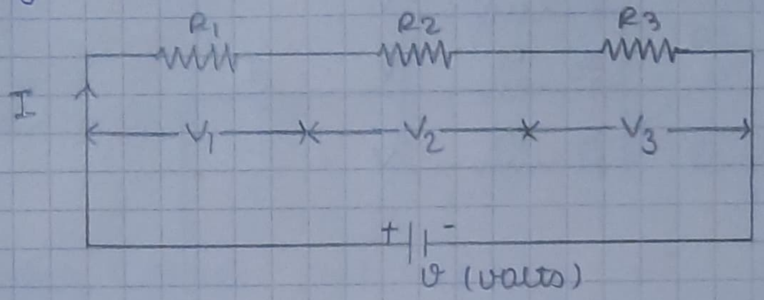
Series Equivalent Resistance

~> let us consider a circuit with three resistare of resistance (R_1, R_2, R_3) and a voltage source (V) which are connected in series.



· As we know that voltage gets divided in series connection & current stays the same.

$$\therefore V = V_1 + V_2 + V_3 \quad \text{——— (1)}$$

According to Ohm's law, ($V = IR$)

$$V_1 = IR_1 \ , \ V_2 = IR_2 \ , \ V_3 = IR_3$$

Now ~~the~~ put the value of V_1, V_2, V_3 in eq (1)

$$\therefore V = IR_1 + IR_2 + IR_3$$

$$\therefore V = I(R_1 + R_2 + R_3)$$

Also, we know that $V_{eq} = IR_{eq}$.

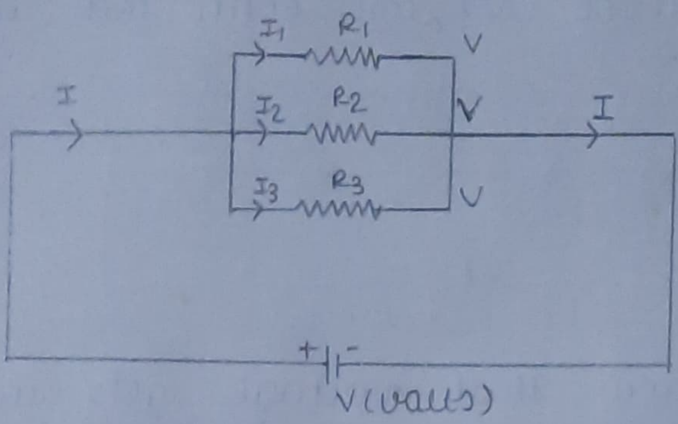
$$\therefore IR_{eq} = I(R_1 + R_2 + R_3)$$

$$\therefore \boxed{R_{eq} = R_1 + R_2 + R_3}$$

* Here individual resistance is always less than the equivalent resistance

Parallel Equivalent Resistance.

→ let us consider a circuit with three resistor of resistance (R_1, R_2, R_3) and a voltage source (V) which are connected in parallel.



• As we know that current gets divided in parallel connection and voltage stays the same.

$$\therefore I = I_1 + I_2 + I_3 \quad \text{——} \quad (1)$$

According to Ohm's law, $(V = IR) \Rightarrow \left(I = \frac{V}{R}\right)$

$$\therefore I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

Now put the value of I_1, I_2, I_3 in eq (1).

$$\therefore I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\therefore I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

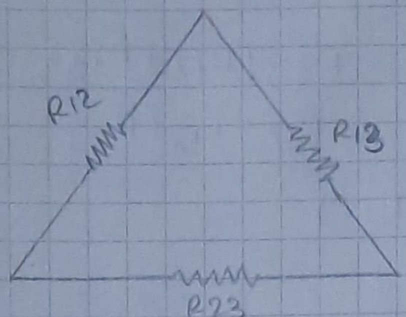
Also, we know that $I_{eq} = \frac{V}{R_{eq}}$

$$\therefore \frac{V}{R_{eq}} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

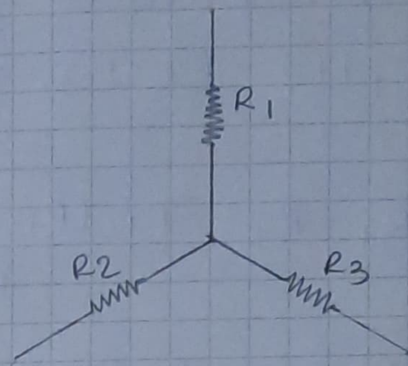
$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

* Here, the individual resistance will be more than the equivalent resistance.

#(3) Delta to Star



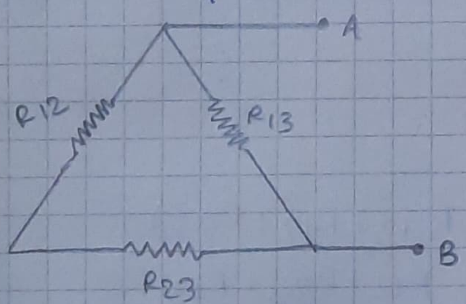
given delta



Equivalent Star

→ Consider three resistances R_{12}, R_{23}, R_{13} connected in delta.

