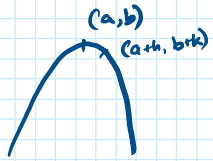
6. Maxima & Minima

19 September 2023 08:50

MAXIMA AND MINIMA

• Maxima

For some $F(x,y)$,



Considering value of $F(x,y)$ at (a,b) here,
(a,b) is a maxima of $F(x,y)$ when

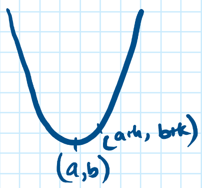
$$F(a+h,\ b+k) - F(a,b) < 0$$

$$\Delta F < 0$$

Irrespective of signs of h and k.

Here $F(a,b)$ gives the maximum value of the function.

• Minima



Again taking (a,b) here,
(a,b) is a minima of $F(x,y)$ when

$$F(a+h,\ b+k) - F(a,b) > 0$$

$$\Delta F > 0$$

Irrespective of signs of h and k.

Here, $F(a,b)$ gives the minimum value of the function.

• Saddle points

The surface ascends in all directions about the maximum point and descends in all directions about the minimum point.
However, there are some points on the surface where the surface ascends in one direction and descends in the other direction.
Such points are called saddle points.

CONDITIONS

Using Taylor's Series expansion of $F(x,y)$ about (a,b), where $x = a+h$ and $y = b+k$

$$F(a+h,\ b+k) = F(a,b) + \frac{1}{1!}\left[h F_x + k F_y\right] + \frac{1}{2!}\left[h^2 F_{xx} + 2hk F_{xy} + k^2 F_{yy}\right]$$

Neglecting higher powers of h and k, as h and k are very small:

$$F(a+h,\ b+k) - F(a,b) = hF_x + kF_y$$

$$\Delta F = hF_x + kF_y$$

$$\Delta F = hF_x + kF_y$$

$$\boxed{\begin{array}{l}\text{Condition for } (a,b) \text{ to be maxima/minima:} \\ \left(\dfrac{\partial F}{\partial x}\right)_{(a,b)} = 0 \quad , \quad \left(\dfrac{\partial F}{\partial y}\right)_{(a,b)} = 0\end{array}}$$

Considering second order term,

$$\Delta F = \frac{1}{2}\left(h^2 F_{xx} + 2hk F_{xy} + k^2 F_{yy}\right)$$

Let $F_{xx} = r$, $F_{xy} = s$, $F_{yy} = t$

$$\Delta F = \frac{1}{2}\left(h^2 r + 2hks + k^2 t\right)$$

HW

why?

$rt - s^2 > 0$ and $r < 0$ $\longrightarrow$ (a,b) is a maximum point

$rt - s^2 > 0$ and $r > 0$ $\longrightarrow$ (a,b) is a minimum point

$rt - s^2 = 0$ $\longrightarrow$ (a,b) is a saddle point

$rt - s^2 < 0$ $\longrightarrow$ Insufficient information to draw any conclusion; further investigation required.