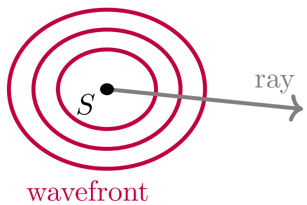
Chapter 17 - Waves II

Sound waves are considered longitudinal waves modeled by the equation:

$$s(x,t) = s_m \cos(kx - \omega t + \phi) \qquad \text{(sound equation)}$$

In 2D space, the travel of sound waves can be visualized by circles emitting out of the source. These **wavefronts** show where the oscillations have the same value. We can show the direction of the waves travel using **rays.**



$$v = \sqrt{\frac{B}{\rho}} = 343 \text{ m/s} \qquad \text{(speed of sound in air)}$$

As sound waves pass through the air, areas of compression and expansion form, in which we can find the speed by finding B, the rate of change of pressure over a volume (bulk modulus), and ρ the density of the medium.

Since the speed of the wave is determined by its medium, not frequency, we can find the wavelength using the same formula from CH16 ($\lambda = \frac{v}{f} = \frac{343}{f}$). We can find the **change in pressure** (Pa) caused by sound waves:

$$\Delta\rho = \Delta\rho_m \sin(kx - \omega t) \qquad \text{(pressure)}$$

Where the pressure amplitude (Pa) given power and displacement is:

$$\Delta\rho_m = (v\rho\omega)s_m \qquad \text{(pressure amplitude)}$$

Moving away from the source, the intensity of sound waves gradually decreases. **Intensity** (W/m^2) is the average power a sound wave carries per unit area:

$$I = \frac{1}{2}\rho v \omega^2 s_m^2 \qquad \text{(intensity)}$$

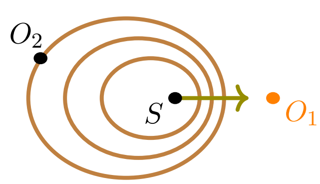
We can find the intensity at any distance r away from the source using the formula:

$$I = \frac{P_s}{4\pi r^2} \qquad \text{(intensity at distance r)}$$

Note that intensity is proportional to the pressure squared ($I \propto \rho^2$). However, we measure **sound level** in decibels (dB) using a logarithmic scale where I_0 is the standard reference intensity (10^{-12} W/m^2).

$$\beta = (10\text{dB}) \log \frac{I}{I_0} \qquad \text{(sound level)}$$

Sources of sound waves are not always stationary, in fact their velocity changes their perceived frequency.



Where v_o is the velocity of the observer, and v_s is the velocity of the source. Frequency changes:

$$f' = f\frac{v \pm v_o}{v \mp v_s} \qquad \text{(Doppler Effect)}$$

The wavefronts show that the frequency increases as Δx decreases, and vice versa.

With this in mind, the top equation is addition if v_0 is going towards the source, while the bottom is negative if v_s is moving towards the observer. The signs switch if they go in the other direction (going away).

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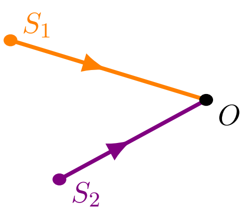
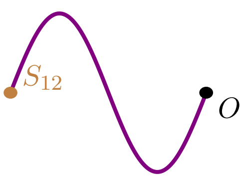
Multiple sound waves can interfere with each other creating both constructive and destructive interference.



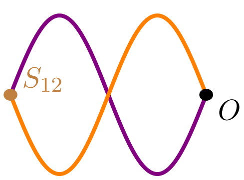
Remembering that the resultant wave of two waves with the same wavelength is $2y_m \cos(\frac{\phi}{2}) \sin(kx - \omega t + \frac{\phi}{2})$, we can correspond the phase offset $(\frac{\phi}{2})$ to the path length difference (ΔL). This gives us:

$$\phi = \frac{\Delta L}{\lambda} 2\pi \qquad \text{(phase difference)}$$



With this equation we can check if it's **fully constructive interference** (creating a larger wave) when ϕ is an integer multiple of π:

$$\phi = m(2\pi) \text{ or } \frac{\Delta L}{\lambda} = 0, 1, 2, ... \qquad \text{(fully constructive)}$$



Fully destructive interference (where the two are out phases creating a smaller wave) occurs when ϕ is an odd multiple of π:

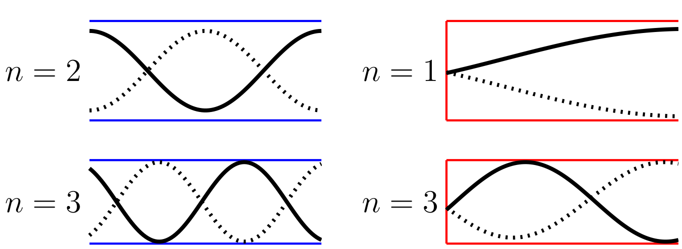
$$\phi = (2m + 1)\pi \text{ or } \frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, ... \qquad \text{(fully destructive)}$$

where m is an integer (0,1,2,...)

When two different frequencies are played together, either interfering constructively (getting louder) or destructively (getting quieter), the difference between the frequencies create **beats:**

$$f_{beat} = |f_1 - f_2| \qquad \text{(beat frequency)}$$

Similar to strings, Sound can create standing waves in a pipe. Now we can find the resonant frequencies:



In a pipe with **both ends open** (blue), the ends of the pipe will always be antinodes. The pipe will work with **any harmonic number:**

$$\lambda = \frac{2L}{n} \qquad (\lambda)$$

$$f = \frac{nv}{2L} \qquad \text{(resonance)}$$

In the pipe with **one end closed** (red), the closed side will be a node, and the open end is an antinode. However the **harmonic number must be an odd integer.**

$$\lambda = \frac{4L}{n} \qquad (\lambda)$$

$$f = \frac{nv}{4L} \qquad \text{(resonance)}$$