} (C_d^u - R)\rho^u = \\ &\sum_{j\in J} f_i + \sum_{j\in J} Q_j (C_d^0 - R)\rho^0 + \sum_{j\in I\setminus J} Q_j \sum_{u=\min\{\{1,2\dots,m\}\mid \emptyset\neq S_j^u\subseteq J\}} (C_d^u - R)\rho^u = \\ &\sum_{j\in J} f_j - R\sum_{j\in J} Q_j + \sum_{j\in I\setminus J} Q_j \sum_{u=\min\{\{1,2,\dots,m\}\mid \emptyset\neq S_j^u\subseteq J\}} (C_d^u - R)\rho^u. \end{aligned} \tag{11}$$

Table 1. Solutions and resulted value of objective function (7a).

Solution	Value of objective function
(0,0,0)	0
(1,0,0)	–5.25
(0,1,0)	**−7.7**
(0,0,1)	–7.65
(1,1,0)	–6.725
(0,1,1)	–7.425
(1,0,1)	–6.85
(1,1,1)	–6

Further, according to (9a):

$$\tilde{Cost}^J = \sum_{j\in J} f_j + \sum_{j\in J} Q_j\gamma_0 + \sum_{j\in I\setminus J} Q_j \sum_{u=\min\{\{1,2,\dots,m\}|\emptyset\neq S_j^u\subseteq J\}} \gamma_u =$$
$$\sum_{j\in J} f_j + 0 + \sum_{j\in I\setminus J} Q_j \sum_{u=\min\{\{1,2,\dots,m\}|\emptyset\neq S_j^u\subseteq J\}} \Big((C_d^u - R)\rho^u + R\Big) =$$
$$\sum_{j\in J} f_j + \sum_{j\in I\setminus J} Q_j \sum_{u=\min\{\{1,2,\dots,m\}|\emptyset\neq S_j^u\subseteq J\}} (C_d^u - R)\rho^u + R\sum_{j\in I\setminus J} Q_j. \qquad (12)$$

Condition (10) follows from (11) and (12). □

As problems (7) and (9) are equivalent, the optimal solution to both problems is the same, and thus we can use the following three-phases algorithm to solve Problem (7). To implement Phase II, we use the mixed integer linear programming solver of MATLAB, *intlinprog* (Mathworks.com 2017).

Algorithm for Solving Problem (7)

Phase I: Express Problem (7) as Problem (9).
Phase II: Solve Problem (9) by any known solution technique to UFLP.
Phase III: Determine the final objective function value of (7a) by setting each $C_{i,j}$ to $C_{i,j} - RQ_i$.

4. Numerical examples

In this section we demonstrate the applicability of our three-phases solution algorithm, and the insights that can be derived from the solution. In the first example we consider the simple network of Example 1, for demonstration purposes. In the second example we show a more realistic scenario, and apply the algorithm on the network of Toronto, Canada.

Example 2 The Insights Derived From the Solution:

Consider the path graph of Example 1. Recall that $S_1 = (\{1\}, \{2\}, \{3\})$, $S_2 = (\{2\}, \{1, 3\})$, and $S_3 = (\{3\}, \{2\}, \{1\})$.

Assume that $Q = (1, 2, 3)$, $R = 2$, $f_i = 2\ \forall\ i \in I$, $\rho_1 = 0.95$, $\rho_2 = 0.8$, $C_d = (0, 0.5, 1)$. Then, $\gamma = (0, 0.575, 1.2)$, and $C_{1,1} = C_{2,2} = C_{3,3} = 0$, $C_{1,2} = 0.575$, $C_{1,3} = 1.2$, $C_{2,1} = C_{2,3} = 1.15$, $C_{3,1} = 3.6$, $C_{3,2} = 1.725$.

The three-phases algorithm suggests to open a single facility at node 2, with a value of −7.7 of the original problem. According to the optimal solution, the facility at node 2 serves $Q_2 + (Q_1 + Q_3)\rho_1 = 4.85$ daily orders. As $Q = 6$, this implies that 19.17% of the orders are lost.

Table 1 presents all feasible solutions along with the resulted values of the original objective function (7a). It is clear that the output of our algorithm (highlighted in the table) is indeed the optimal solution.

