Serie 5 Trigonometrica De Fourier

Navarro Rodriguez

1: Determinar los valores de las constantes $A, B, C \in \mathbb{R}$ de tal manera que las funciones $f_1(t) = A$, $f_2(t) = B \operatorname{sen} \frac{3}{2}(t)$ y $f_3(t) = C \operatorname{sen}(t)$ definidas para $-2\pi \leq t \leq 2\pi$, sean ortonormales

$$\int_{-2\pi}^{2\pi} A \cdot B \operatorname{sen}\left(\frac{3}{2}t\right) dt = -\frac{2}{3} AB \cos\left(\frac{3}{2}t\right)\Big|_{-2\pi}^{2\pi} = \left[-\frac{2}{3}AB\cos\frac{3}{2}(2\pi) - \left(-\frac{2}{3}AB\cos\frac{3}{2}(-2\pi)\right)\right]$$

$$= -\frac{2}{3}AB\cos 3\pi + \frac{2}{3}AB\cos 3\pi = 0$$

$$\int_{-2\pi}^{2\pi} A \cdot C \operatorname{sen}(t)\, dt = -AC\cos(t)\Big|_{-2\pi}^{2\pi} = \left[-AC\cos(2\pi) - \left(-AC\cos(-2\pi)\right)\right]$$

$$= -AC + AC = 0$$

$$\int_{-2\pi}^{2\pi} B\operatorname{sen}\left(\frac{3}{2}t\right) \cdot C\operatorname{sen}(t)\, dt = \frac{BC}{2}\int_{-2\pi}^{2\pi} \cos\left(t\left(\frac{3}{2}-1\right)\right) dt - \frac{BC}{2}\int_{-2\pi}^{2\pi} \cos\left(t\left(\frac{3}{2}+1\right)\right) dt$$

$$\frac{BC}{2}\, 2\operatorname{sen}\left(\frac{1}{2}t\right)\Big|_{-2\pi}^{2\pi} - \frac{BC}{2} \cdot \frac{2}{5}\operatorname{sen}\left(\frac{5}{2}t\right)\Big|_{-2\pi}^{2\pi} = \left[BC(\operatorname{sen}\pi) - \left(BC(\operatorname{sen}(-\pi))\right)\right]$$

$$-5BC(\operatorname{sen}5\pi) - \left(-5BC(\operatorname{sen}-5\pi)\right) = 0$$

Ortonormalidad

$$\int_{-2\pi}^{2\pi} A \cdot A\, dt = A^2 t\Big|_{-2\pi}^{2\pi} = A^2 4\pi = 1 \rightarrow A = \sqrt{\frac{1}{4\pi}}$$

$$\int_{-2\pi}^{2\pi} B^2 \operatorname{sen}^2\left(\frac{3}{2}t\right) dt = B^2 \cdot \frac{1}{2}\left(t - \frac{1}{3}\operatorname{sen}(3t)\right)\Big|_{-2\pi}^{2\pi}$$

$$= B^2 \cdot \frac{1}{2}\left(2\pi - \frac{1}{3}\operatorname{sen}3(2\pi)\right) - \left(\frac{1}{2}\left(-2\pi - \frac{1}{3}\operatorname{sen}(3(-2\pi))\right)\right) = \frac{B^2\pi}{2} = 1 \rightarrow B = \sqrt{\frac{2}{\pi}}$$

$$\int_{-2\pi}^{2\pi} C^2 \operatorname{sen}^2(t)\, dt = C^2 \frac{1}{2}\left(t - \frac{1}{2}\operatorname{sen}(2t)\right)\Big|_{-2\pi}^{2\pi} = C^2 \frac{1}{2}\left(2\pi - \frac{1}{2}\operatorname{sen}(2(2\pi)) - \left(-2\pi - \frac{1}{2}\operatorname{sen}(2(-2\pi))\right)\right)$$

$$= 2\pi C^2 = 1 \rightarrow C = \sqrt{\frac{1}{2\pi}}$$

2: Comprobar que el conjunto de funciones $\left\{\overset{f_1}{1}, \overset{f_2}{\operatorname{sen} \frac{2}{3}(t)}, \overset{f_3}{\operatorname{sen} \frac{2\pi}{3}(t)}\right\}$ definidas en el intervalo $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$, donde $n \in \mathbb{N}$ y $n \neq 1$, es ortogonal

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} 1 \cdot \operatorname{sen}\left(\frac{2}{3}t\right) dt$$

f_1 contra f_2

$u = \frac{2}{3}t$

$du = \frac{2}{3} dt$

$\frac{3}{2} du = dt$

$$= -\frac{3}{2}\cos\frac{2}{3}t \Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} = -\frac{3}{2}\cos\left(\frac{2}{3}\left(\frac{3\pi}{2}\right)\right) - \left[-\frac{3}{2}\cos\frac{2}{3}\left(-\frac{3\pi}{2}\right)\right]$$

$$= -\frac{3}{2}\cos(\pi) + \frac{3}{2}\cos(\pi) = -\frac{3}{2}(-1) + \frac{3}{2}(-1) = \frac{3}{2} - \frac{3}{2} = 0 \,//$$

