Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
automatica
Automatica 44 (2008) 1097–1103
www.elsevier.com/locate/automatica

Brief paper

Optimal soft landing control for moon lander ☆

Xing-Long Liu[a], Guang-Ren Duan[a,*], Kok-Lay Teo[b]

[a]*Center for Control Systems and Guidance Technology, Harbin Institute of Technology, Harbin, PR China*
[b]*Department of Mathematics and Statistics Curtin University of Technology, Perth, W.A., Australia*

Received 15 August 2006; received in revised form 11 July 2007; accepted 30 August 2007
Available online 21 February 2008

Abstract

This paper considers the soft landing problem of a moon lander. The main purpose is to design the optimal control law to ensure the soft landing of the lander with the least fuel consumption. It is formulated as a constrained optimal control problem, where specific requirements of this soft landing problem are all incorporated in the problem formulation. Then, by utilizing the specific features of the problem, this optimal soft landing problem is transformed into an equivalent standard optimal control problem subject to continuous state inequality constraints. A computational method is developed, based on the control parameterization in conjunction with a time scaling transform and the constraint transcription method, to design the optimal controller for this new constrained optimal control problem. Numerical results are presented for illustration.

Keywords: Optimal soft landing; Lunar lander; Optimal control; Control parameterization technique; Time scaling transform; Constraints transcription technique

1. Introduction

Lunar landing is a challenging problem. It has attracted significant interests from many scientists and engineers in the past four decades. See, for example, Kian and Ernst (2004), Nagarajan, Seetharama, and Kasturirangan (1996), Klarer (1997), Liu and Duan (2005, 2006) and the relevant references cited therein. A moon lander is launched by a rocket from the earth surface into the earth parking orbit. After entering into the earth parking orbit, the lander will regulate its attitude, communicate with the earth center and compute the optimal location and time to leave the parking orbit and fly to the moon. Then, through the acceleration of the propeller, the lander will leave the earth parking orbit and move into the space between the earth and the moon. During this period of time, the flight depends completely on the moon gravitation. When the lander approaches the moon parking orbit, the propeller of the lander will be again in operation so that the lander will safely enter the moon parking orbit along the tangential direction with the right attitude (Klarer, 1997). After that, the lander will check its state, communicate with the earth center and compute the optimal location and time to leave the moon parking orbit. The lander will land softly on the moon surface through the application of the reverse force of the propeller (Liu & Duan, 2005, 2006).

In this paper, the main purpose is to design the optimal control law to ensure the soft landing of the lander with the least possible fuel consumption. This soft landing problem is formulated as a constrained optimal control problem, where specific requirements of this soft landing are all incorporated in the problem formulation. Then, by taking into consideration of the specific features of the problem, this optimal soft landing control problem is transformed into an equivalent standard optimal control problem involving continuous state inequality constraints. There are some methods available in the literature which can be used to solve this constrained optimal control problem. However, they are, in general, not very efficient. Examples of these methods are Goh and Teo (1988), Teo, Goh, and Wong (1991), Miele (1980), and Sakawa and Shindo (1980) and the references cited therein. Relevant software packages are

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Xiaohua Xia under the direction of Editor Mituhiko Araki.

* Corresponding author. Tel.: +86 45186412204; fax: +86 45186418134.
E-mail addresses: liuxinglonglxl@126.com (X.-L. Liu), g.r.duan@hit.edu.cn (G.-R. Duan), k.l.teo@curtin.edu.au (K.-L. Teo).

0005-1098/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2007.08.021

*MISER*3.2 (Jennings, Teo, & Goh, 1997), *SCOS* (John, 2001, 2004; Kenneth, Frode, & Edvall, 2006), *RIOTS_*95 (Schwartz & Polak, 1996; Schwartz, Polak, & Chen, 1997). In this paper, the constraint transcription method (Teo, Rehbock, & Jennings, 1993) is used to approximate the continuous state inequality constraints as corresponding sequences of inequality constraints in canonical form. In this way, we obtain a sequence of optimal control problems with canonical inequality constraints. We then develop an efficient computational method based on the control parameterization technique (Teo et al., 1991) in conjunction with a time scaling transform (Lee, Teo, & Jennings, 1997). This computational method can make use of the optimal control software package, MISER 3.2.

2. Problem formulation

The soft landing process is to start from the perilune, which is the lowest point in the transfer orbit of the moon, to the moon surface. Assuming that the lander has a propeller with a continuous variable thrust, the selenocentric polar coordination system as shown in Fig. 1 is used as the coordinates of the moon lander locomotion.

Let the center of the moon be the origin of the coordinates. The oY coordinate is from the center of the moon to the perilune. r is the distance between the moon lander and the center of the moon. θ is the angle between oY and or. $F(t)$ is the thrust force of the propeller, satisfying $0 \leqslant F(t) \leqslant F_{\max}$, for all $t \geqslant 0$, and $\psi(t) \geqslant 0$, for all $t \geqslant 0$ is the angle from the vertical line of or to the direction of the thrust.