Now equivalent resistance b/w A and B



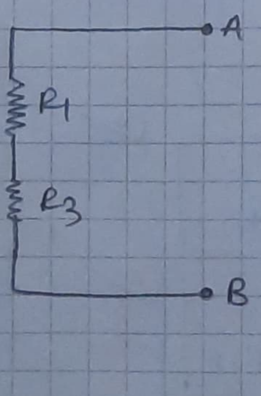
Here, R_{13} is parallel with $(R_{12}+R_{23})$

$$R_{13} = \frac{R_{13}(R_{12}+R_{23})}{R_{13}+R_{12}+R_{23}} \quad \text{— (1)}$$

Similarly,

$$R_{12} = \frac{R_{12}(R_{13}+R_{23})}{R_{12}+R_{13}+R_{23}} \text{ — (2)} \quad \& \quad R_{23} = \frac{R_{23}(R_{12}+R_{13})}{R_{23}+R_{12}+R_{13}} \text{ — (3)}$$

Now consider same two terminals of equivalent star - connection



Between A and B resistance $= R_1 + R_3$ — (4)

Now Comparing eq (1) & (4)

$$R_1 + R_3 = \frac{R_{13}(R_{12}+R_{23})}{R_{13}+R_{12}+R_{23}} \quad \text{— (5)}$$

$$R_1 + R_2 = \frac{R_{12}(R_{13}+R_{23})}{R_{12}+R_{13}+R_{23}} \quad \text{— (6)}$$

$$R_2 + R_3 = \frac{R_{23}(R_{12}+R_{13})}{R_{23}+R_{12}+R_{13}} \quad \text{— (7)}$$

Now, subtract eq ⑦ from eq ⑥

$$\therefore R_1 + \cancel{R_2} - \cancel{R_2} - R_3 = \frac{R_{12}(R_{13}+R_{23})}{R_{12}+R_{13}+R_{23}} - \frac{R_{23}(R_{12}+R_{13})}{R_{12}+R_{13}+R_{23}}$$

$$\therefore R_1 - R_3 = \frac{R_{12}(R_{13}+R_{23}) - R_{23}(R_{12}+R_{13})}{R_{12}+R_{23}+R_{13}}$$

$$\therefore R_1 - R_3 = \frac{R_{12}R_{13} + \cancel{R_{12}R_{23}} - \cancel{R_{23}R_{12}} - R_{23}R_{13}}{R_{12}+R_{23}+R_{13}}$$

$$\therefore R_1 - R_3 = \frac{R_{12}R_{13} - R_{23}R_{13}}{R_{12}+R_{23}+R_{13}}$$

$$\therefore R_1 - R_3 = \frac{R_{13}(R_{12}-R_{23})}{R_{12}+R_{23}+R_{13}} \quad — ⑧$$

Now add eq ⑧ and eq ⑤

$$\therefore R_1 - \cancel{R_3} + R_1 + \cancel{R_3} = \frac{R_{13}(R_{12}-R_{23})}{R_{12}+R_{23}+R_{13}} + \frac{R_{13}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{13}}$$

$$\therefore 2R_1 = \frac{R_{13}(R_{12}-R_{23}) + R_{13}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{13}}$$

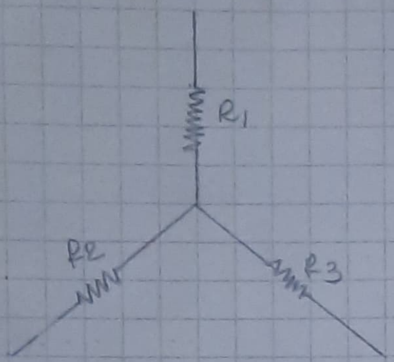
$$\therefore 2R_1 = \frac{R_{13}R_{12} - \cancel{R_{13}R_{23}} + R_{13}R_{12} + \cancel{R_{13}R_{23}}}{R_{12}+R_{23}+R_{13}}$$

$$\therefore \cancel{2}R_1 = \frac{\cancel{2}R_{13}R_{12}}{R_{12}+R_{23}+R_{13}} \Rightarrow \boxed{R_1 = \frac{R_{12}R_{13}}{R_{12}+R_{23}+R_{13}}}$$

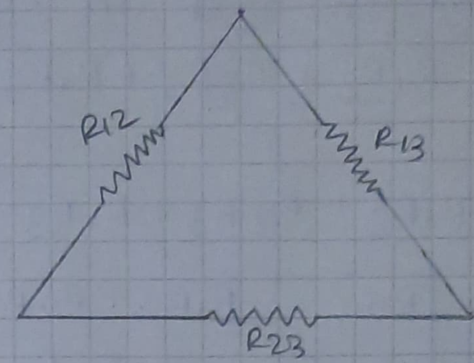
$$\boxed{R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{13}}}$$

$$\boxed{R_3 = \frac{R_{13}R_{23}}{R_{12}+R_{23}+R_{13}}}$$

Star to Delta



given star



equivalent delta

→ Consider three resistances R_1, R_2, R_3 connected in star.

