$$x^2+2y^2-3z=F$$

$$\begin{cases} x-y+z=-1 & ; g_1 \\ -2x+12y+3z=7 & ; g_2. \end{cases}$$

$$\left.\begin{aligned} G &= f+pg \\ G &= f+p_1g_1+p_2g_2 \end{aligned}\right\}$$ ф-ия Лагранжа

$$G=x^2+2y^2-3z+p_1(x-y+z)+p_2\times$$
$$\times(-2x+12y+3z)$$

$$G'_x=2x+p_1-2p_2=0 \qquad G'_z=-3+p_1+3p_2=0$$

$$G'_y=4y-p_1+12p_2=0$$

$$\begin{cases} x-y+z+1=0 \\ -2x+12y+3z-7=0 \end{cases}$$

$$p_1=144/55 \qquad x=-13/11$$

$$p_2=7/55 \qquad y=3/11$$

Диффер-л 2$^{го}$ порядка:  $z=5/11$

$$d^2G=G''_{xx}(dx)^2+G''_{yy}(dy)^2+G''_{zz}(dz)^2$$
$$+2G''_{xy}dx\,dy+2G''_{xz}dx\,dz+2G''_{yz}dy\,dz$$

Точка $\left(-\frac{13}{11}; \frac{3}{11}; \frac{5}{11}\right)$

минимум ф-ии, т.к.

$d^2 G > 0$

В этой точке $f\left(-\frac{13}{11}; \frac{3}{11}; \frac{5}{11}\right)$

~

$y = f(x)$ – ф-ия

$R \to R$

функционал:

$y = \mathcal{J}(f(x))$ $\qquad \Delta f = f(t+\Delta t) - f(t)$

$R \to \mathcal{H}$ набор функций

Приращ. функционала

$$\Delta \mathcal{J} = \mathcal{J}(\underbrace{x(t) + \delta x(t)}_{\text{вариация}}) - \mathcal{J}(x(t))$$

вариация $\delta x(t) =$
$= x(t) - x_0(t)$

$f(t) = 2t^2 + 1$

$\Delta f(t) = 2(t+\Delta t)^2 + 1 - 2t^2 + 1 =$

$\cancel{2t^2} + 2t\Delta t - 2\Delta t^2 + 1 - \cancel{2t^2} + 1 =$

$= 2 - 2\Delta t^2 + 2t\Delta t$

$\mathcal{J} = \int_{t_0}^{t_k} (2x^2(t) + 1)\,dt = \int_{t_0}^{t_k} [2(x + \delta x)^2 +$

$$+1]dt - \int_{t_0}^{t_k}[2x^2+1]\,dt = \int_{t_0}^{t_k}[2x^2+$$

$$+4x\delta x+2\delta x^2+1-2x^2-1]\,dt =$$

$$= \int_{t_0}^{t_k}[4x\delta x+2\delta x^2]\,dt.$$

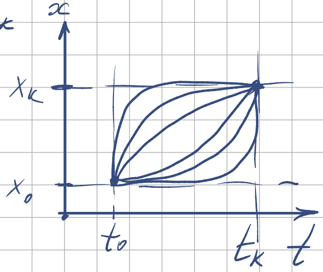
$$J = \int_{t_0}^{t_k} V(x(t), \dot{x}(t), t)\,dt$$

$$x(t_0) = x_0$$

$$x(t_k) = x_k$$

$$V'_x - \frac{d}{dt}(V'_{\dot{x}}) = 0$$

Ур-е Эйлера–Лагранжа




$$V_x = \frac{d}{dt}(V_{\dot{x}}) = 0$$

$x^*$ – оптим. тракт.

$x^* = C_1, C_2$ — const

$$\begin{cases} x^*(t_0) = x_0 \\ x^*(t_k) = x_k \end{cases}$$

Условие Лежандра:

$\left(\frac{\partial^2 V}{\partial \dot{x}^2}\right)\Big|_{x^*} > 0$ MIN* — оптим. тракт.

$< 0$ MAX* $= C_1 \ldots$

$$J = \int_1^2 \underbrace{[\dot{x}^2(t) - 2t\,x(t)]}_{V}\,dt$$

$$x(1) = 0$$

$$x(2) = -1$$

Ур-е Э-Л: $V'_x - \frac{d}{dt}\left(V'_{\dot{x}}\right) = 0$

$$V'_x = -2t$$

$$V'_{\dot{x}} = 2\dot{x}$$

$$\left(\frac{\partial V}{\partial x}\right)_* - \frac{d}{dt}\left(\frac{\partial V}{\partial \dot{x}}\right) = 0$$

$$-2t - \frac{d}{dt}(2\dot{x}) = 0$$

$$-2t - 2\ddot{x} = 0$$

$$\ddot{x} = -t$$

$$\dot{x} = \int -t\,dt = -\frac{1}{2}t^2 + C_1$$

$$x = \int\left(-\frac{1}{2}t^2 + C_1\right)dt =$$

$$= -\frac{1}{6}t^3 + C_1 t + C_2$$

$$\begin{cases} x^*(1) = 0 \\ x^*(2) = -1 \end{cases} \qquad \begin{cases} -\frac{1}{6} + C_1 + C_2 = 0 \\ -\frac{2^3}{6} + C_1 \cdot 2 + C_2 = -1 \end{cases}$$

$$J = \int_0^1 \dot{x}^3 dt$$

$$x(0) = 0$$
$$x(1) = 1$$

$$\frac{d}{dt}(V_{\dot{x}}) = 0$$

$$V_{\dot{x}} = C \Rightarrow$$

$$x^* = C_1 t + C_2$$

$$\begin{cases} 0 = C_1 \cdot 0 + C_2 \rightarrow C_2 = 0 \\ 1 = C_1 + C_2 \Rightarrow C_1 = 1 \end{cases}$$

$$x^* = t$$

По усл-ю Лежандра найдём минимум это или максимум.

$$V = \dot{x}^3$$
$$V' = 3\dot{x}^2$$
$$V'' = 6\dot{x}$$

$$\left.\frac{\partial^2 V}{\partial \dot{x}^2}\right|_* = (6\dot{x})_* = 6 \cdot 1 = 6 > 0$$

min

$$x^* = t \Rightarrow \dot{x}^* = 1$$

$$\Rightarrow x^* = C_1 t + C_2$$