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} 1 \cdot \operatorname{sen}\left(\frac{2\pi}{3}t\right) dt$$

f_1 contra f_3

$u = \frac{2\pi}{3}t$

$du = \frac{2\pi}{3} dt$

$\frac{3}{2\pi} du = dt$

$$= -\frac{3\pi}{2}\cos\left(\frac{2\pi}{3}(t)\right)\Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} = -\frac{3\pi}{2}\cos\left(\frac{2\pi}{3}\left(\frac{3\pi}{2}\right)\right) - \left[-\frac{3\pi}{2}\cos\left(\frac{2\pi}{3}\left(-\frac{3\pi}{2}\right)\right)\right]$$

$$= -\frac{3\pi}{2}\cos(\pi^2) + \frac{3\pi}{2}\cos(\pi^2) = 0 \,//$$

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} \operatorname{sen}\left(\frac{2}{3}t\right) \cdot \operatorname{sen}\left(\frac{2\pi}{3}t\right) dt$$

f_2 contra f_3

$$= -\frac{3}{2}\cos\left(\frac{2}{3}t\right) \cdot -\frac{3\pi}{2}\cos\left(\frac{2\pi}{3}t\right)\Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \left[-\frac{3}{2}\cos\frac{2}{3}\left(\frac{3\pi}{2}\right) - \left(-\frac{3}{2}\cos\frac{2}{3}\left(-\frac{3\pi}{2}\right)\right)\right] \cdot \left[-\frac{3\pi}{2}\cos\frac{2\pi}{3}\left(\frac{3\pi}{2}\right) - \left(-\frac{3\pi}{2}\cos\frac{2\pi}{3}\left(-\frac{3\pi}{2}\right)\right)\right]$$

$$= \left[-\frac{3}{2}(-1) + \frac{3}{2}(-1)\right] \cdot \left[-\frac{3\pi}{2}\cos(\pi^2) + \frac{3\pi}{2}\cos(\pi^2)\right]$$

$$= [\,0\,] \cdot [\,0\,] = 0 \,//$$

$\therefore$ El conjunto de funciones es ortogonal //

3: Determinar los valores de las constantes $A, B, C \in \mathbb{R}$ de tal manera que el conjunto de funciones ortogonales $[A, B \operatorname{sen} \frac{4}{3}(t), C \operatorname{sen} \frac{2\pi}{3}(t)]$ definidas en el intervalo $[-\frac{3\pi}{2}, \frac{3\pi}{2}]$, donde $n \in \mathbb{N}$ y $n \neq 2$ sea ortonormal

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} A \cdot B \operatorname{sen}\left(\frac{4}{3}t\right) dt = -\frac{3}{4} AB\cos\left(\frac{4}{3}t\right)\Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} = \left[-\frac{3}{4}AB\cos\frac{4}{3}\left(\frac{3\pi}{2}\right) - \left(-\frac{3}{4}AB\cos\frac{4}{3}\left(-\frac{3\pi}{2}\right)\right)\right]$$

$$= -\frac{3}{4}AB + \frac{3}{4}AB = 0$$

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} A \cdot C \operatorname{sen}\left(\frac{2\pi}{3}t\right) dt = -\frac{3}{2} AC\cos\left(\frac{2\pi}{3}t\right)\Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} = \left[-\frac{3}{2}AC\cos\frac{2\pi}{3}\left(\frac{3\pi}{2}\right) - \left(-\frac{3}{2}AC\cos\frac{2\pi}{3}\left(-\frac{3\pi}{2}\right)\right)\right]$$

$$= -\frac{3}{2}AC + \frac{3}{2}AC = 0$$

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} B\operatorname{sen}\left(\frac{4}{3}t\right) \cdot C\operatorname{sen}\left(\frac{2\pi}{3}t\right) dt = \frac{BC}{2}\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} \cos\left(t\left(\frac{4}{3}-\frac{2\pi}{3}\right)\right) dt - \frac{BC}{2}\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} \cos\left(t\left(\frac{4}{3}+\frac{2\pi}{3}\right)\right) dt$$

$$\frac{BC}{2} \cdot \frac{3}{4-2\pi} \operatorname{sen}\left(t\left(\frac{4-2\pi}{3}\right)\right)\Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} - \frac{BC}{2} \cdot \frac{3}{4+2\pi} \operatorname{sen}\left(t\left(\frac{4+2\pi}{3}\right)\right)\Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}}$$

$$\frac{3BC}{8-4\pi} \operatorname{sen}(4-\pi^2) - \left(\frac{3BC}{8-4\pi}\operatorname{sen}(-4-2\pi)\right) - \left[\frac{3BC}{8+4\pi}\operatorname{sen}(4+\pi^2) - \frac{3BC}{8+4\pi}\operatorname{sen}(-4+\pi^2)\right]$$

$$= 0$$

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} A \cdot A\, dt = A^2 t\Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} = A^2 \cdot \frac{6\pi}{2} = 1 \rightarrow A = \sqrt{\frac{2}{6\pi}}$$