we know that

$$R_1 = \frac{R_{12} \cdot R_{13}}{R_{12}+R_{23}+R_{13}} \quad \text{—— (1)} \qquad R_2 = \frac{R_{12} \cdot R_{23}}{R_{12}+R_{23}+R_{13}} \quad \text{—— (2)}$$

$$R_3 = \frac{R_{13} \cdot R_{23}}{R_{12}+R_{23}+R_{13}} \quad \text{—— (3)}$$

Multiply eq (1) & (2)

$$R_1 \cdot R_2 = \frac{(R_{12} \cdot R_{13})(R_{12} \cdot R_{23})}{(R_{12}+R_{23}+R_{13})^2} = \frac{R_{12}^2 \cdot R_{13} \cdot R_{23}}{(R_{12}+R_{23}+R_{13})^2} \quad \text{—— (4)}$$

Similarly,

$$R_2 \cdot R_3 = \frac{R_{23}^2 \cdot R_{12} \cdot R_{13}}{(R_{12}+R_{23}+R_{13})^2} \quad \text{— (5)} \qquad R_3 \cdot R_1 = \frac{R_{13}^2 \cdot R_{12} \cdot R_{23}}{(R_{12}+R_{23}+R_{13})^2} \quad \text{— (6)}$$

Add eq (4), (5) & (6).

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2 R_{13} R_{23} + R_{23}^2 R_{12} R_{13} + R_{13}^2 R_{12} R_{23}}{(R_{12}+R_{23}+R_{13})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{13} R_{23} \cancel{(R_{12}+R_{23}+R_{13})}}{(R_{12}+R_{23}+R_{13})\cancel{(R_{12}+R_{23}+R_{13})}}$$

$$\therefore R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{13} R_{23}}{R_{12}+R_{23}+R_{13}} \quad \text{—— (7)}$$

Substitute eq (I) in eq (7)

$$\therefore R_1R_2 + R_2R_3 + R_1R_3 = \frac{R_{12}R_{13}(R_{23})}{R_{12}+R_{23}+R_{13}}$$

$$\therefore R_1R_2 + R_2R_3 + R_1R_3 = R_1 \cdot R_{23}$$

$$\therefore \boxed{R_{23} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}}$$

$$\therefore \boxed{R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}}$$

$$\therefore \boxed{R_{13} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}}$$

Temperature Co-efficient of Resistance.

~ let R_0 be the resistance at 0°C and R_T be the resistance at T°C then change in resistance (ΔR) is directly proportional to,

i) The orginal resistance.

$$\Delta R \propto R_0 \quad — (1)$$

ii) The change in temperature

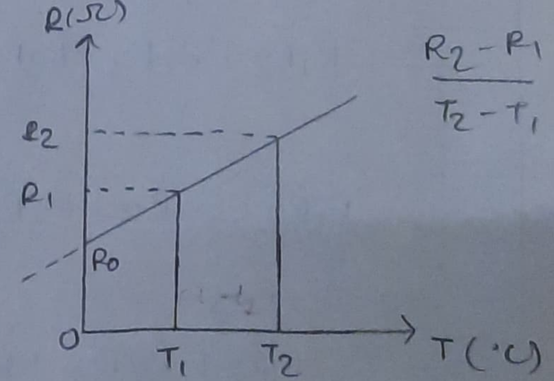
$$\Delta R \propto \Delta T \quad — (2)$$

Combining ~~Comparing~~ eq (1) & (2).

$$\Delta R \propto R_0 \Delta T$$

$$\therefore \Delta R = \alpha . R_0 \Delta T$$

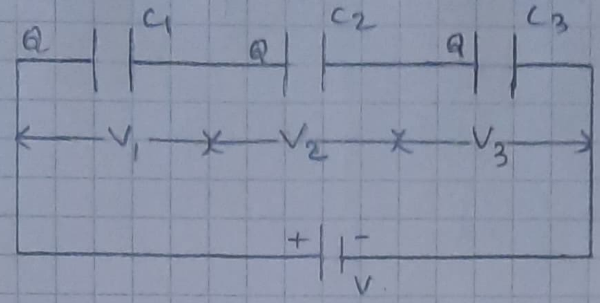
$$\therefore \boxed{\alpha = \frac{\Delta R}{R_0 \Delta T}} \; °C^{-1} / K^{-1}.$$



* $\frac{\Delta R}{\Delta T}$ slope of the graph

Series Equivalent Capacitance

~> let us consider a circuit with three capacitor with capacitance C_1, C_2, C_3 and a voltage source (V) connected in series.



~> As we know that current stays same in series connection so the charge on each capacitor is also same but potential difference is different across each capacitor

$$V = V_1 + V_2 + V_3 \quad \text{—— (1)}$$

But we know that $(Q = CV) \Rightarrow \left(V = \frac{Q}{C}\right)$

$$\therefore V_1 = \frac{Q}{C_1}, \quad V_2 = \frac{Q}{C_2}, \quad V_3 = \frac{Q}{C_3}$$

Put the value of V_1, V_2 and V_3 in eq (1)

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\therefore V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