With reference to the above polar coordinates, the dynamics of the moon lander is (see Wang, Li, & Hui, 2000) given by

$$\begin{cases} \dot{r} = v, \\ \dot{v} = \frac{F}{m} \sin\psi - \mu/r^2 + r\omega^2, \\ \dot{\theta} = \omega, \\ \dot{\omega} = -\left(\dfrac{F}{m}\cos\psi + 2v\omega\right)/r, \\ \dot{m} = -F/C \end{cases} \tag{1}$$

with initial conditions

$$r(0) = r_0, \quad v(0) = 0, \quad \theta(0) = 0, \quad \omega(0) = \omega_0, \quad m(0) = m_0, \tag{2}$$

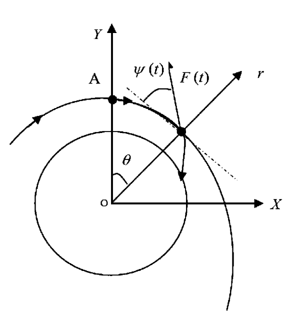

Fig. 1. Polar coordinates of lunar soft landing in the longitudinal plane.

where $v(t)$ is the velocity along the direction of or, μ is the gravitational force of the moon, C is a constant denoting the exhaust velocity coefficient of the propeller, $m(t)$ satisfying $m(t) > 0$ for all $t \geqslant 0$ is the mass of the lander, and $\omega(t)$ is the angular velocity of the lander with reference to the center of the moon. We assume that μ is a constant, which means that the moon has a homogeneous gravity field.

For soft landing, the following terminal conditions are to be satisfied.

$$r(t_f) = r_f, \quad v(t_f) = v_f, \quad \omega(t_f) = 0. \tag{3}$$

Here, $t_f > 0$ is a given time, r_0 is the distance between the moon center and the perilune, r_f is the radius of the moon, and v_f, which is the velocity of the lander when approaching the moon surface, must be less than or equal to a given positive value. The total fuel consumption can be expressed as

$$J_1 = m_0/m_{t_f}, \tag{4}$$

where m_0 and m_{t_f} are, respectively, the initial mass and terminal mass of the moon lander. Clearly, the function (4) and $J_2 = \ln(m_0/m_{t_f})$ have the same optimal solution. On the other hand, we have

$$\ln\left(\frac{m_0}{m_{t_f}}\right) = -\int_0^{t_f} \frac{\mathrm{d}(\ln m(t))}{\mathrm{d}t}\mathrm{d}t = C^{-1}\int_0^{t_f} \frac{F(t)}{m(t)}\,\mathrm{d}t. \tag{5}$$

Thus, the objective of achieving the least fuel consumption is equivalent to minimization of the following cost function.

$$J = \int_0^{t_f} \frac{F(t)}{m(t)}\,\mathrm{d}t. \tag{6}$$

We observe that the angle θ and the mass of the lander m do not influence the safety of the soft landing. Thus, we can ignore these state variables in system (1). Define $u_x = F\cos\psi/m$, $u_y = F\sin\psi/m$, $S = r - r_f$, $v_y = v$ and $v_x = r\omega$. From the definitions of u_x and u_y, the cost function (6) is equivalent to

$$J_2 = \int_0^{t_f} \sqrt{u_x^2(t) + u_y^2(t)}\,\mathrm{d}t. \tag{7}$$

Here, S denotes the distance between the lander and the moon surface, v_y is the vertical velocity to the moon surface, v_x is the horizontal velocity along the moon surface, u_y and u_x are, respectively, the force accelerations corresponding to v_y and v_x.

Theorem 1. *For a given $F(t)$, the constraints $0 \leqslant F(t) \leqslant F_{\max}$ for all $t \geqslant 0$ are satisfied if and only if*

$$0 \leqslant m_0\sqrt{u_x^2 + u_y^2}\mathrm{e}^{-\beta(t)/C} \leqslant F_{\max} \quad \textit{for all } t \geqslant 0, \tag{8}$$

where $m_0 = m(0)$, $C > 0$

$$\dot{\beta}(t) = \sqrt{u_x^2(t) + u_y^2(t)} \quad \textit{and} \quad \beta(0) = 0. \tag{9}$$

Proof. From (1) and (2), we have

$$\frac{F(t)}{m(t)} = F(t)/\left(m_0 - \int_0^t F(\tau)C^{-1}\,\mathrm{d}\tau\right). \tag{10}$$