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} B^2\operatorname{sen}^2\left(\frac{4}{3}t\right) dt = B^2 \cdot \frac{1}{2}t - \frac{3}{16}\operatorname{sen}\left(\frac{8}{3}t\right)\Big|_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}}$$

$$= B^2 \cdot \frac{1}{2}\left(\frac{3\pi}{2}\right) - \frac{3}{16}\operatorname{sen}\left(\frac{8}{3}\left(\frac{3\pi}{2}\right)\right) - \left(\frac{1}{2}\left(-\frac{3\pi}{2}\right)\frac{3}{16}\operatorname{sen}\left(\frac{8}{3}\left(-\frac{3\pi}{2}\right)\right)\right) = \frac{B^2}{2} \cdot \frac{3\pi}{2} = 1 \rightarrow B = \sqrt{\frac{4}{3\pi}}$$

$$\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} C^2\operatorname{sen}^2\left(\frac{2\pi}{3}t\right) dt = C^2 \cdot \frac{t}{2} - \frac{3}{8\pi}\operatorname{sen}\left(\frac{4\pi}{3}t\right)\Big|_{-\frac{3\pi}{2}}^{3\pi/2}$$

$$= C^2 \cdot \frac{\frac{3\pi}{2}}{2} - \frac{3}{8\pi}\operatorname{sen}\left(\frac{4\pi}{3}\cdot\frac{3\pi}{2}\right) - \left(\frac{-\frac{3\pi}{2}}{2} - \frac{3}{8\pi}\operatorname{sen}\left(\frac{4\pi}{3}\left(-\frac{3\pi}{2}\right)\right)\right) = \frac{C^2 6\pi}{4} = 1 \rightarrow C = \sqrt{\frac{4}{6\pi}}$$

4: Determinar los valores de las constantes $A, B, C \in \mathbb{R}$ de tal manera que las funciones $f_1(t) = A$, $f_2(t) = B\cos 9\pi t$ y $f_3(t) = C \operatorname{sen} 6\pi t$ definidas para $-\frac{1}{3} \leq t \leq \frac{1}{3}$, sean ortonormales.

$$\int_{-\frac{1}{3}}^{\frac{1}{3}} A \cdot B\cos 9\pi t \, dt = \frac{AB}{9\pi} \operatorname{sen}(9\pi t)\Big|_{-\frac{1}{3}}^{\frac{1}{3}} = \frac{AB}{9\pi}\left[\operatorname{sen}(3\pi) - \operatorname{sen}(-3\pi)\right] = 0$$

$$\int_{-\frac{1}{3}}^{\frac{1}{3}} A \cdot C \operatorname{sen} 6\pi t \, dt = \frac{AC}{6\pi}\cos(6\pi t)\Big|_{-\frac{1}{3}}^{\frac{1}{3}} = \frac{AC}{6\pi}\left[\cos(2\pi) - \cos(-2\pi)\right] = 0$$

$$\int_{-\frac{1}{3}}^{\frac{1}{3}} B\cos(9\pi t) \cdot C \operatorname{sen}(6\pi t)\, dt = \frac{BC}{2}\int_{-\frac{1}{3}}^{\frac{1}{3}} \operatorname{sen}(6\pi + 9\pi)\, dt - \frac{BC}{2}\int_{-\frac{1}{3}}^{\frac{1}{3}} \operatorname{sen}(9\pi - 6\pi)t\, dt$$

$$= \frac{BC}{2} \cdot -\frac{1}{15\pi}\cos(15\pi t) - \frac{BC}{2} \cdot -\frac{1}{3\pi}\cos(3\pi t)\Big|_{-\frac{1}{3}}^{\frac{1}{3}}$$

$$-\frac{BC}{30\pi}\left[\cos(5\pi) - \cos(5\pi)\right] + \frac{BC}{6\pi}\left[\cos(\pi) - \cos(-\pi)\right] = 0$$

Ortonormalidad

$$3\int_{-\frac{1}{3}}^{\frac{1}{3}} A^2\, dt = 1 \;;\; 3A^2 t\Big|_{-1/3}^{1/3} = 1 \;;\quad 3A^2\left[\tfrac{1}{3} - \left(-\tfrac{1}{3}\right)\right] = 1 \quad 3A^2\left(\tfrac{2}{3}\right) = 1 \quad A = \frac{1}{\sqrt{2}}$$

$$3\int_{-\frac{1}{3}}^{\frac{1}{3}} B \cos^2(9\pi t)\, dt = 3B^2\left[\frac{t}{2} - \frac{1}{36\pi}\operatorname{sen}(18\pi t)\right]\Big|_{-\frac{1}{3}}^{\frac{1}{3}}$$

$$\frac{3B^2}{2}\left[\frac{9\pi}{2}\left(\frac{2}{3}\right) + \frac{1}{4}\left[\operatorname{sen}(6\pi) - \operatorname{sen}(-6\pi)\right]\right] = \frac{3B^2}{2} \cdot \frac{2}{3} = 1 \quad B^2 = 1$$

$$3\int_{-\frac{1}{3}}^{\frac{1}{3}} C \operatorname{sen}^2(6\pi t)\, dt = 3C^2 \cdot \left[\frac{t}{2} - \frac{1}{24\pi}\operatorname{sen}(12\pi t)\right]\Big|_{-\frac{1}{3}}^{\frac{1}{3}}$$

$$\frac{3C^2}{2}\left[3\pi\left(\frac{2}{3}\right) - \frac{1}{4}\left[\operatorname{sen}(4\pi) - \operatorname{sen}(-4\pi)\right]\right] = C^2 = 1 \qquad C = 1$$