Note that even though in Problem (7) we assume that the parcel locker facilities have an unlimited capacity, we can use the solution to obtain a realistic estimate on the required size of the facilities. For example, if all customers use the service for standard-sized parcels, with a weight not larger than 20 lb., and a volume of 18 in. × 14 in. × 8 in., then the size of the locker should be a bit larger than that volume, and, as the facility at node 2 serves 4.85 daily orders, it should have five standard lockers. We note that this estimation assumes that all parcels are picked up in one day, and does not consider any seasonality in the problem.

Figure 1. A parcel locker.

Finally, note that we can easily estimate the robustness of the final solution. For example, analysis of the robustness with respect to the facilities' setup costs reveals that if f_3 is changed into 1.95, then the problem will have two optimal solutions: Opening a single facility at node 2 or opening a single facility at node 3, both with the same objective function value of -7.7. However, if all nodes have the same setup cost, then opening a single facility at node 2 continues to be the optimal solution as long as $f_i > 1.72\ \forall\, i \in I$. When $f_i = 1.72$ the optimal solution is to open two facilities at nodes 2 and 3, with an objective function value of -7.985. This solution continues to be the optimal one as long as $f_i > 0.57\ \forall\, i \in I$. Then, it is optimal to open facilities at all nodes.

Example 3 The Parcel Locker Facilities Location Problem in Toronto:

In this example we utilize the same data that are used in Berman, Krass, and Wang (2006), to represent the Toronto metropolitan area in Canada. Specifically, the area of Toronto is partitioned into 96 FSA's (Forward Sortation Area - the first three digits of the Canadian postal code), and each such FSA is represented as a node in the network. Each node has a population that represents the number of people that resides and works at the corresponding area. There is an edge between two nodes if the corresponding areas share a boundary. That is, Toronto is represented as a network having $n = 96$ nodes, with a total population of 2.5 million people.

We note that this data are accurate for Toronto of 2006. Nowadays, in 2017, there are 102 FSA's in Toronto, and a total population of 2.79 million people.

Since we do not know the actual values of the other parameters in the problem, we next estimate them.

We consider the daily profit out of a planning horizon of a single year.

According to recent statistics (Friend 2015; McKinnon 2015), 76% of Canadians shopped online during 2015, and about a quarter of that percentage were 'frequent' shoppers, which shopped online for four to ten times per year. Assuming a similar ordering behaviour across all regions, the fraction of population that shops online from each node is $0.76 \cdot 0.25 = 0.19$ on average. Thus, we multiply the populations' values of the nodes by 0.19 to get the P_i parameter for $i \in I$. Further, since the ordering frequency is four to ten times per year (i.e. seven times on average), we assume that the daily frequency is $F_i = \frac{7}{365} \simeq 0.019$ for $i \in I$. Thus, $Q_i = (\text{Original population of node } i) \cdot 0.19 \cdot 0.019$ for $i \in I$, and the total number of daily orders is $\sum_{i \in I} Q_i = 9,025$. Note that in estimating the ordering frequency we ignore issues of socioeconomic classes and customers' willingness to travel to pick up a package, and thus the ordering is fixed among all areas.

We assume that $D_1 = 5$ km, $D_2 = 15$ km, and $D_3 = 20$ km. We further assume that $\rho_0 = 1$, $\rho_1 = 0.9$, $\rho_2 = 0.8$, $\rho_3 = 0.7$, and that customers are not willing to travel more than 20 km to a parcel locker.

The parcel lockers service in Toronto is currently being offered by Amazon (Amazon.ca 2017) and the Canada Post (Canada Post Corporation 2017). According to Amazon.com (2017), a standard shipping rate in Canada is \$2.5 per shipment plus \$12 per libra, and there is no additional fee for shipping to parcel lockers. Therefore, we assume that $R = \$5$. We also assume that $C_d^1 = \$1$, $C_d^2 = \$2$, $C_d^3 = \$3$.

