Ольга Юрьевна

4 пр.

$y = 3\cos x \qquad y = \sqrt{x}$

$y' = -3\sin x \qquad y = \frac{1}{2\sqrt{x}}$

$y = 6 + x + 3x^2 - \sin x - 2x^{1/3} + x^{-2} - 11\operatorname{ctg} x$

$y' = 1 + 6x - \cos x - \frac{2}{3}x^{-2/3} - 2x^{-3} + \frac{11}{\sin^2 x}$

$\operatorname{ctg} x = -\frac{1}{\sin^2 x}$

$y = \frac{1}{2}x^{2/3}\cos x$

$y' = \frac{1}{2}\left(\frac{2}{3}x\cos x - x^{2/3}\sin x\right)$

$\int \frac{\sin x}{\cos x}dx = \left\{\begin{array}{l}\cos x = t\\ dt = -\sin x\, dx\end{array}\right\} =$

$= -\int \frac{dt}{t} = -\ln t = -\ln(\cos x) + C$

$\int \frac{2x^3 - \sqrt{x^5} + 1}{\sqrt{x}}dx = \int \frac{2x^3}{\sqrt{x}}dx - \int \frac{\sqrt{x^5}}{\sqrt{x}}dx + \int \frac{1}{\sqrt{x}}dx = 2\int x^{\frac{5}{2}}dx -$

$- \int x^2 dx + \int x^{-\frac{1}{2}}dx =$

$= \left| \int x^n dx = \frac{x^{n+1}}{n+1} + C \right| = 2 \cdot \frac{2}{7} x^{\frac{7}{2}} - \frac{x^3}{3} + 2\sqrt{x} + C.$

$$y = \frac{3^x + 5}{\cos x} \Rightarrow y' = \frac{3^x \ln 3 \cos x + \sin x (3^x + 5)}{\cos^2 x}$$

$(a^x)' = a^x \ln a$

$y = \sin(3x-5)$

$y' = \cos(3x-5) \cdot 3$

$y = (2x+1)^5$

$y' = 5(2x+1)^4 \cdot 2 = 10(2x+1)^4$

$y = \sqrt[3]{x^2 + \operatorname{tg} x + 15} \Rightarrow$

$y' = \frac{1}{3}(x^2 + \operatorname{tg} x + 15)^{-\frac{2}{3}} \cdot \left(2x + \frac{1}{\cos^2 x}\right)$

$$\int (9x^2 + 24x^3 + 16x^4) dx = 9\frac{x^3}{3} + 24\frac{x^4}{4} + 16\frac{x^5}{5} + C$$

$$= 3x^3 + 6x^4 + \frac{16}{5}x^5 + C$$

$$\int \frac{1}{5-2x} dx = \left| \begin{array}{l} t = 2x \\ dt = 2dx \end{array} \right| = \frac{1}{2} \int \frac{2dx}{5-2x} =$$

$$= -\frac{1}{2} \frac{t}{dt} = -\frac{1}{2} \ln(5-2x) + C$$

$$\int \frac{\ln x}{x^2}dx = \left\{ \begin{array}{l} u = \ln x \quad du = \frac{dx}{x} \\ \int dv = \frac{1}{x^2}dx \end{array} \right.$$

$$V = \int \frac{1}{x^2}dx = -\frac{1}{x}$$

$$\int u\,dv = uv - \int v\,du$$

$$= -\frac{1}{x}\ln x - \int -\frac{1}{x^2}dx = -\frac{1}{x}\ln x - \frac{1}{x} + C$$