5: Obtener la serie de Fourier trigonometrica de la función

$$f(t)=\begin{cases} k & \text{si } -L\le t\le 0 \\ 0 & \text{si } 0\le t\le L \end{cases}$$

donde $k\neq 0$ y $L>0$. Escribir la forma de la serie cuando a) $k=-1$ y $L=1$

b) $k=1$ y $L=\pi$

$$a_0=\frac{1}{1}\int_{-1}^{0}-1\,dt=0-(-1)=1$$

$$a_n=\frac{2}{2}\int_{-1}^{0}-1\cos\left(\frac{2n\pi t}{2}\right)dt=\int_{-1}^{0}-\cos(\pi n t)\,dt=-\frac{\operatorname{sen}(n\pi t)}{n\pi}\Big|_{-1}^{0}=0$$

$$b_n=\frac{2}{2}\int_{-1}^{0}-1\operatorname{sen}(n\pi t)\,dt=\frac{\cos(n\pi t)}{n\pi}\Big|_{-1}^{0}=\frac{1}{n\pi}\left(1-(-1)^n\right)$$

$$f(t)=-\frac{1}{2}+\sum_{n=1}^{\infty}\left(\frac{1}{n\pi}\left(1-(-1)^n\right)\right)\operatorname{sen}(n\pi t)$$

b) $$f(t)=\frac{1}{2}+\sum_{n=1}^{\infty}\left(\frac{-1}{n\pi}\right)\left(1-(-1)^n\right)\operatorname{sen}(n\pi t)$$

6: Obtener la serie de Fourier trigonometrica de la función

$$f(t)\begin{cases} 0 & \text{si } -L\le t<0 \\ -\pi e^{2t} & \text{si } 0\le t\le L \end{cases}$$

donde $L>0$. Escribir la forma de la serie cuando $L=1$

$$a_0=\frac{1}{1}\int_{-1}^{0} 0\,dt+\frac{1}{1}\int_0^1 -\pi e^{2t}\,dt=-\pi\int_0^1 e^{2t}dt=-\frac{\pi}{2}e^{2t}\Big|_0^1$$

$$-\frac{\pi}{2}e^{2(1)}-\left(-\frac{\pi}{2}e^{2(0)}\right)=-\frac{\pi}{2}e^2+\frac{\pi}{2}$$ //

$$a_n=\frac{2}{2}\int_0^1 -\pi e^{2t}\cos(n\pi t)\,dt=\frac{-\pi}{2}\left[e^{2t}\cos(n\pi t)\right]\Big|_0^2$$

$$=-\frac{\pi}{2}\left(e^2(-1)^n-1\right)\left(\frac{4}{\pi^2n^2+4}\right)+e^{2t}\,\mathrm{sen}(n\pi t)$$ //

$$b_n=\frac{2}{2}\int_0^1 -\pi e^{2t}\,\mathrm{sen}(n\pi t)\,dt=\frac{e^2\pi^2(-1)^n n-\pi^2 n}{\pi^2n^2+4}$$ //

$$f(1)=\left(\frac{\pi-e^2\pi}{4}\right)+\sum_{n=1}^{\infty}\left[\frac{2\pi(e^2(-1)^n-1)\cos(n\pi t)}{n^2\pi^2+4}+\frac{n\pi^2(e^2(-1)^n-1)\,\mathrm{sen}(n\pi t)}{n^2\pi+4}\right]$$ //

7: Obtener la serie trigonométrica de Fourier de la función $f(t) = -t - 2L, \quad -L \le t \le L$ donde $L > 0$. Escribir la forma de la serie cuando $L = 1$

$$a_0 = \frac{2}{2L}\int_{-L}^{L} (-t-2L)\,dt = \frac{1}{L}\left(\frac{-t^2}{2} - 2Lt\right)\Big|_{-L}^{L} = \frac{1}{L}\left(-\frac{L^2}{2} - 2L^2 + \frac{L^2}{2} + (-2L^2)\right) = -4L = -4$$

$$a_n = \frac{2}{2}\int_{-1}^{1} (-t-2)\cos\left(\frac{2n\pi t}{2L}\right) dt = \int_{-1}^{1} [t\cos(n\pi t) - 2\cos(n\pi t)]\,dt$$

$$= \frac{-t\,\text{sen}\,n\pi t}{n\pi} - \frac{\cos n\pi t}{n^2\pi^2} - \frac{2\,\text{sen}(n\pi t)}{n\pi}\Big|_{-1}^{1} = \frac{(-1)^n - (-1)^n}{n^2\pi^2} = 0$$

$$b_n = \frac{2}{2}\int_{-1}^{1} (-t-2)\,\text{sen}(n\pi t)\,dt = \int_{-1}^{1} [-t\,\text{sen}(n\pi t) - 2\,\text{sen}(n\pi t)]\,dt$$