In Figure 1 we can view a standard parcel locker. Its price is ranging from \$1000 to \$6000 (Parcel locker 2017), therefore, we assume that the fixed setup cost of a locker is $\$\frac{3500}{365} \simeq \$10$. Further, following TheRedPin (2016), the real-estate prices in Toronto are the highest in the downtown area, the cheapest in Mimico and Islington, and average in the other parts of the city. Therefore, we assume that the fixed and varying setup prices are \$20 in Mimico and Islington, \$60 in the downtown area, and \$40 otherwise.

Table 2. Demonstration of the final solution quality.

The exchanged nodes	Value of objective function	The percentage of lost orders	The objective function worsening percentage
(16, 17)	−$16, 648	5.12	0.92
(35, 54)	−$16, 712	4.8	0.54
(89, 90)	−$16, 501	5.85	1.8

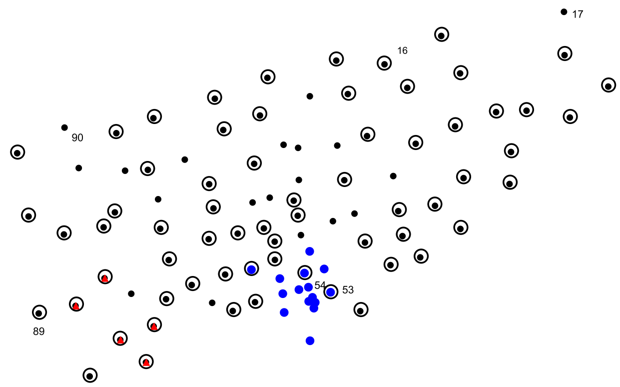
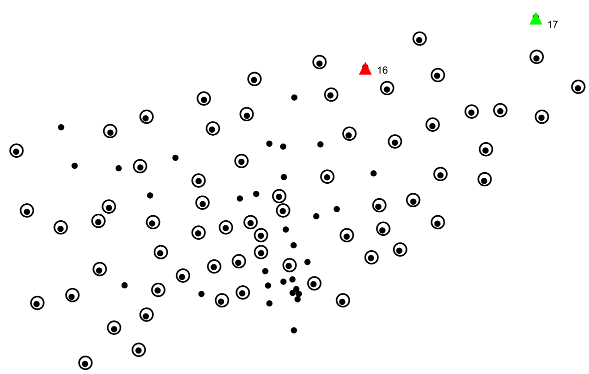
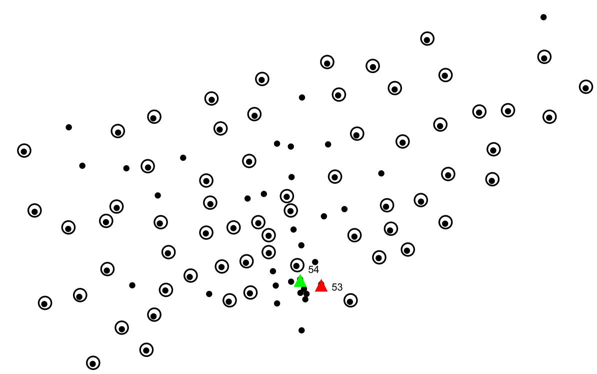


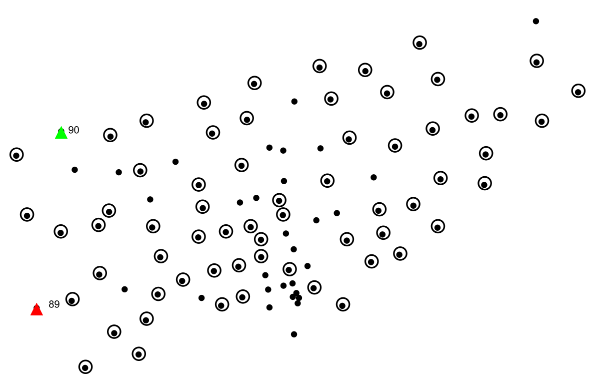
Figure 2. Solution.



(a) Exchanging Nodes 16 and 17.



(b) Exchanging Nodes 53 and 54



(c) Exchanging Nodes 89 and 90.

Figure 3. The Solution and the Exchanged Nodes.