Define $a(t) = F(t)/m(t)$ where $a(0) = F(0)/m_0$. Then, (10) is equivalent to

$$m_0 - \int_0^t F(\tau)C^{-1}\,\mathrm{d}\tau = F(t)/a(t). \tag{11}$$

Differentiating both sides of (11), we obtain

$$\dot{F}(t) = (\dot{a}(t)/a(t) - a(t)C^{-1})F(t). \tag{12}$$

Thus,

$$F(t) = C_1 a(t)\mathrm{e}^{-\int_0^t a(\tau)C^{-1}\,\mathrm{d}\tau}, \tag{13}$$

where $C_1 = m_0$. The conclusion of the theorem follows readily from (13), the definitions of u_x and u_y, and (8). □

Now, we see that, for soft landing, system (1) is equivalent to

$$\begin{cases} \dot{S}(t) = v_y(t), \\ \dot{v}_y(t) = u_y(t) - \mu/(S(t)+r_f)^2 + v_x^2/(S(t)+r_f), \\ \dot{v}_x(t) = -u_x(t) - v_x(t)v_y(t)/(S(t)+r_f), \end{cases} \tag{14}$$

with initial and terminal conditions

$$\begin{aligned} &S(0) = r_0 - r_f, \quad v_x(0) = r_0\omega_0, \quad v_y(0) = 0, \\ &S(t_f) = 0, \quad v_x(t_f) = 0, \quad v_y(t_f) = v_f. \end{aligned} \tag{15}$$

In practice, the propeller only produces continuous forces. Thus, both u_x and u_y are to be generated by their respective virtual input signals, which are

$$\dot{u}_x(t) = w_x(t), \quad \dot{u}_y(t) = w_y(t), \tag{16}$$

where w_x and w_y are the respective rates of change of u_x and u_y. The initial conditions for (16) are

$$u_x(0) = \zeta_x, \quad u_y(0) = \zeta_y(t), \tag{17}$$

where ζ_x and ζ_y are parameters to be determined.

In reality, it is not necessary for the velocities v_x and v_y at the terminal time t_f to be exactly equal to the given terminal velocities for soft landing. It is acceptable if the terminal velocities of the moon lander are within some given ranges of values. The state S denotes the distance between the lander and the moon surface. It is not required to be exactly equal to zero at the terminal time. During the design process of the optimal controller, the lunar radius is artificially increased slightly so that the moon lander reaches the "terminal" state $S(t_f)$ before landing on the moon surface. Then, it is suspended in the air at the terminal time. At this stage, a simple vertical control can be used to achieve the task of the soft landing. Thus, the terminal constraints specified in (15) can be appended into the cost function (7) to form a new cost function by using the penalty function idea as shown below.

$$\begin{aligned} J = {} & k_1 S^2(t_f) + k_2 v_x^2(t_f) + k_3(v_y(t_f) - v_y)^2 \\ & + \int_0^{t_f} \sqrt{u_x^2(t) + u_y^2(t)}\,\mathrm{d}t \end{aligned} \tag{18}$$

where k_1, k_2 and k_3 are penalty parameters. We can adjust these parameters to achieve the required accuracy of satisfying the terminal constraints.

We write the continuous state inequality constraints as

$$\begin{aligned} &g_1(t) = m_0\sqrt{u_x^2(t) + u_y^2(t)}\mathrm{e}^{-\beta(t)/C} - F_{\max} \leqslant 0, \\ &g_2(t) = -S(t) \leqslant 0. \end{aligned} \tag{19}$$

We may now state the corresponding optimal soft landing control problem as:

Problem (P): Subject to the system described by (14), (16) and (9) with initial and terminal conditions (15) and (17), find control functions w_x and w_y such that the cost function (18) is minimized subject to the continuous state inequality constraints (19), where the penalty parameters k_1, k_2 and k_3 are to be appropriately adjusted.

3. A computation method

We shall develop an efficient computational method for solving Problem (P) as follows.

Let the time interval $[0, t_f]$ be partitioned into n_p subintervals with $n_p + 1$ partition points denoted by $\tau_0^p, \tau_1^p, \ldots, \tau_{n_p}^p$ such that

$$\tau_0^p = 0, \tau_{n_p}^p = t_f \quad \text{and} \quad \tau_{k-1}^p < \tau_k^p \ \text{for } k = 1, \ldots, n_p, \tag{20}$$

where n_p satisfies $n_{p+1} > n_p$. We now approximate the control functions in the form of piecewise constant functions as

$$\begin{aligned} w_x^p(t) &= \sum_{k=1}^{n_p} \sigma_{x,k}^p \chi_{[\tau_{k-1}^p, \tau_k^p)}(t), \\ w_y^p(t) &= \sum_{k=1}^{n_p} \sigma_{y,k}^p \chi_{[\tau_{k-1}^p, \tau_k^p)}(t), \end{aligned} \tag{21}$$

where $\chi_{[\tau_{k-1}^p, \tau_k^p)}(t)$ denotes the indicator function of $[\tau_{k-1}^p, \tau_k^p)$ defined by

$$\chi_{[\tau_{k-1}^p, \tau_k^p)}(t) = \begin{cases} 1, & t \in [\tau_{k-1}^p, \tau_k^p), \\ 0 & \text{elsewhere} \end{cases} \tag{22}$$

$\sigma_{x,k}^p$, $\sigma_{y,k}^p$, $k = 1, 2, \ldots, n_p$ are control parameters, and τ_k^p, $k = 0, 1, \ldots, n_p$ are switching time points such that (20) are satisfied.