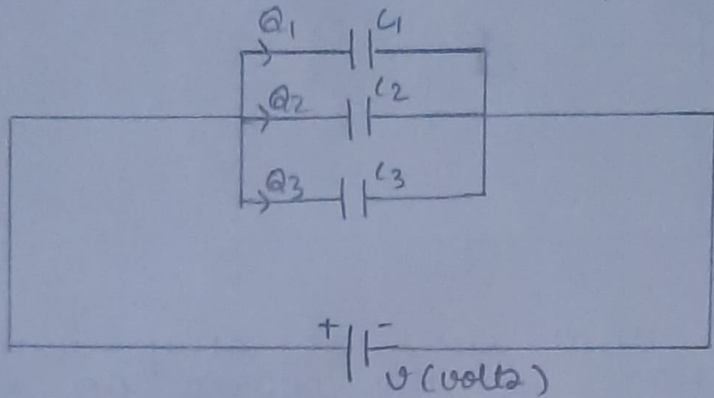
Also, we know that $V_{eq} = \frac{Q}{C_{eq}}$

$$\therefore \frac{\cancel{Q}}{C_{eq}} = \cancel{Q}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\therefore \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Parallel Equivalent Capacitance

~> let us consider a circuit with three capacitor with capacitance of C_1, C_2, C_3 and a voltage source (V) are connected in parallel.



~> As we know that in parallel connection potential difference stays the same but current is different so charge across all the capacitor is different

$$Q = Q_1 + Q_2 + Q_3 \quad — (1)$$

But we know that, $(Q = CV)$

$$\therefore Q_1 = C_1V, \quad Q_2 = C_2V, \quad Q_3 = C_3V$$

Put the value of Q_1, Q_2, Q_3 in eq (1).

$$\therefore Q = C_1V + C_2V + C_3V.$$

$$\therefore Q = V(C_1 + C_2 + C_3)$$

Also, we know that $Q_{eq} = C_{eq} \cdot V$.

$$C_{eq}\cancel{V} = \cancel{V}(C_1 + C_2 + C_3)$$

$$\therefore \boxed{C_{eq} = C_1 + C_2 + C_3}$$

Uniform Dielectric Medium

⇝ Consider a capacitor having two parallel plate of area $A\ m^2$ separated by a dielectric ~~med~~ of thickness d meters with relative ~~permittivity~~ permittivity ε_r.

$\therefore$ Electric flux density, $D = \dfrac{Q}{A}$ —— (1)

Electric field intensity, $E = \dfrac{V}{d}$ —— (2)

Here, Electric flux density can also be defined as,

$$D = \varepsilon E$$

$$\therefore \frac{Q}{A} = \varepsilon \frac{V}{d} \quad \text{(From eq (1) \& (2))}$$

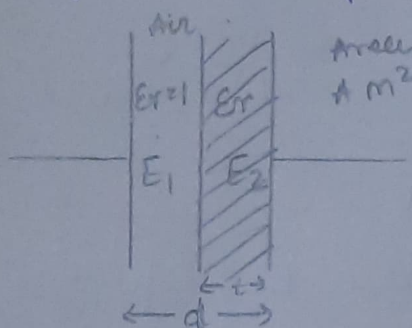
$$\therefore \frac{Q}{V} = \varepsilon \frac{A}{d}$$

$$\therefore C = \varepsilon \frac{A}{d} \quad \left(\because \frac{Q}{V} = C\right) \ \& \ (\varepsilon = \varepsilon_0 \varepsilon_r)$$

$$\therefore \boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}} \ \text{'F' (Farad)}$$

Medium Partly Air.

~ Here, the medium consists partly air and partly dielectric slab of thickness t with relative permittivity ε_r.



We know that, $D = \varepsilon E$

$$\therefore E_1 = \frac{D}{\varepsilon_0} \text{ for air}$$

$$\therefore E_2 = \frac{D}{\varepsilon_0 \varepsilon_r} \text{ for medium.}$$

~ Potential difference between plates is,

$$V = V_1 + V_2$$

$$\therefore V = E_1 (d-t) + E_2 t \qquad \left(\because E = \frac{V}{d} \Rightarrow V = Ed\right)$$

$$\therefore V = \frac{D}{\varepsilon_0}(d-t) + \frac{D}{\varepsilon_0 \varepsilon_r}(t)$$

$$\therefore V = \frac{D}{\varepsilon_0}\left[(d-t) + \frac{t}{\varepsilon_r}\right]$$

$\therefore$ But $D = \frac{Q}{A}$

$$\therefore V = \frac{Q}{A\varepsilon_0}\left[d - \left(t - \frac{t}{\varepsilon_r}\right)\right]$$

$$\therefore A\varepsilon_0 = \frac{Q}{V}\left[d - \left(t - \frac{t}{\varepsilon_r}\right)\right]$$

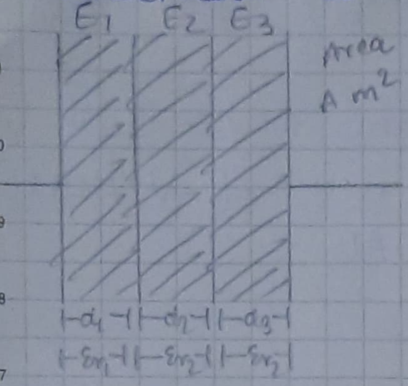
But $C = \frac{Q}{V}$

$$\therefore A\varepsilon_0 = C\left[d - \left(t - \frac{t}{\varepsilon_r}\right)\right]$$

$$\therefore \boxed{C = \frac{\varepsilon_0 A}{\left[d - \left(t - \frac{t}{\varepsilon_r}\right)\right]} \ F}$$