$$xy' = y$$

$$x\frac{dy}{dx} = y$$

$$x\,dy = y\,dx$$

$$\int \frac{dy}{y} = \int \frac{dx}{x} \quad \ln|C|$$

$$\ln y = \ln x + C$$

$$y = e^{\ln x + C}$$

$$y = \ln xC$$

$$xy' = y - xe^{y/x}$$

} однородное

$$x\frac{dy}{dx} = y - xe^{y/x}$$

$$y = tx \qquad t = t(x)$$

$$y' = \dot{t}x + t \qquad t' = \dot{t}x$$

$$x\left(\frac{dx}{dt}x+t\right)=tx-xe^{t}$$

$$\frac{dt}{dx}=-e^{t}x$$

$$\int\frac{dt}{-e^{t}}=\int\frac{dx}{x}\rightarrow e^{-t}=\ln xC$$

$$e^{-y/x}=\ln xC$$

$$y''-2y'+10y=0$$

$$s^2-2s+10=0$$

$D=4-40\rightarrow$ мнимые корни

$=-36$

$$S_1=\frac{2\pm\sqrt{36}}{2}=\boxed{1\pm i}$$

$$e^{\alpha x}(C_1\cos\beta x-C_2\sin\beta x)$$

$$S_{1,2}=\alpha\pm\beta i$$

$$y=e^{x}(C_1\cos 3x-C_2\sin 3x)$$

$y'' - 4y = 8x^3$ } Неоднородное

$y'' - 4y = 0$ $\qquad$ $y = y_{oo} + \tilde{y}$

$s^2 - 4 = 0$

$s = \pm 2$ $\qquad$ $y_{oo} = C_1 e^{s_1 x} + C_2 e^{s_2 x}$

$y_{oo} = e^{2x} C_1 + C_2 e^{-2x}$

По виду правой части $\qquad$ $y_{oo} = C_1 e^{s_1 x} + C_2 x e^{s_2 x}$

$8x^3$ восстановим неодн. решение

$\tilde{y} = Ax^3 + Bx^2 + Cx + D$

$\tilde{y}' = 3Ax^2 + 2Bx + C$

$\tilde{y}'' = 6Ax + 2B$

$6Ax + 2B - 4Ax^3 - 4Bx^2 - 4Cx - 4D = 8x^3$

$-4Ax^3 = 8x^3 \Rightarrow A = -2$

$2B - 4D = 0 \rightarrow 2B = 4D \quad \frac{1}{2}B = D$

$6Ax - 4Cx = 0 \Rightarrow -12x = 4Cx$

$C = -3$

$-4Bx^2 = 0$

$\rightarrow B = 0 \rightarrow D = 0.$

$$\tilde{y} = -2x^3 - 3x$$

$$y = C_1 e^{2x} + C_2 e^{-2x} - 2x^3 - 3x$$

$$(1)\ \begin{cases} x' = -2x + 4y \\ y' = -x + 3y \end{cases} \qquad \begin{aligned} x(0) &= 3 \\ y(0) &= 0 \end{aligned}$$

$$3y - y' = -x \qquad x = y' - 3y$$

$$x' = y'' - 3y'$$

$$y = y(t)$$
$$x = x(t)$$

$$(1):\ -2x + 4y = y'' - 3y'$$

$$y'' - 3y' - 4y = -2x$$

$$y'' - 3y' - 4y = 0$$

$$s^2 - 3s - 4 = 0$$

$$D = 9 - 16 = -7$$

$$s_1 = \frac{3 \pm \sqrt{7}\, i}{2} = \frac{3}{2} \pm \frac{\sqrt{7}}{2} i$$

$$y_{oo} = C_1 \cos$$

$$y'' - 3y' = -2(y' - 3y) + 4y$$

$$y'' - 3y' = -2(y' - 3y) + 4y$$

$$y'' - 3y' = -2y' + 6y + 4y$$

$$\boxed{-y'' + y' + 2y = 0}$$

$$-s^2 + s + 2 = 0$$

$$D = 1 + 8 = 9$$

$$s = \frac{1+3}{2} = 2 \qquad s_2 = -1$$

$$y = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

$$y = C_1 e^{2t} + C_2 e^{-t}$$

$$y' = 2C_1 e^{2t} - C_2 e^{-t}$$

$$x = 2C_1 e^{2t} - C_2 e^{-t} - 3(C_1 e^{2t} + C_2 e^{-t})$$

$$x = 4C_1 e^{-t} + C_2 e^{2t}$$

$$\begin{cases} 4C_1 + C_2 = 3 \\ C_1 + C_2 = 0 \end{cases} \Rightarrow \begin{matrix} C_1 = 1 \\ C_2 = -1 \end{matrix}$$