Chapter 16 - Waves I

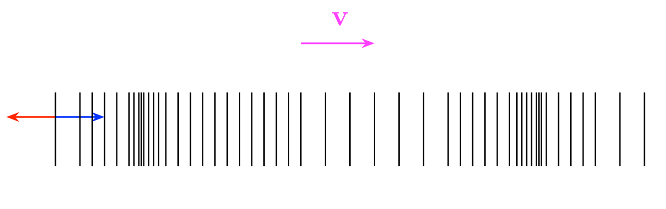
There are three main types of waves:

1. **Mechanical waves:** governed by Newton's laws, and can only exist within physical materials, associated with water, sound and seismic waves

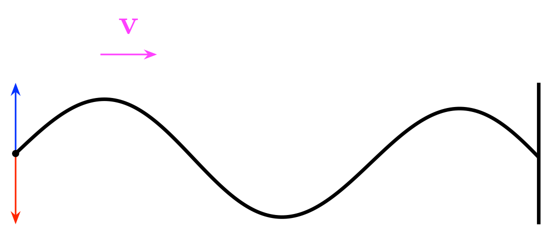
2. **Electromagnetic waves:** requires no physical medium, and travel at speed c ($3 \times 10^8 m/s$) in a vacuum, associated with light, microwaves, and x-rays

3. **Matter waves:** governed by Quantum Physics, particles travel exhibiting both wave-like and particle-like behavior, associated with electrons, protons, atoms and matter

Mechanical waves are further classified into how they displace the medium they travel in. For example, a speaker playing music displaces air by moving its diaphragm back and forth. This forward movement pushes air creating an area of compression. When it moves back, it creates a new area where air is spread out.



This is known as a **longitudinal wave** where they displace the medium parallel to the wave's direction. Now, imagine a rope with one end in your hand and the other tied to a pole. When you move your hand up and down in a harmonic motion, the rope forms in the shape of wave as it travels along its length.

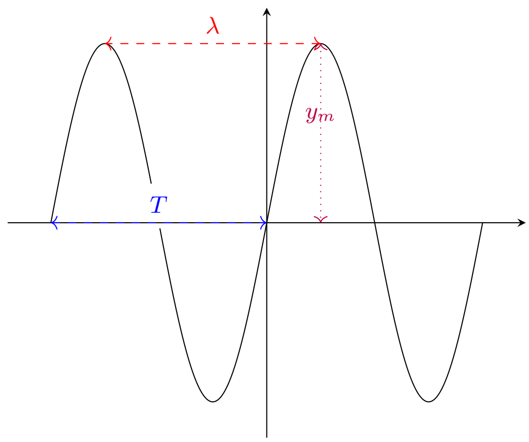


In the case of the rope, the motion is a **transverse wave**, where the displacement of the medium is perpendicular to the direction the wave travels.

Chapter 16 - Waves I

Both transverse and longitudinal waves are sinusoidal waves where we can find the displacement in the positive x direction modeled by the **sinusoidal function**:

$$\underbrace{y(x,t)}_{\text{displacement}} = \overbrace{y_m}^{\text{amplitude}} sin(\underbrace{kx - \omega t + \phi}_{\text{phase}}) \tag{displacement}$$



Where:

- $y(x,t)$: displacement (m)
- y_m: amplitude (m)
- λ: wavelength (m)
- f: frequency (Hz)
- T: period (s)
- ω: angular frequency (rad/s)
- k: wave number (m^{-1})
- ϕ: phase constant (rad)

The speed of a wave is dependent on the medium its traveling through as waves travel differently in air than in a rope. In fact the velocity of a string under pressure can be calculated by finding the **linear density**

$$\mu = \frac{m}{l} \tag{linear density}$$

Where m is the mass of the string, and l is the length of the string. With linear density and the tension force, now we can find the **velocity.**

$$v = \sqrt{\frac{F_T}{\mu}} \tag{wave velocity}$$

Linear density μ can also be used to find the **average power:**

$$P_{avg} = \frac{1}{2}\mu v \omega^2 y_m^2 \tag{average power}$$

Addition formulas:

$$k = \frac{2\pi}{\lambda} \tag{angular wave number}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \tag{frequency}$$

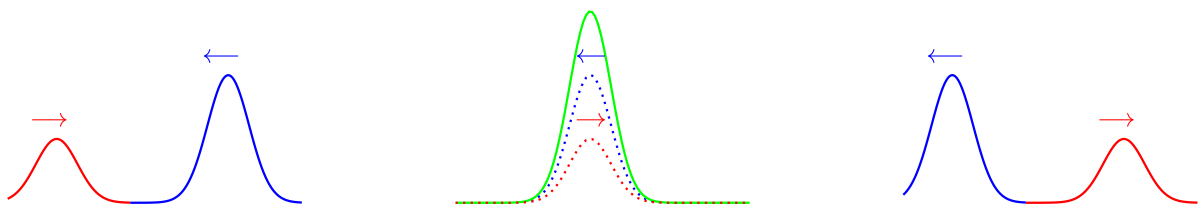
$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \tag{wave velocity}$$

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t) \tag{transverse velocity}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \tag{wave equation}$$

Chapter 16 - Waves I

More than one wave can travel on the same region. In fact, when they overlap the waves follow the Principle of Superposition where the net displacement is the sum of the displacement from wave one and wave two. However, this only affects the displacement, the travel of the wave is untouched.