Given this data and assumptions, we used our three phases solution algorithm to solve the Parcel Locker Facilities Location Problem in Toronto. The solution suggests to open 65 facilities. The facilities' locations are presented in Figure 2.

Black dots, circled dots, red dots, and blue dots represent, respectively, demand nodes of the network, the nodes of the solution where a facility is opened, the nodes of Mimico and Islington areas, and the nodes of the downtown area. Given this solution, the value of the objective function of Problem (9a) is \$3420, the value of Problem (7a), is $-\$16,803$, and the relative percentage of lost orders is 4.35%.

Further, the algorithm opens 2 facilities out of the 14 nodes in the downtown area of Toronto, and 5 facilities out of the 5 nodes in Mimico and Islington areas.

As Problem (7) is NP-hard, we do not have a guarantee that the final solution of our algorithm will always be the optimal one. A proof can be obtained through an exhaustive search across all possible solutions, but this would require checking an enormous number of solutions.

To demonstrate the quality of the final solution, we selected in random three pairs of relatively close nodes, where one of them had an open facility and the other had not, and exchanged them. In all three cases, the exchange led to a worse value of the objective function, see Figure 3 and Table 2. In Figure 3, black dots, circled dots, the red triangle, and the green triangle represent, respectively, demand nodes of the network, the nodes of the solution where a facility is opened, the node of the solution which has a facility, and the neighbor node which has not.

For sake of comparison only, according to the Amazon site (Amazon.com 2017), Amazon has opened 40 parcel lockers in 25 FSA locations in Toronto. Further, the Canada Post installs and maintains parcel lockers for free at buildings (with 60 and more units) that request them.

5. Concluding remarks and directions for a future work

In this work we considered the problem of designing a parcel locker network as a solution to the Logistics Last Mile Problem: Choosing the optimal number, location, and sizes of parcel lockers facilities so as to maximize consignment companying' profits.

As the E-commerce market continues to increase, parcel locker networks become promising contributors toward solving the Logistics Last Mile Problem. They can be advantageous to cities, by reducing the city logistics flows, taking advantage of consolidation opportunities; to logistics carriers, by reducing the number of failed deliveries and the number of vehicles and deliverers needed to cover a geographic area; to retailers, by offering convenient delivery locations for their consumers, and for the consumers, by offering flexibility in collection hours, security, and savings as compared with regular home delivery.

We believe that the need in parcel lockers usage will soon become unavoidable, and hope that this research will contribute to companies that seek to offer this service.

The current work has some limitations. The first is our static approach. We basically ignore the dynamic aspects of the problem, e.g. the seasonality of ordering rates, the variability of customers' willingness to use the service, the variability of operational setup prices, etc. The second is that we do not try to determine the optimal mix of different-size lockers (Faugere and Montreuil 2017). We implicitly assume that all parcel lockers have a fixed and given size. The third is that we focus only on solving the problem of delivery to the end-consumer, even though our solution may also be used to solve the reversed problem, of picking up the parcels from the consumers' locations and transferring them to their destination (Cheng, Liao, and Hua 2017).

This opens the door to further research. First, to deal with the dynamic aspect of the problem, we can try to encourage customers to collect their parcels as close as possible to the delivery time using bonuses and/or fines. This is a well-known practice, which is being implemented, e.g. by port authorities. The port authorities specify a certain free-of-charge time period to clear out the cargo by consignees from port premises. Thereafter, certain charges are applied at predefined rates. Another possible route is to couple our optimization model with a simulation model, to dynamically optimize a parcel locker network design over a certain period of time. One may also try to choose the optimal combination of different-size lockers in order to increase further the consignment companying' profits. Finally, the concept of parcel lockers we presented here may also serve the purpose of picking up parcels from consumers and that would open up another interesting line of research that will address the simultaneous flows of parcels to and from the consumers using the same locker sites.

Disclosure statement

No potential conflict of interest was reported by the authors.

Note

1. Mr. Alex Walker is the Chairman of the Board and Chief Financial Officer of Dream Payments Corp. in Toronto, Canada. He served as the Chief Executive Officer of Cube Route Inc. since 2004–2007. Cube Route Inc. provides on-demand logistics service to manage delivery and field service fleets.

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