Let Γ^p be the set of all vectors τ^p satisfying (20). $\sigma_x^p = (\sigma_{x,1}^p, \ldots, \sigma_{x,n_p}^p)$, $\sigma_y^p = (\sigma_{y,1}^p, \ldots, \sigma_{y,n_p}^p)$ and $\tau^p = (\tau_1^p, \ldots, \tau_{n_p}^p)$. We consider Problem ($P$) with its control functions w_x^p and w_y^p expressed, respectively, by (21) to be referred to as Problem (P_p). For each $p \geqslant 1$, Problem (P_p) is an optimal parameter selection problem. The gradient formulae of the cost function (18) with respect to the control parameter vectors σ_x^p and σ_y^p as well as the initial condition parameters ζ_x and ζ_y can be easily obtained. See Theorem 5.2.1 of Teo et al. (1991). In fact, the gradient formulae of the cost function with respect to the switching vector τ^p can also be derived by using an argument similar to that given for Theorem 5.3.1 of

Teo et al. (1991). However, these gradient formulae are not effective for numerical calculation. For details, see comments given in Lee et al. (1997). In this paper, we will employ the idea of a time scaling transform (Lee et al., 1997) to map the variable switching time points into pre-assigned fixed knots.

We introduce a transformation which will map $t \in [0, t_f]$ into $s \in [0, 1]$:

$$\mathrm{d}t(s)/\mathrm{d}s = \vartheta^p(s) \tag{23}$$

with initial and terminal condition $t(0) = 0$ and $t(1) = t_f$, where ϑ^p is given by

$$\vartheta^p(s) = \sum_{i=1}^{n_p} \delta_i^p \chi_{[\varsigma_{i-1}^p, \varsigma_i^p)}(s) \tag{24}$$

$\varsigma_i^p \geqslant 0$, $\delta_i^p \geqslant 0$, $i = 0, 1, \ldots, n_p$, are pre-assigned time points in the new time horizon [0, 1]. The function $\vartheta^p(s)$, with possible discontinuity points at $s = \varsigma_k^p = k/n_p$, $k = 0, 1, \ldots, n_p$, is called a time scaling control. Let δ_k^p, $k = 1, \ldots, n_p$, be referred to collectively as δ^p, and Ω^p be the set of all such δ^p. Define $\widetilde{\omega}_x^p(s) = w_x^p(t(s))$ and $\widetilde{\omega}_y^p(s) = w_y^p(t(s))$. Then, $\widetilde{\omega}_x^p(s)$ and $\widetilde{\omega}_y^p(s)$ are determined uniquely by σ_x^p, σ_y^p and δ^p.

Define $\widetilde{u}_x(s) = u_x(t(s))$, $\widetilde{u}_y(s) = u_y(t(s))$, $\widetilde{S}(s) = S(t(s))$, $\widetilde{v}_x(s) = v_x(t(s))$, $\widetilde{v}_y(s) = v_y(t(s))$ and $\widetilde{\beta}(s) = \beta(t(s))$. Then, Problem (P_p), after the time scaling transform and the application of the constraint transcription method to the constraints (19), becomes:

Problem $(P_{p,\varepsilon,\lambda_1,\lambda_2})$: Given the dynamical system:

$$\begin{cases} \dot{\widetilde{S}}(s) = \vartheta^p(s)\widetilde{v}_y(s), \\ \dot{\widetilde{v}}_y(s) = \vartheta^p(s)\left(\widetilde{u}_y(s) - \dfrac{\mu}{(\widetilde{S}(s)+r_f)^2} + \dfrac{\widetilde{v}_x^2(s)}{\widetilde{S}(s)+r_f}\right), \\ \dot{\widetilde{v}}_x(s) = \vartheta^p(s)(-\widetilde{u}_x(s) - \widetilde{v}_x(s)\widetilde{v}_y(s)/(\widetilde{S}(s)+r_f)), \\ \dot{\widetilde{u}}_x(s) = \vartheta^p(s)\widetilde{\omega}_x^p(s), \\ \dot{\widetilde{u}}_y(s) = \vartheta^p(s)\widetilde{\omega}_y^p(s), \\ \dot{\widetilde{\beta}}(s) = \vartheta^p(s)\sqrt{\widetilde{u}_x^2(s)+\widetilde{u}_y^2(s)}, \\ \dot{t}(s) = \vartheta^p(s) \end{cases} \tag{25}$$