We can calculate the displacement of the **traveling wave** by adding the displacements of each of the two waves.

$$y'(x,t) = y_1(x,t) + y_2(x,t) \qquad \text{(displacement of waves)}$$

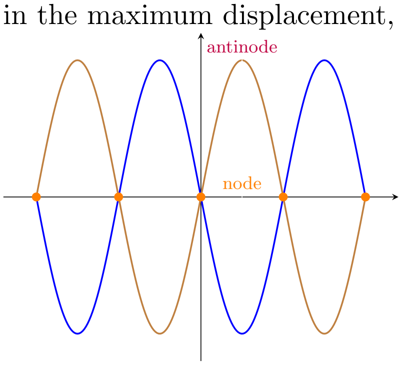
Now if two waves are traveling in the same direction where one of them is shifted by ϕ radians, but have the **same amplitude and wavelength,** we can calculate the resultant wave:

$$y'(x,t) = 2y_m \cos\left(\frac{\phi}{2}\right)\ sin(kx - \omega t + \frac{\phi}{2}) \qquad \text{(traveling wave)}$$

The Principle of Superposition also applies to waves traveling in the opposite direction. For a wave traveling in the positive x direction $y_1(x,t) = y_m sin(kx - \omega t)$ and another traveling in the negative x direction $y_1(x,t) = y_m \sin(kx + \omega t)$, the resultant wave is now a **standing wave**, given by the equation:

$$y'(x,t) = 2y_m \sin(kx)\cos(\omega t) \qquad \text{(standing wave)}$$

Standing waves bring characteristics that traveling waves do not have. These waves have nodes where the two waves intersect, resulting in zero displacement. Where the two waves are the farthest apart resulting in the maximum displacement, are known as antinodes.



Nodes can be found when $\sin(kx) = 0$ giving the function:

$$x = n\frac{\lambda}{2} \qquad \text{(node position)}$$

and antinodes can be found using:

$$x = (n + \frac{1}{2})\frac{\lambda}{2} \qquad \text{(antinode position)}$$

where n is a non-negative integer such as 0,1,2,....

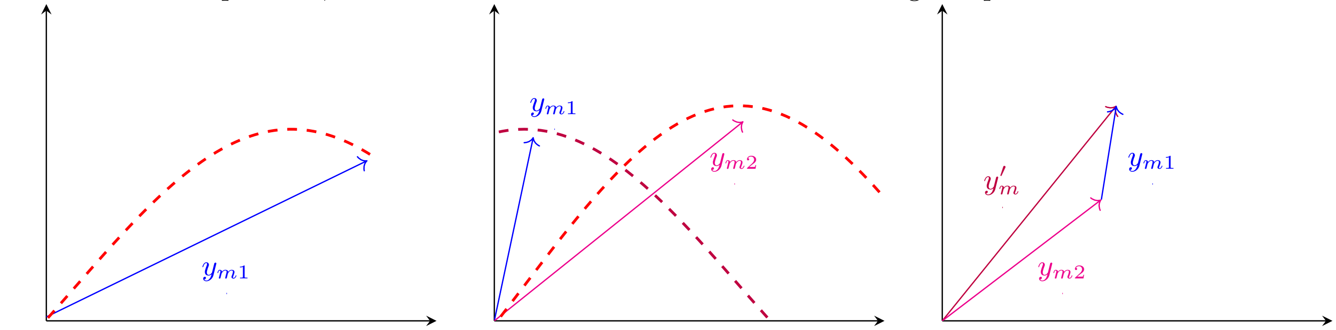
Nodes can be used to find **resonant frequencies**. Resonance is when standing waves reaches its maximum amplitude at a specific frequency. At this frequency, the energy transfer to the medium is at its most efficient.

$$f = \frac{v}{\lambda} = n\frac{v}{2L} \qquad \text{(Resonant frequency)}$$

Where n is a harmonic number. **Harmonic numbers** refers to the mode of vibration, representing the number of half-wavelengths (amount of antinodes in the system).

$$\lambda = \frac{2L}{n} \qquad \text{(wavelength } \lambda\text{)}$$

The previous formulas only work if the two waves have the same amplitude. If two waves happen to have different amplitudes, we can treat the direction of each wave at a given point as a vector.



Phasors allow us to treat each point as a vector, where we can add the two phasors using vector addition. Let $y_1 = y_{m1}\sin(kx + \omega t)$ and $y_2 = y_{m2}\sin(kx + \omega t + \phi)$. By adding the horizontal and vertical components of each vector we get the resultant vector:

$$y'_m = \sqrt{(y'_{mh})^2 + (y'_{mv})^2}$$

Where y'_{mh} is the horizontal component and y'_{mv} is the vertical component

$$y'_{mh} = y_{m1}\cos(0) + y_{m2}\cos(\phi)$$

$$y'_{mv} = y_{m1}\sin(0) + y_{m2}\sin(\phi)$$

In fact we can simplify this equation to:

$$y'_m = \sqrt{(y_{m1})^2 + (y_{m2})^2 + 2y_{m1}y_{m2}\cos(\phi)} \qquad \text{(net amplitude)}$$

The new phase constant is given by:

$$\beta = \tan^{-1}\left(\frac{y_{m2}\sin(\phi)}{y_{m1} + y_{m2}\cos(\phi)}\right) \qquad \text{(net phase constant)}$$

Now the net wave function has the form of:

$$y'(x,t) = y'_m \sin(kx - \omega t + \beta)$$