Composite Medium

→ Here, there are three composite medium with thickness d and relative permittivity ε_r.



we know that, $D = \varepsilon E$

$$\therefore E_1 = \frac{D}{\varepsilon_0 \varepsilon_{r_1}}$$

$$\therefore E_2 = \frac{D}{\varepsilon_0 \varepsilon_{r_2}}$$

$$\therefore E_3 = \frac{D}{\varepsilon_0 \varepsilon_{r_3}}$$

→ Potential difference b/w plates is,

$$V = V_1 + V_2 + V_3$$

But $E = \frac{V}{d} \Rightarrow V = E.d.$

$$\therefore V = E_1 d_1 + E_2 d_2 + E_3 d_3$$

$$\therefore V = \frac{D}{\varepsilon_0 \varepsilon_{r_1}} \cdot d_1 + \frac{D}{\varepsilon_0 \varepsilon_{r_2}} \cdot d_2 + \frac{D}{\varepsilon_0 \varepsilon_{r_3}} \cdot d_3$$

$$\therefore V = \frac{D}{\varepsilon_0} \left[\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}} \right]$$

Also, $D = \frac{Q}{A}$

$$\therefore V = \frac{Q}{A \varepsilon_0} \left[\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}} \right]$$

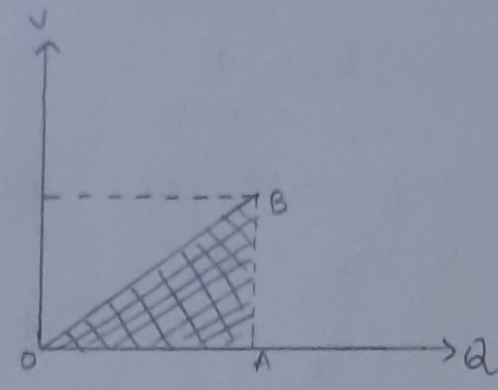
$$\therefore \varepsilon_0 A = \frac{Q}{V} \left[\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}} \right]$$

But, $C = \frac{Q}{V}$

$$\therefore \boxed{C = \frac{\varepsilon_0 A}{\left[\frac{d_1}{\varepsilon_{r_1}} + \frac{d_2}{\varepsilon_{r_2}} + \frac{d_3}{\varepsilon_{r_3}} \right]}}$$

Energy Stored in Capacitor.

- When an uncharged capacitor is connected to D.C. supply, the voltage across it rises gradually & it starts storing energy.
- The energy required to charge the capacitor with Q coulombs at a potential difference of V volts is given by the area of triangle OAB.



∴ Energy stored in a capacitor = Area of $\triangle$OAB

$$\therefore \quad E = \frac{1}{2} \times OA \times AB$$

$$\therefore \quad E = \frac{1}{2} \times Q \times V$$

$$\therefore \quad E = \frac{1}{2} \times (CV) \times V \qquad [\because Q = CV]$$

$$\therefore \quad \boxed{E = \frac{1}{2} \times C \times V^2} \text{ Joule (J)}$$

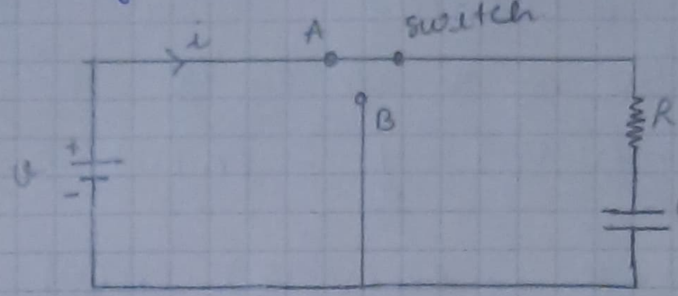
OR

$$\therefore \quad E = \frac{1}{2} \times C \times \frac{Q^2}{C^2}$$

$$\therefore \quad E = \frac{1}{2} \times \frac{Q^2}{C} \text{ Joule}$$

Charging of a Capacitor

→ Consider a circuit consisting of a resistor, and a capacitor connected in series with a switch A and battery of V volts. Current is flowing through the circuit through switch A.



→ At any instant during charging,

V_C = Potential difference across capacitor.

i = charging current

$$i = \frac{dq}{dt} = \frac{d(C \cdot V_C)}{dt} \quad (\because Q = C \cdot V_C)$$

∴ According to KVL,

$$V = V_C + V_R \quad \text{—— (1)}$$

$$\therefore V = V_C + iR$$

$$\therefore V = V_C + R \cdot \frac{d(C \cdot V_C)}{dt}$$

$$\therefore V = V_C + RC \cdot \frac{dV_C}{dt}$$

$$\therefore \frac{dV_C}{V - V_C} = \frac{dt}{RC} \quad \text{—— (2)}$$

integrating both side

$$\int \frac{dV_C}{V - V_C} = \int \frac{dt}{RC} + K \quad ; \; K = \text{constant of integration}$$

$$\therefore \frac{\ln(V - V_C)}{-1} = \frac{t}{RC} + K$$

$$\therefore -\ln(V - V_C) = \frac{t}{RC} + K \quad \text{—— (3)}$$

At the instant of closing the switch.