with initial conditions:

$$\widetilde{S}(0) = r_0 - r_f, \quad \widetilde{v}_x(0) = r_0\omega_0, \quad \widetilde{v}_y(0) = 0,$$
$$\widetilde{u}_x(0) = \zeta_x, \quad \widetilde{u}_y(0) = \zeta_y, \quad t(0) = 0, \quad \widetilde{\beta}(0) = 0, \tag{26}$$

where ϑ^p is determined uniquely by δ^p, find control parameter vectors σ_x^p and σ_y^p, initial condition parameter constants ζ_x and ζ_y, and time scaling parameter vector $\delta^p \in \Omega^p$ such that the cost function:

$$J = k_1\widetilde{S}^2(1) + k_2\widetilde{v}_x^2(1) + k_3(\widetilde{v}_y(1) - v_f)^2 + C^{-1}\int_0^1 \vartheta^p(s)\sqrt{\widetilde{u}_x^2(s)+\widetilde{u}_y^2(s)}\,\mathrm{d}s \tag{27}$$

is minimized subject to the constraint:

$$\sum_{k=1}^{n_p} \delta_k^p/n_p = t_f \tag{28}$$

and the canonical inequality constraints:

$$F_{i,\varepsilon,\lambda_1,\lambda_2}^p(\sigma_x^p, \sigma_y^p, \xi_x, \xi_y, \delta^p) = -\lambda_i + \int_0^1 L_\varepsilon(g_i(s, \sigma_x^p, \sigma_y^p, \xi_x, \xi_y, \delta^p))\,\mathrm{d}s \leqslant 0 \tag{29}$$

where $\lambda_i \geqslant 0$, $i = 1, 2$, are adjustable constant values, g_i, $i = 1, 2$, are defined by (19), and

$$L_\varepsilon(g(s)) = \begin{cases} 0 & \text{if } g_i(s) < -\varepsilon, \\ (g_i(s)+\varepsilon)^2/4\varepsilon & \text{if } -\varepsilon \leqslant g_i(s) < \varepsilon, \\ g_i(s) & \text{if } g_i(s) > \varepsilon, \end{cases} \tag{30}$$

where $\varepsilon > 0$ is an adjustable constant. The following theorem shows the relationship between the continuous state inequality constraints (19) and their approximate canonical inequality constraints.

Theorem 2. *For any $\varepsilon > 0$, let $(\sigma_x^{p,\varepsilon,\lambda_1,\lambda_2}, \sigma_y^{p,\varepsilon,\lambda_1,\lambda_2})$ be the control parameter vectors, $(\xi_x^{p,\varepsilon,\lambda_1,\lambda_2}, \xi_y^{p,\varepsilon,\lambda_1,\lambda_2})$ the initial condition parameters, and $\delta_x^{p,\varepsilon,\lambda_1,\lambda_2}$ the time scaling control parameter vector. Then, there exist $\lambda_i(\varepsilon)$, $i = 1, 2$, such that for all λ_i satisfying $0 < \lambda_i < \lambda_i(\varepsilon)$, if $\Pi = (\sigma_x^{p,\varepsilon,\lambda_1,\lambda_2}, \sigma_y^{p,\varepsilon,\lambda_1,\lambda_2}, \xi_x^{p,\varepsilon,\lambda_1,\lambda_2}, \xi_y^{p,\varepsilon,\lambda_1,\lambda_2}, \delta^{p,\varepsilon,\lambda_1,\lambda_2})$ satisfies the constraints*

$$F_{i,\varepsilon,\lambda_1,\lambda_2}^p(\Pi) = \int_0^1 L_\varepsilon(g_i(s,\Pi))\,\mathrm{d}s - \lambda_i \leqslant 0 \tag{31}$$

then it satisfies the continuous constraints (29).

Proof. The proof is similar to that given for Theorem 8.5.1 of Teo et al. (1991). □

During the computation, we assign, for each $i = 1, 2$, initial ε_i and λ_i. Then, check whether the continuous state inequality constraints (29) are satisfied or not. If they are satisfied, decrease ε_i and λ_i. If they are not satisfied, then by Theorem 2 we see that the reduction of λ_i needs only be carried out a finite number of steps, and the continuous state inequalities constraints (19) will be satisfied. The process is repeated until ε_i are smaller than or equal to a given tolerance.

4. Numerical simulations

In this section, simulation results are given. It is well known that the lunar radius, namely r_f, is 1738000 m. The gravitational force of the moon, μ, is 4902.75 km^3/S^2. We assume that the mass of lander is 600 kg. At the perilune, r_0 is 1 753 000 m and ω_0 is 9.6410^{-4} rad/s. C is 300 g_{E} where g_{E} is the earth gravitation acceleration. Let the terminal time, t_f, be 500 s, and the terminal speed be 0 m/s.

