Taylor's and McLaurin's Series

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TAYLOR'S & MCLAURIN'S SERIES

TAYLOR'S → infinite series about a point a, that progresses in powers of $(x-a)$

→ special case of Taylor's infinite series that progresses in powers of x

Taylor's Series for $y = f(x)$ about $x = a$

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \dots$$

McLaurin's Series

Put $x = a = 0$.

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$$

EXAMPLE:

Find McLaurin's series expansion for $f(x) = e^x$

Soln:

$f(x) = e^x$ $\quad f(0) = 1$

$f'(x) = e^x$ $\quad f'(0) = 1$

$f''(x) = e^x$ $\quad f''(0) = 1$

Thus we have

$$e^x = 1 + \frac{x}{1!}(1) + \frac{x^2}{2!}(1) + \dots$$

Find McLaurins series expansion for $f(x) = \tan^{-1}(x)$

Soln: $f(x) = \tan^{-1}(x)$ $\quad f(0) = 0$

Soln: $f(x) = \tan^{-1}(x)$

$f'(x) = \frac{1}{1+x^2}$

$f''(x) = \frac{-2x}{(1+x^2)^2}$

$f(0) = 0$

$f'(0) = 1$

$f''(0) = 0$