$$t = 0, \quad V_C = 0$$

$$\therefore -\ln(V) = 0 + K$$

$$\therefore K = -\ln(V)$$

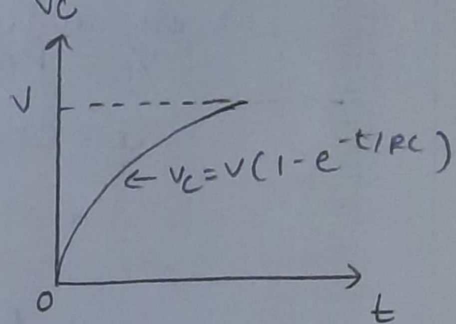
put the value of K in eq ③

$$-\ln(V - V_C) = \frac{t}{RC} - \ln(V)$$

$$\therefore \frac{-t}{RC} = \ln(V - V_C) - \ln(V)$$

$$\therefore e^{-t/RC} = \frac{V - V_C}{V}$$

$$\therefore \boxed{V_C = V(1 - e^{-t/RC})}$$



similarly at any instant

$$V_C = \frac{q}{C}$$

$$\therefore \frac{q}{C} = \frac{Q}{C}(1 - e^{-t/RC})$$

$$\therefore \text{Now, } V - V_C = \cancel{V_R}\, iR$$

$$\therefore i = \frac{V - V_C}{R}$$

$$\therefore \cancel{i = \frac{V - Ve^{-t/RC}}{R}} \quad i = \frac{Ve^{-t/RC}}{R}$$

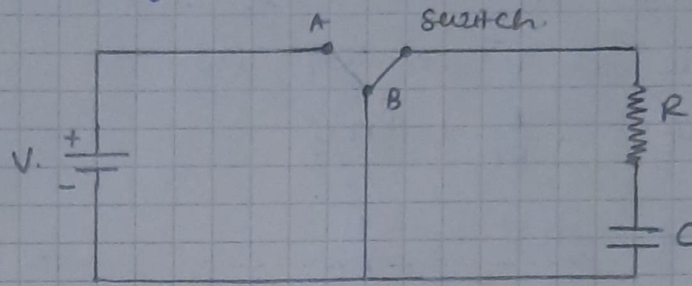
$$\therefore \cancel{i = \frac{V}{R}} \quad i = \frac{V}{R} e^{-t/RC}$$

[∵ initial charging current $= \frac{V}{R}$ (I_0)]

$$\therefore \boxed{i = I_0 e^{-t/RC}}$$

Discharging of a Capacitor

~> Consider a circuit consisting of a resistor, and a capacitor connected in series with a switch B and battery of V volts. Current is flowing through the circuit through switch B.



~> At any instant during discharging,

V_C = PD across capacitor

i = discharging current

$$\therefore i = \frac{dq}{dt} = \frac{d(C \cdot V_C)}{dt} \qquad [\because Q = C \cdot V_C]$$

According to KVL,

$$0 = V_C + V_R \quad \text{--- (1)}$$

$$\therefore 0 = V_C + iR$$

$$\therefore 0 = V_C + R \cdot \frac{d(C \cdot V_C)}{dt}$$

$$\therefore 0 = V_C + RC \cdot \frac{dV_C}{dt}$$

$$\therefore \frac{dV_C}{V_C} = -\frac{dt}{RC} \quad \text{--- (2)}$$

integrating both side

$$\int \frac{dV_C}{V_C} = -\int \frac{dt}{RC} + K$$

$$\therefore \ln V_C = -\frac{t}{RC} + K \quad \text{--- (3)}$$

→ At the instant of closing the switch,

$t = 0 \quad V_c = V$

$\ln(V) = 0 + K$

$\therefore K = \ln V.$

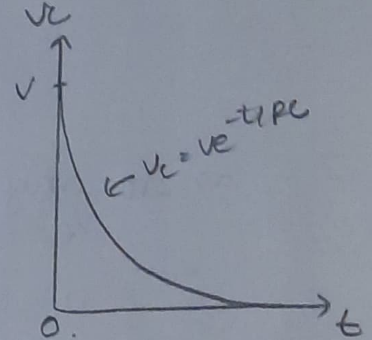
Put the value of K in eq ③.

$\ln V_c = -\frac{t}{RC} + \ln V.$

$\therefore \ln V_c - \ln V = -\frac{t}{RC}$

$\therefore \frac{V_c}{V} = e^{-t/RC}$

$\therefore \boxed{V_c = Ve^{-t/RC}}$



Similarly at any instant,

$V_c = \frac{q}{C}$

$\therefore \frac{q}{C} = \frac{Q}{C} e^{-t/RC}$

Now, $i = -\frac{V_R}{R}$

$\therefore V_R = V_C$

$\therefore i = -\frac{V_C}{R}$

$\therefore i = -\frac{V}{R} e^{-t/RC}$

$\therefore \boxed{i = -I_0 e^{-t/RC}}$

[$\because -I_0$ = initial discharging current]