$$= \frac{t\cos(n\pi t)}{n\pi} - \frac{\text{sen}(n\pi t)}{n^2\pi^2} + \frac{2\cos(n\pi t)}{n\pi}\Big|_{-1}^{1} = \frac{(-1)^n}{n\pi} + \frac{(-1)^n}{n\pi} = \frac{2(-1)^n}{n\pi}$$

$$f(t) = -2 + \sum_{n=0}^{\infty} \frac{2}{n\pi}(-1)^n\,\text{sen}(n\pi t)$$

Escaneado con CamScanner

8: Obtener la serie de Fourier trigonometrica de la función

$$f(t) = \begin{cases} t & \text{si } -1 \leq t \leq 0 \\ -t & \text{si } 0 \leq t \leq 1 \end{cases} \qquad T = 2$$

$$a_0 = \frac{1}{2}\int_{-1}^{0} t\,dt + \frac{1}{2}\int_{0}^{1} -t\,dt = \frac{t^2}{4}\Big|_{-1}^{0} - \frac{t^2}{4}\Big|_{0}^{1} = -\frac{1}{4} + -\frac{1}{4} = -\frac{1}{2}$$

$$a_n = \frac{2}{2}\int_{-1}^{0} t\cos(n\pi t)\,dt + \frac{2}{2}\int_{0}^{1} -t\cos(n\pi t)\,dt = \frac{t\,\text{sen}(n\pi t)}{n\pi} + \frac{\cos n\pi t}{n^2\pi^2}\Big|_{-1}^{0}$$

$$-\frac{t\,\text{sen}\,n\pi t}{n\pi} + \frac{\cos n\pi t}{n^2\pi^2}\Big|_{0}^{1}$$

$$= \frac{1}{n^2\pi^2} - \frac{(-1)^n}{n^2\pi^2} - \frac{(-1)^n}{n^2\pi^2} + \frac{1}{n^2\pi^2} = \frac{2}{n^2\pi^2}\left[1-(-1)^n\right]$$

$$b_n = \frac{2}{2}\int_{-1}^{0} t\,\text{sen}(n\pi t)\,dt - \int_{0}^{1} t\,\text{sen}(n\pi t)\,dt = \frac{-t\cos(n\pi t)}{n\pi} + \frac{\text{sen}(n\pi t)}{n^2\pi^2}\Big|_{-1}^{0}$$

$$+ \frac{t\cos(n\pi t)}{n\pi} - \frac{\text{sen}(n\pi t)}{n^2\pi^2}\Big|_{0}^{1}$$

$$= \frac{\cos(n\pi)(-1)}{n\pi} + \frac{\cos(n\pi)(1)}{n\pi} = 0$$

$$f(t) = -\frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}\left[1-(-1)^n\right]\cos(n\pi t)$$

$$\operatorname{sen}(A)\cdot\cos(B) = \frac{1}{2}\operatorname{sen}(A-B) + \frac{1}{2}\operatorname{sen}(A+B)$$

9: Obtener la serie trigonometrica de Fourier de la función

$f(t) = \operatorname{sen}\frac{1}{2}t \quad -2\pi \le t \le 2\pi$ y trazar la grafica de la función a la que converge para $t \in (-2\pi, 2\pi)$ $T = 2\pi$

$$a_0 = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} \operatorname{sen}\left(\frac{1}{2}t\right)dt = -\frac{1}{2\pi}\,2\cos\left(\frac{1}{2}t\right)\Big|_{-2\pi}^{2\pi} = -\frac{2}{2\pi}\cos\left(\frac{1}{2}t\right)\Big|_{-2\pi}^{2\pi}$$

$$= -\frac{2}{2\pi}\cos\frac{1}{2}(2\pi) - \left(-\frac{2}{2\pi}\cos\frac{1}{2}(-2\pi)\right) = -\frac{1}{2\pi} - \left(-\frac{1}{2\pi}\right) = -\frac{1}{2\pi} + \frac{1}{2\pi} = 0_{//}$$

$$a_n = \frac{2}{2\pi}\int_{-2\pi}^{2\pi} \operatorname{sen}\left(\frac{1}{2}t\right)\cdot\cos\left(\frac{2n\pi}{2\pi}t\right)dt = \frac{1}{\pi}\int_{-2\pi}^{2\pi}\operatorname{sen}\left(\frac{1}{2}t\right)\cdot\cos(nt)\,dt$$

$$= \frac{1}{\pi}\cdot\frac{1}{2}\int_{-2\pi}^{2\pi}\operatorname{sen}\left(\frac{1}{2}t - nt\right)dt + \frac{1}{\pi}\cdot\frac{1}{2}\int_{-2\pi}^{2\pi}\operatorname{sen}\left(\frac{1}{2}t + nt\right)dt$$

$$= \frac{1}{2\pi}\cdot\frac{-2}{1-2n}\cos\left(t\left(\frac{1}{2}-n\right)\right)\Big|_{-2\pi}^{2\pi} + \frac{1}{2\pi}\cdot\frac{-2}{1+2n}\cos\left(t\left(\frac{1}{2}+n\right)\right)\Big|_{-2\pi}^{2\pi}$$

