# Serie 6 Integral de Fourier

Navarro Rodriguez

1.- Obtener la integral de Fourier trigonometrica de la función $f(t) = k|H(t) - H(t-a)|$ donde $k \neq 0$ y $a > 0$

$$F_c(\omega) = \int_{-\infty}^{\infty} f(t)\cos(\omega t)\,dt$$

$$= \int_0^a k\cos(\omega t)\,dt = \frac{k}{\omega}\operatorname{sen}(\omega t)\Big|_{t=0}^{t=a} = \frac{k}{\omega}\operatorname{sen}(a\omega)$$

$$F_s(\omega) = \int_{-\infty}^{\infty} f(t)\operatorname{sen}(\omega t)\,dt = \int_0^a k\operatorname{sen}(\omega t)\,dt$$

$$= -\frac{k}{\omega}\cos(\omega t)\Big|_{t=0}^{t=a} = -\frac{k}{\omega}(\cos(a\omega) - 1) = \frac{k}{\omega}(1-\cos(a\omega))$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{k}{\omega}\operatorname{sen}(a\omega)\cos(\omega t) + \frac{k}{\omega}(1-\cos(a\omega))\operatorname{sen}(\omega t)\right]d\omega$$

$$+ \frac{i}{2\pi}\int_{-\infty}^{\infty} \frac{k}{\omega}\operatorname{sen}(a\omega)\operatorname{sen}(\omega t) - \frac{k}{\omega}(1-\cos(a\omega)\cos(\omega t)]\,d\omega$$

2. Obtener la integral de Fourier trigonométrica de la función

$$f(t)=\begin{cases} 0 & \text{si} \quad t<-a \\ 2 & \text{si} \quad -a\le t\le a \\ 0 & \text{si} \quad t>a \end{cases}$$

donde $a>0$. Escribir la forma de la integral cuando a) $a=1$ b) $a=\pi$

$$F_c(\omega)=\int_{-\infty}^{\infty} f(t)\cos(\omega t)\,dt=\int_{-a}^{a} 2\cos(\omega t)\,dt=\frac{2}{\omega}\operatorname{sen}(\omega t)\Big|_{t=-a}^{t=a}$$

$$=\frac{2}{\omega}\operatorname{sen}(\omega a)-\frac{2}{\omega}\operatorname{sen}(-\omega a)$$

$$F_s(\omega)=\int_{-\infty}^{\infty} f(t)\operatorname{sen}(\omega t)\,dt=\int_{-a}^{a} 2\operatorname{sen}(\omega t)\,dt=-\frac{2}{\omega}\cos(\omega t)\Big|_{t=-a}^{t=a}$$

$$=-\frac{2}{\omega}\cos(\omega a)-\left[-\frac{2}{\omega}\cos(-\omega a)\right]=-\frac{2}{\omega}\cos(\omega a)+\frac{2}{\omega}\cos(-\omega a)$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{2}{\omega}\operatorname{sen}(\omega a)-\frac{2}{\omega}\operatorname{sen}(-\omega a)\right]\cos(\omega t)+\left[-\frac{2}{\omega}\cos(\omega a)+\frac{2}{\omega}\cos(-\omega a)\right]\operatorname{sen}(\omega t)\right\}d\omega$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{2}{\omega}\operatorname{sen}(\omega a)-\frac{2}{\omega}\operatorname{sen}(-\omega a)\right]\operatorname{sen}(\omega t)-\left[-\frac{2}{\omega}\cos(\omega a)+\frac{2}{\omega}\cos(-\omega a)\right]\cos(\omega t)\right\}d\omega$$

a) $$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{2}{\omega}\operatorname{sen}(\omega)-\frac{2}{\omega}\operatorname{sen}(-\omega)\right]\cos(\omega t)+\left[-\frac{2}{\omega}\cos(\omega)+\frac{2}{\omega}\cos(-\omega)\right]\operatorname{sen}(\omega t)\right\}d\omega$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{2}{\omega}\operatorname{sen}(\omega)-\frac{2}{\omega}\operatorname{sen}(-\omega)\right]\operatorname{sen}(\omega t)+\left[-\frac{2}{\omega}\cos(\omega)+\frac{2}{\omega}\cos(-\omega)\right]\cos(\omega t)\right\}d\omega$$

b) $$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{2}{\omega}\operatorname{sen}(\omega\pi)-\frac{2}{\omega}\operatorname{sen}(-\omega\pi)\right]\cos(\omega t)+\left[-\frac{2}{\omega}\cos(\omega\pi)+\frac{2}{\omega}\cos(-\omega\pi)\right]\operatorname{sen}(\omega t)\right\}d\omega$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{2}{\omega}\operatorname{sen}(\omega\pi)-\frac{2}{\omega}\operatorname{sen}(-\omega\pi)\right]\operatorname{sen}(\omega t)-\left[-\frac{2}{\omega}\cos(\omega\pi)+\frac{2}{\omega}\cos(-\omega\pi)\right]\cos(\omega t)\right\}d\omega$$