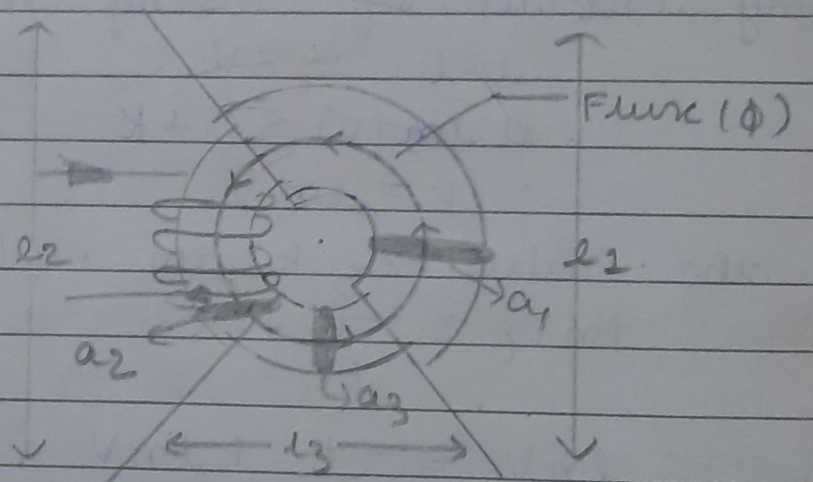
$I_0 = -\frac{V_R}{R}$

Q.1 Explain series and parallel magnetic circuit.

→ Series Magnetic Circuit.

A magnetic circuit which has only one path for the flux is known as series magnetic circuit.

- Let coil wound on ring has N turns carrying a current of I amperes.



The total reluctance of the magnetic circuit,

$$S_T = S_1 + S_2 + S_3$$
$$= \frac{l_1}{\mu_1 a_1} + \frac{l_2}{\mu_2 a_2} + \frac{l_3}{\mu_3 a_3}$$

Here, MMF is different same as potential difference in series electric circuit

$$\phi_T = \frac{(MMF)_T}{S_T} = \frac{(NI)_T}{S_T} = \frac{NI}{S_1+S_2+S_3}$$

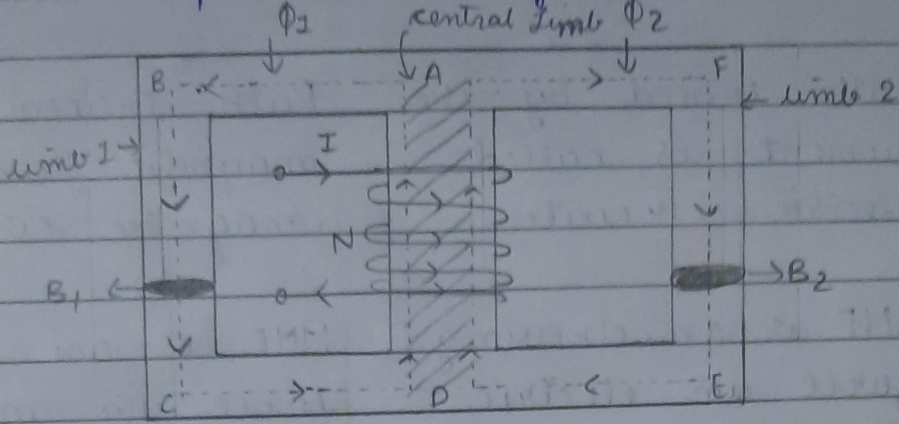
$$\therefore NI = \phi_T (S_1+S_2+S_3)$$

$$\therefore NI = S_1\phi + S_2\phi + S_3\phi$$

$$\therefore \boxed{(MMF)_T = (MMF)_1 + (MMF)_2 + (MMF)_3}$$

→ Parallel Magnetic Circuit

A magnetic circuit which has more than one path for the flux is known as parallel magnetic circuit.



$$\text{Flux} = \frac{\text{MMF}}{\text{reluctance}}$$

$$\therefore \text{MMF} = \phi \times S$$

For path ABCDA, $NI = \phi_1 S_1 + \phi S_C$

For path AFEDA, $NI = \phi_2 S_2 + \phi S_C$

Here, $S_1 = \frac{l_1}{\mu a_1}$, $S_2 = \frac{l_2}{\mu a_2}$, $S_C = \frac{l_C}{\mu a_C}$

$a_1 = a_2 = a_C$ = area of cross-section

$\therefore$ Total MMF = MMF required by central limb + MMF required by any one of outer limbs

$$\therefore \boxed{(NI)_T = (NI)_{AD} + (NI)_{ABCD} / (NI)_{AFED}}$$

Also, $\phi_1 S_1 = \phi_2 S_2$

Efficiency.

i) Half - wave Recitifier

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\%$$

we know that,

$$P_{dc} = I^2_{dc}(R_L) \quad \& \quad P_{ac} = I^2_{rms}(R_L + r_f)$$

$$\therefore \eta = \frac{I^2_{dc}(R_L)}{I^2_{rms}(R_L + r_f)} \times 100\%$$

Also, $I_{dc} = \frac{I_m}{\pi}$ $\quad I_{rms} = \frac{I_m}{2}$

$$\therefore \eta = \frac{\left(\frac{I_m}{\pi}\right)^2 (R_L)}{\left(\frac{I_m}{2}\right)^2 (R_L + r_f)} \times 100\%$$

$$\therefore \eta = \frac{4 \cancel{I_m^2} (R_L)}{\pi^2 \cancel{I_m^2} (R_L + r_f)} \times 100\%$$

$$\therefore \eta = \frac{4}{9.85} \times \frac{R_L}{R_L + r_f} \times 100\%$$

$$\therefore \eta = \frac{0.406 R_L}{R_L + r_f} \times 100\%$$

$$\therefore \eta = \frac{0.406}{1 + \frac{r_f}{R_L}} \times 100\%$$

★ ~~Here~~, ~~we~~ Here the efficiency will be maximum if r_f is negligible as compared to R_L.