$$= \frac{1}{2\pi}\cdot\frac{-2}{1-2n}\cos\left(2\pi\left(\frac{1}{2}-n\right)\right) - \left[\frac{1}{2\pi}\cdot\frac{-2}{1-2n}\cos\left(-2\pi\left(\frac{1}{2}-n\right)\right)\right] + \frac{1}{2\pi}\cdot\frac{-2}{1+2n}\cos\left(2\pi\left(\frac{1}{2}+n\right)\right) - \left[\frac{1}{2\pi}\cdot\frac{-2}{1+2n}\cos\left(-2\pi\left(\frac{1}{2}+n\right)\right)\right]$$

$$= \frac{-2}{2\pi-4n\pi}\left[-1-(-1)\right] + \frac{-2}{2\pi-4n\pi}\left[-1-(-1)\right] = 0_{//}$$

$$b_n = \frac{2}{2\pi}\int_{-2\pi}^{2\pi}\operatorname{sen}\left(\frac{1}{2}t\right)\cdot\operatorname{sen}\left(\frac{2n\pi}{2\pi}t\right)dt = \frac{1}{\pi}\int_{-2\pi}^{2\pi}\operatorname{sen}\left(\frac{1}{2}t\right)\cdot\operatorname{sen}(nt)\,dt$$

$$\frac{1}{\pi}\cdot\frac{1}{2}\int_{-2\pi}^{2\pi}\cos\left(t\left(\frac{1}{2}-n\right)\right)dt - \frac{1}{\pi}\cdot\frac{1}{2}\int_{-2\pi}^{2\pi}\cos\left(t\left(\frac{1}{2}+n\right)\right)dt$$

$$\frac{1}{2\pi}\cdot\frac{2}{1-2n}\operatorname{sen}\left(t\left(\frac{1}{2}-n\right)\right)\Big|_{-2\pi}^{2\pi} - \frac{1}{2\pi}\cdot\frac{2}{1+2n}\operatorname{sen}\left(t\left(\frac{1}{2}+n\right)\right)\Big|_{-2\pi}^{2\pi}$$

$$= \frac{1}{2\pi}\cdot\frac{2}{1-2n}\operatorname{sen}\left(2\pi\left(\frac{1}{2}-n\right)\right) - \left[\frac{1}{2\pi}\cdot\frac{2}{1+2n}\operatorname{sen}\left(-2\pi\left(\frac{1}{2}-n\right)\right)\right] + \frac{1}{2\pi}\cdot\frac{2}{1+2n}\operatorname{sen}\left(2\pi\left(\frac{1}{2}+n\right)\right) - \left[\frac{1}{2\pi}\cdot\frac{2}{1+2n}\operatorname{sen}\left(-2\pi\left(\frac{1}{2}+n\right)\right)\right]$$

$$= 0_{//} \qquad f(t) = \operatorname{sen}\frac{1}{2}t_{//}$$

10: Obtener la serie seno de Fourier de la función $f(t) = k$ para $0 \leq t \leq 2$, donde $k \neq 0$. Escribir la forma de la serie cuando $k = -1$

$$a_0 = \frac{1}{2}\int_0^2 k\, dt = \frac{1}{2}k \cdot t \Big|_0^2 = \frac{k}{2} \cdot 2 - \frac{k}{2} \cdot 0 = k = -1$$

$$a_n = \frac{2}{2}\int_0^2 k \cdot \cos\left(\frac{2n\pi}{2} t\right) dt = 1\int_0^2 k\cos(n\pi t)\, dt = \frac{k}{n\pi}\operatorname{sen}(n\pi t) + \frac{k}{n^2\pi^2}\cos(n\pi t)\Big|_0^2$$

$$= \frac{k}{n\pi}\operatorname{sen}(2n\pi) + \frac{k}{n^2\pi^2}\cos(2n\pi) - \left[\frac{k}{n\pi}\operatorname{sen}(0\pi n) + \frac{k}{n^2\pi^2}\cos(0\pi n)\right]$$

$$= \frac{k}{n^2\pi^2} - \left[\frac{k}{n^2\pi^2}\right] = 0$$

$$b_n = \frac{2}{2}\int_0^2 k\operatorname{sen}\left(\frac{2n\pi}{2} t\right) dt = 1\int_0^2 k\operatorname{sen}(n\pi t)\, dt = k\left(\frac{-\cos(n\pi t)}{n\pi}\right)\Big|_0^2$$

$$= \frac{\cos 2n\pi}{n\pi} - \frac{\cos(0)}{n\pi} = \frac{(-1)^n - 1}{n\pi}$$

$$f(t) = \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n\pi} \operatorname{sen}(n\pi t)$$