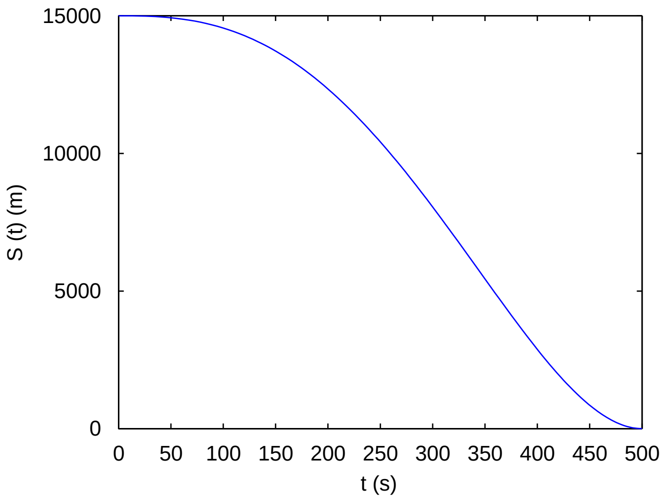

Fig. 2. Height curve $S(t)$ (m).

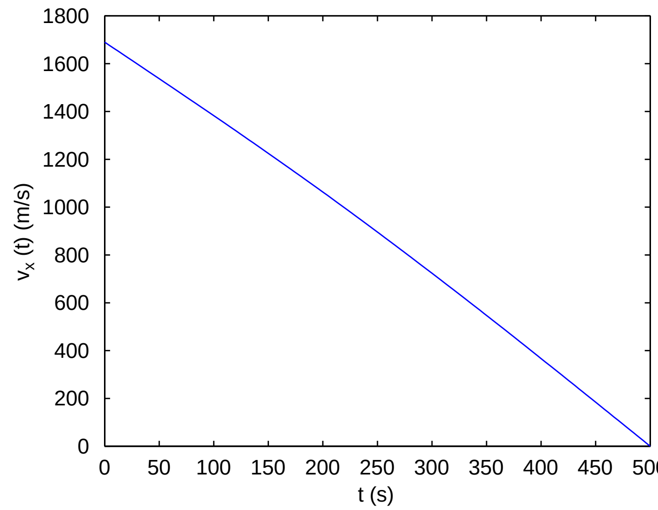

Fig. 4. Horizontal velocity $v_x(t)$ (m/s).

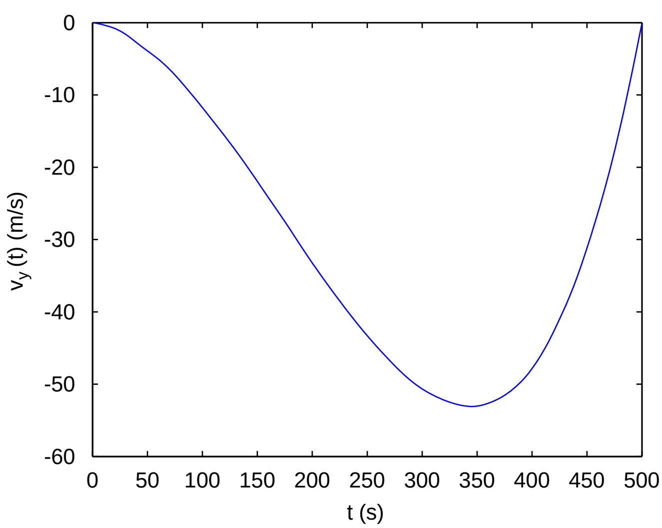

Fig. 3. Vertical velocity $v_y(t)$ (m/s).

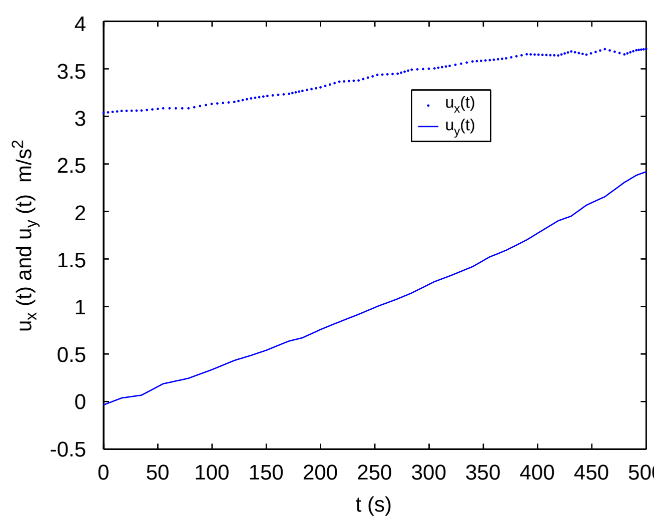

Fig. 5. Control signal $u_x(t)$, $u_y(t)$ $(\mathrm{m/s^2})$.