$$\therefore \boxed{\eta = 40.6\%}$$

ii) Full - wave Rectifier

$$\eta = \frac{P_{dc}}{P_{ac}} \times 100\%$$

We know that,

$P_{dc} = I^2_{dc} (R_L)$ $\quad$ $P_{ac} = I^2_{rms} (R_L + r_f)$

$$\therefore \eta = \frac{I^2_{dc} (R_L)}{I^2_{rms} (R_L + r_f)} \times 100\%$$

But $I_{dc} = \frac{2I_m}{\pi}$ & $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$\therefore \eta = \frac{\left(\frac{2I_m}{\pi}\right)^2 R_L}{\left(\frac{I_m}{\sqrt{2}}\right)^2 (R_L + r_f)} \times 100\%$$

$$\therefore \eta = \frac{4I_m^2 \times 2}{\pi^2 I_m^2} \times \frac{(R_L)}{(R_L + r_f)} \times 100\%$$

$$\therefore \eta = \frac{8}{\pi^2} \times \frac{R_L}{R_L + r_f} \times 100\%$$

$$\therefore \eta = \frac{8}{9.85} \times \frac{R_L}{R_L + r_f} \times 100\%$$

$$\therefore \eta = \frac{0.812 R_L}{R_L + r_f} \times 100\%$$

$$\therefore \eta = \frac{0.812}{1 + \frac{r_f}{R_L}} \times 100\%$$

* Here, the efficiency will be max. if we neglect r_f as compared to R_L

$$\therefore \boxed{\eta = 81.2\%}$$

Ripple Factor.

i) Half - wave Rectifier

$$I_{rms} = \sqrt{I^2_{dc} + I^2_{ac}}$$

$$\therefore I_{ac} = \sqrt{I^2_{rms} - I^2_{dc}}$$

Now divide both side by I_{dc}.

$$\therefore \frac{I_{ac}}{I_{dc}} = \frac{1}{I_{dc}}\left(\sqrt{I^2_{rms} - I^2_{dc}}\right)$$

$\left[\because \frac{I_{ac}}{I_{dc}} = \text{Ripple Factor}\right]$

$$\therefore Y = \frac{1}{I_{dc}}\left(\sqrt{I^2_{rms} - I^2_{dc}}\right)$$

$$\therefore Y = \sqrt{\frac{I^2_{rms} - I^2_{dc}}{I^2_{dc}}}$$

$$\therefore Y = \sqrt{\left(\frac{I^2_{rms}}{I^2_{dc}}\right) - 1}$$

But, $I_{rms} = \frac{I_m}{2}$ & $I_{dc} = \frac{I_m}{\pi}$

$$\therefore Y = \sqrt{\left(\frac{\frac{I_m^2}{4}}{\frac{I_m^2}{\pi^2}}\right) - 1}$$

$$\therefore Y = \sqrt{\left(\frac{\pi^2}{4}\right) - 1}$$

$$\therefore Y = \sqrt{2.46 - 1}$$

$$\therefore Y = \sqrt{1.46}$$

$$\therefore \boxed{Y = 1.21}$$

ii) Full-wave Rectifier

$$I^2_{rms} = \sqrt{I^2_{dc} + I_{ac}^2}$$

$$\therefore I^2_{ac} = \sqrt{I^2_{rms} - I^2_{dc}}$$

Divide By I^2_{dc} both side

$$\therefore \frac{I_{ac}}{I_{dc}} = \frac{1}{I_{dc}}\left(\sqrt{I^2_{rms} - I^2_{dc}}\right)$$

$\left[\because \frac{I_{ac}}{I_{dc}} = \text{Ripple Factor}\right]$

$$\therefore \gamma = \frac{1}{I_{dc}}\left(\sqrt{I^2_{rms} - I^2_{dc}}\right)$$

$$\therefore \gamma = \sqrt{\frac{I^2_{rms} - I^2_{dc}}{I^2_{dc}}}$$

$$\therefore \gamma = \sqrt{\frac{I^2_{rms}}{I^2_{dc}} - 1}$$

But $I_{rms} = \frac{I_m}{\sqrt{2}}$ & $I_{dc} = \frac{2I_m}{\pi}$

$$\therefore \gamma = \sqrt{\left(\frac{\frac{I_m^2}{2}}{\frac{4I_m^2}{\pi^2}}\right) - 1}$$

$$\therefore \gamma = \sqrt{\left(\frac{\pi^2}{8}\right) - 1}$$

$$\therefore \gamma = \sqrt{1.23 - 1}$$

$$\therefore \gamma = \sqrt{0.23}$$

$$\therefore \boxed{\gamma = 0.48}$$