11: Obtener la serie seno de Fourier de la función $f(t) = \pi t$ para $0 \leq t \leq L$, donde $L > 0$. Escribir la forma de la serie cuando

a) $L = 1$

b) $L = \pi$

$$b_n = \frac{2}{L}\int_0^L \pi t \operatorname{sen}\left(\frac{n\pi t}{L}\right) dt = \frac{2\pi}{L}\int_0^L t \operatorname{sen}\left(\frac{n\pi t}{L}\right) dt =$$

$$= \frac{2\pi}{L}\left[-\frac{Lt}{n\pi}\cos\left(\frac{n\pi t}{L}\right) - \frac{t^2}{n^2\pi^2}\operatorname{sen}\left(\frac{n\pi t}{L}\right)\right]_0^1$$

$$= \frac{2\pi}{L}\left(\frac{-1(\cos(n\pi))}{n\pi} - \frac{t^2}{n^2\pi^2}\operatorname{sen}(n\pi) + 0 + 0\right) = \frac{2\pi}{L}\left(\frac{-1^2}{n\pi}(-1)^n\right) = (-1)^n \frac{-2\pi t}{n\pi}$$

$$= -\frac{2L}{n}(-1)^n \qquad f(t) = -\sum_{n=1}^{\infty} \frac{2L(1)^n}{n}\operatorname{sen}\left(\frac{n\pi t}{L}\right)$$

a) $f(t) = -\sum_{n=1}^{\infty} \frac{2}{n}(-1)^n \operatorname{sen}(n\pi t)$

b) $f(t) = \sum_{n=1}^{\infty} \frac{-2\pi}{n}(-1)^n \operatorname{sen}(nt)$

12: Obtener la serie trigonométrica de Fourier de la función periódica $f(t) = A\cos\pi t \quad 0 \le t \le 1$ con periodo $T=1$, donde $A \neq 0$.

$$a_0 = \frac{1}{1}\int_0^1 A\cos(\pi t)\,dt = 1\int_0^1 A\cos(\pi t)\,dt = \frac{A}{\pi}\operatorname{sen}\pi t\Big|_0^1 = 0$$

$$a_n = 2\int_0^1 A\cos(\pi t)\cos(2n\pi t)\,dt = A\int_0^1 [\cos((1-2n)\pi t) + \cos((1+2n)\pi t)]\,dt$$

$$= A\left[\frac{\operatorname{sen}(1-2n)\pi t}{(1-2n)\pi} + \frac{\operatorname{sen}(1+2n)\pi t}{(1+2n)\pi}\right]\Big|_0^1 = 0$$

$$b_n = 2\int_0^1 A\cos(\pi t)\operatorname{sen}(2n\pi t)\,dt = A\int_0^1 [\operatorname{sen}((2n-1)\pi t) + \operatorname{sen}((2n+1)\pi t)]\,dt$$

$$= A\frac{(-\cos((2n-1)\pi t)}{(2n-1)\pi} - \frac{\cos((2n+1)\pi t)}{2n+1\pi}\Big|_0^1 = A\left[\frac{2}{(2n-1)\pi} + \frac{2}{(2n+1)\pi}\right]$$

$$= \frac{A(8n)}{\pi(4n^2-1)}$$

$$f(t) = \frac{A8}{\pi}\sum_{n=1}^{\infty}\frac{n}{(4n^2-1)}\operatorname{sen}(2n\pi t)$$

13: Obtener la serie de Fourier compleja de la función $f(t) = \pi t^3$ para $-\frac{T}{2} \leq t \leq \frac{T}{2}$ y $f(t) = f(t+T)$.

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} \pi t^3 e^{\frac{-i2n\pi t}{T}} =$$

$$= \frac{\pi}{T}\left(-\frac{T}{i2n\pi} t^3 e^{\frac{-2n\pi t i}{T}} + \frac{3t^2T^2 e^{\frac{-2\pi n t i}{T}}}{4\pi^2 n^2} + \frac{6tT^3 e^{\frac{-2n\pi t i}{T}}}{8\pi^3 n^3 i} - \frac{6T^4 e^{\frac{-2n\pi t i}{T}}}{16\pi^4 n^4}\right)\Bigg|_{-T/2}^{T/2}$$

$$= \frac{\pi}{T}\left[\frac{-T^4}{8in\pi} e^{\frac{-2n\pi t i}{2t}} + \frac{6T^4 e^{-n\pi i}}{8n^3\pi^3 i}\right] = \frac{\pi T^3}{8in\pi}\left(-(-1)^n \cdot - \frac{6(-1)^n}{n^2\pi^2}\right)$$

$$+ \frac{iT^3}{8n}\left((-1)^n + \frac{6(-1)^n}{n^2\pi^2}\right)$$

$$f(t) = \sum_{n=-\infty}^{\infty}\left(\frac{iT^3}{8n}\left[\frac{6(-1)^n}{n^2\pi^2} - (-1)^n\right] e^{\frac{i2\pi n t}{T}}\right)$$

14: Obtener la serie compleja de Fourier de la función $f(t) = t^2$ definida para $0 \le t \le 1$ con $f(t) = f(t+1)$. Dibujar el espectro de amplitud o frecuencia.

$$C_n = \frac{1}{2}\int_{-T/2}^{T/2} f(t) e^{\frac{-i2n\pi t}{T}} dt = \int_0^1 t^2 e^{-i2n\pi t} dt$$