The least fuel consumption of soft landing problem with the given parameter values are solved using the computational method presented in Section 3 for which the optimal control software MISER3.2 is used. The computer used is Pentium (R) 1.6 GHz with 512M memory. The results obtained are:

$$m(t_f) = 327\,\mathrm{kg}, \quad v_x(t_f) = 0.2\,\mathrm{m/s}, \quad v_y(t_f) = -0.1\,\mathrm{m/s}. \tag{32}$$

Figs. 2–4 describe the state trajectories and Fig. 5 depicts the control signals which are the horizontal and vertical accelerations generated by the propeller. Fig. 2 shows the trajectory $S(t)$ from the perilune to the lunar surface. It is in the perilune that the distance between the lander and the lunar surface is 15 000 m. The variation of $S(t)$ is relatively mild, as the vertical velocity is small at the beginning. As the centrifugal force decreases, the lander is accelerated to the lunar center. Thus, the vertical velocity is increased at the initial stage as shown in Fig. 3. For the soft landing purpose, the vertical velocity must be v_f when the lander lands on the surface of the moon. Therefore, the vertical velocity is decreased towards the end of the time horizon as shown in Fig. 3. Fig. 4 describes the variation of the horizontal velocity which is continuously decreased at all time.

For comparison, we have also solved the optimal control problem, which consists of the cost function (7), system (14) and the continuous state constraint inequality (19), by SOCS (Sparse Optimal Control Software) (Kenneth et al., 2006) and *RIOTS_95* (Schwartz et al., 1997). SOCS generates similar optimal control signals. However, it takes much longer time than the proposed approach. The basic idea underlying SOCS is the transformation of the optimal control problem into a nonlinear programming problem by the discretization technique (John, 2001, 2004), and then enhancing the precision by decreasing the sampling time. When SOCS is used to solve the optimal control problem (using the same computer as the one that we run the MISER 3.2) with the time horizon divided into 30 pieces, it converges rapidly to the optimal result but with very poor precision. When the number of pieces is increased to 900, a similar precision is achieved as that obtained by our approach. However, the computational time is increased by more than 100 times. The performance of *RIOTS*_95 (Schwartz & Polak, 1996) is almost the same as that of SOCS. The computational time taken by our approach is less than 30% of that of SOCS and *RIOTS*_95.

5. Conclusions

This paper deals with the soft landing problem of a moon lander with least fuel consumption. By utilizing specific features associated with the least fuel consumption soft landing problem, we obtained an equivalent standard optimal control problem subject to two continuous state inequality constraints. An efficient computational method was developed based on the control parameterization method in conjunction with a time scaling transform, where the constraint transcription method is used to approximate the continuous state inequality constraints. Numerical simulation showed that the proposed computational method is highly effective and efficient.

Acknowledgments

This work was supported by the Chinese Nature Science Foundation under Grant no. 60475027 and Program for Changjiang Scholars and Innovative Research Team in University. It was also supported by a research grant from the Australian Research Council.

References

Goh, C. J., & Teo, K. L. (1988). Control parameterization: A unified approach to optimal control problems with general constraints. *Automatica*, *24*, 3–18.

Jennings, L. S., Teo, K. L., & Goh, C. J. (1997). MISER3.2 Optimal Control Software: Theory and User Manual, Department of Mathematics, University of Western Australia, Australia. ⟨http://www.cado.maths.uwa.edu.au⟩.

John, B. T. (2001). *Practical methods for optimal control using nonlinear programming*. Philadelphia, PA: SIAM.

John, B. T. (2004). Sparse Optimal Control Software (SCOS), Mathematics and Engineering Analysis Technical Document mct-tech-01-014, release 6.2, Boeing Information and Support Services, POBOX 3707, Seattle.

Kenneth, H., Frode, M., & Edvall, M. M. (2006). USER'S GUIDE FOR TOMLAB/SCOS, ⟨http://tomopt.com/tomlab/products/socs/⟩.

Kian, Y., & Ernst, M. (2004). Analysis of parking orbits and transfer trajectories for mission design of CIS-lunar space stations. *Acta Astronautica*, *55*, 759–771.

Klarer, P. (1997). Small scale intelligence for lunar exploration. *Control Engineering Practice*, *5*, 859–863.

Lee, H. W. J., Teo, K. L., & Jennings, L. S. (1997). Control parametrization enhancing technique for time optimal control problem. *Dynamical System Application*, *6*, 243–261.

Liu, X. L., & Duan, G. R. (2005). Robust guidance and control of lunar lander. *Proceedings of SPIE, international conference on space information technology*, *Wuhan, China*, 59854J-59854N.

Liu, X. L., & Duan, G. R. (2006). Nonlinear optimal control for the soft landing of Lunar lander. *Proceedings of 1st international symposium on systems and control in aerospace and astronautics*, *Harbin, China*, (pp. 1382–1387).

Miele, A. (1980). Gradient algorithms for the optimization of dynamic systems. In: C.T., Leondes (Ed.), *Control and dynamic systems: Advances in theory and applications*, New York: Academic Press. (Vol. 16, pp. 1–52).

Nagarajan, N., Seetharama, M., & Kasturirangan, K. (1996). Modified Kalman filter based autonomous orbit determination using horizon scanner measurements. *Acta Astronautica*, *38*, 769–782.

Sakawa, Y., & Shindo, Y. (1980). On the global convergence of an algorithm for optimal control. *IEEE Transactions Automatic Control*, *25*, 1149–1153.

Schwartz, A., & Polak, E. (1996). Consistent approximations for optimal control problems based on Runge–Kutta integration. *SIAM Journal of Control Optimization*, *34*, 24–35.

Schwartz, A., Polak, E., & Chen, Y. (1997). Recrsive Integration Optimal Trajectory Solver 95: A Matlab Toolbox for Solving Optimal Control Problems, Stanford University, CA, ⟨http://www.schwartz-home.com/adam/RIOTS/⟩.

Teo, K. L., Goh, C. J., & Wong, K. H. (1991). *A unified computational approach to optimal control problems*. England: Longman Scientific and Technical.

Teo, K. L., Rehbock, V., & Jennings, L. S. (1993). A new computational algorithm for functional inequality constrained optimization problems. *Automatica*, *29*, 789–792.

Wang, D. Y., Li, T. S., & Hui, Y. (2000). A sub-optimal fuel guidance law for lunar soft landing. *Journal of Astronautics*, *21*, 55–63.

Xing-Long Liu was born in Heilongjiang Province, PR China, on October 18, 1981. He received his B.Sc. degree in Control Science and Engineering, and both his M.Sc. and Ph.D. degrees in Navigation, Guidance and Control from Harbin Institute of Technology, PR China. His main research interests include robust control, nonlinear feedback control, switched systems and guidance and control of aerocrafts.

Guang-Ren Duan was born in Heilongjiang Province, PR China, on April 5, 1962. He received his B.Sc. degree in Applied Mathematics, and both his M.Sc. and Ph.D. degrees in Control Systems Theory. From 1989 to 1991, he was a post-doctoral researcher at Harbin Institute of Technology, where he became professor of control systems theory in 1991. Prof. Duan visited the University of Hull, UK, and the University of Sheffield, UK from December 1996 to October 1998, and worked at the Queen's University of Belfast, UK from October 1998 to October 2002. Since August 2000, he has been elected Specially Employed Professor at Harbin Institute of Technology sponsored by the Cheung Kong Scholars Program of the Chinese government. He is currently the Director of the Center for Control Theory and Guidance Technology at Harbin Institute of Technology. He is the author and co-author of over 400 publications. His main research interests include robust control, eigenstructure assignment, descriptor systems, missile autopilot control and magnetic bearing control. Dr. Duan is a Chartered Engineer in the UK, a Senior Member of IEEE and a Fellow of IEE.

Kok Lay Teo (M'74–SM'87) received the B.Sc. degree in telecommunications engineering from Ngee Ann Technical College, Singapore, and the M.A.Sc. and Ph.D. degrees in electrical engineering from the University of Ottawa, Canada. He was with the Department of Applied Mathematics, University of New South Wales, Australia, the Department of Industrial and Systems Engineering, National University of Singapore, Singapore, the Department of Mathematics, the University of Western Australia, Australia. In 1996, he joined the Department of Mathematics and Statistics, Curtin University of Technology, Australia, as Professor. He then took up the position of Chair Professor of Applied Mathematics and Head of Department of Applied Mathematics at the Hong Kong Polytech-

nic University, China, from 1999 to 2004. He is currently Professor of Applied Mathematics and Head of Department of Mathematics and Statistics at Curtin University of Technology. He has published five books and over 300 journal papers. He has a software package, MISER3.3, for solving general constrained optimal control problems. He is Editor-in-Chief of the Journal of Industrial and Management Optimization, and Dynamics of Continuous, Discrete and Impulsive Systems, Series B. He also serves as an associate editor of a number of international journals, including Automatica, Nonlinear Dynamics and Systems Theory, Journal of Global Optimization, Optimization and Engineering, Discrete and Continuous Dynamic Systems (Series A and Series B), Dynamics of Continuous, Discrete and Impulsive Systems, Series A, Optimization Letters, and Journal of Inequality and Applications. His research interests include both the theoretical and practical aspects of optimal control and optimization, and their practical applications such as in signal processing in telecommunications, and financial portfolio optimization.