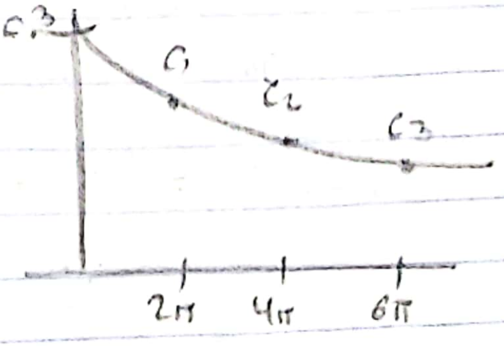
$$= \frac{-t^2 e^{-i2n\pi t}}{2n\pi i} + \frac{2te^{-i2n\pi t}}{4n^2\pi^2} + \frac{2e^{-i2n\pi t}}{8n^3\pi^3 i}\Big|_0^1$$

$$= \frac{-1}{2n\pi i}(1-0) + \frac{2(1-0)}{4n^2\pi^2} + \frac{2(1-1)}{8n^3\pi^3 i} = \frac{-1}{2n\pi i} + \frac{1}{2n^2\pi^2}$$

$$= \frac{1}{2n^2\pi^2} + \frac{1}{2n\pi} \longrightarrow f(t) = \sum_{n=\infty}^{\infty}\left(\frac{1}{2n^2\pi^2} + \frac{i}{2n\pi}\right)e^{-i2n\pi t}$$

$$C_0 = \int_0^1 t^2 dt = \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}$$

n	C_n	$\lvert C_n \rvert$	Arg(n)
1	$\frac{1}{2\pi^2} + \frac{i}{2\pi}$	0.167	1.7626
2	$\frac{1}{8\pi^2} + \frac{i}{2\pi}$	0.0806	1.4129
3	$\frac{1}{18\pi^2} + \frac{i}{6\pi}$	0.0533	1.4650



15: Dibujar el espectro de amplitud o de frecuencia de la función $f(t)=2t^3$ definida para $0\le t\le L$, donde $L>0$ y $f(t)=f(t+L)$.

$$C_0=\frac{1}{L}\int_0^1 2t^3\,dt=\frac{1}{2L}t^4\Big|_0^1=\frac{L^3}{2}$$

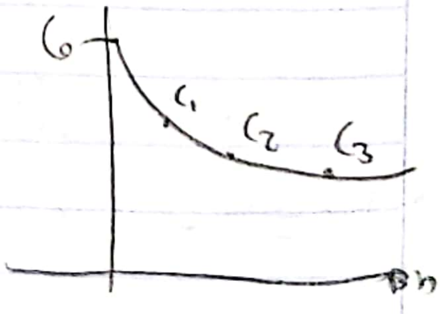
$$C_n=\frac{1}{L}\int_0^1 2t^3 e^{-\frac{2n\pi i}{L}}\,dt=\frac{2}{L}\left(\frac{t^3Le^{-\frac{2\pi n t i}{L}}}{-2\pi n i}+\frac{3t^2L^2e^{-\frac{2n\pi i t}{L}}}{4\pi^2n^2}+\frac{6tL^3e^{\frac{2n\pi i t}{L}}}{8\pi^3n^3 i}-\frac{6L^4e^{-\frac{2\pi n t i}{L}}}{16\pi^4n^4}\right)\Bigg|_0^L$$

$$\frac{2}{L}\left(\frac{-L^4e^{-2n\pi i}}{2\pi n i}+\frac{3L^4e^{-2\pi n i}}{4\pi^2n^2}+\frac{6L^4e^{-2\pi n i}}{16\pi^4n^4}+\frac{6L^4}{16\pi^4n^4}\right)$$

$$=\frac{-2L^3e^{-2\pi n i}}{2\pi n L}+\frac{6L^3e^{-2\pi n i L}}{4\pi^2n^2}+\frac{12L^3e^{-2\pi n i}}{8\pi^3n^3 i}$$

$$=\frac{2L^3e\,i^{-2n\pi i}}{2\pi n}+\frac{6L^3e^{-2n\pi i}}{4\pi^2n^2}-\frac{6L^3e^{-2n\pi i}}{4n^3\pi^3}i$$

$$=\frac{L^3 i}{\pi n}+\frac{3L^3}{2\pi^2n^2}-\frac{6L^3 i}{4n^3n^3}\qquad C_0=\frac{L^3}{2}$$



$$f(t)=\sum_{n=0}^{\infty}\left(\frac{3L^3}{2\pi^2n^2}+\frac{L^3 i}{n\pi}-\frac{3L^3 i}{2\pi^3n^3}\right)e^{-\frac{2n\pi i t}{L}}$$

n	C_n	$\lvert C_n\rvert$	$Arg(C_n)$
1	$\frac{3L^3}{2\pi^2}+\frac{iL^3}{\pi}-\frac{3L^3 i}{2\pi^3}$	$0.304\,L^3$	1.05
2	$\frac{3L^3}{8\pi^2}+\frac{iL^3}{2\pi}-\frac{3L^3 i}{16\pi^3}$	$0.157\,L^3$	1.32
3	$\frac{L^3}{6\pi^2}+\frac{iL^3}{3\pi}-\frac{3L^3}{27(2\pi^3)}$	$0.105\,L^3$	1.41

Escaneado con CamScanner