3: Obtener la integral de Fourier trigonométrica de la función

$$f(t)=\begin{cases} t^3 & \text{si } |t|<a \\ 0 & \text{si } |t|\geq a \end{cases}$$

donde $a>0$. Escribir la forma de la integral cuando a) $a=1$ b) $a=\pi$

$$F_c(\omega)=\int_{-a}^{a} t^3\cos(\omega t)\,dt=\frac{1}{\omega}t^3\text{sen}(\omega t)+\frac{3}{\omega^2}t^2\cos(\omega t)-\frac{6}{\omega^3}t\,\text{sen}(\omega t)-\frac{6}{\omega^4}\cos(\omega t)\Big|_{t=-a}^{t=a}$$

$$=\frac{1}{\omega}[a^3\text{sen}(a\omega)-(-a)^3\text{sen}(-a\omega)]+\frac{3}{\omega^2}[a^2\cos(a\omega)-(-a)^2\cos(-a\omega)]$$

$$=-\frac{6}{\omega^2}[a\,\text{sen}(a\omega)-(-a)\text{sen}(-a\omega)]+\frac{6}{\omega^4}[\cos(a\omega)-\cos(-a\omega)] \rightarrow F_c(\omega)=0$$

$$F_s(\omega)=\int_{-a}^{a} t^3\text{sen}(\omega t)\,dt=-\frac{1}{\omega}t^3\cos(\omega t)+\frac{3}{\omega^2}t^2\text{sen}(\omega t)+\frac{6}{\omega^3}t\cos(\omega t)-\frac{6}{\omega^4}\text{sen}(\omega t)\Big|_{t=-a}^{t=a}$$

$$=-\frac{1}{\omega}[a^3\cos(a\omega)-(-a)^3\cos(-a\omega)]+\frac{3}{\omega^2}[a^2\text{sen}(a\omega)-(-a)^2\text{sen}(-a\omega)]$$
$$+\frac{6}{\omega^3}[a\cos(a\omega)-(-a)\cos(-a\omega)]-\frac{6}{\omega^4}[\text{sen}(a\omega)-\text{sen}(a\omega)]$$

$$=-\frac{2}{\omega}a^3\cos(a\omega)+\frac{6}{\omega^2}a^2\text{sen}(a\omega)+\frac{12}{\omega^3}a\cos(a\omega)-\frac{12}{\omega^4}\text{sen}(a\omega)$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[-\frac{2a^3}{\omega}+\frac{12}{\omega^3}a\right]\cos(a\omega)+\left[\frac{6a^2}{\omega^2}-\frac{12}{\omega^4}\right]\text{sen}(a\omega)\right\}\text{sen}(\omega t)\,d\omega$$
$$-\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{-2a^3}{\omega}+\frac{12}{\omega^3}a\right]\cos(a\omega)+\left[\frac{6a^2}{\omega^2}-\frac{12}{\omega^4}\right]\text{sen}(a\omega)\right\}\cos(\omega t)\,d\omega$$

a) $$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[-\frac{2}{\omega}+\frac{12}{\omega^3}\right]\cos(\omega)+\left[\frac{6}{\omega^2}-\frac{12}{\omega^4}\right]\text{sen}(\omega)\right\}\text{sen}(\omega t)\,d\omega-\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{-2}{\omega}+\frac{12}{\omega^3}\right]\cos(\omega)\right.$$
$$\left.+\left[\frac{6}{\omega^2}-\frac{12}{\omega^4}\right]\text{sen}(\omega)\right\}\cos(\omega t)\,d\omega$$

b) $$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{-2\pi^3}{\omega}+\frac{12\pi}{\omega^3}\right]\cos(\omega\pi)+\left[\frac{6\pi^2}{\omega^2}-\frac{12}{\omega^4}\right]\text{sen}(\omega\pi)\right\}\text{sen}(\omega t)\,d\omega$$
$$-\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{-2\pi^3}{\omega}+\frac{12\pi}{\omega^3}\right]\cos(\omega\pi)+\left[\frac{6\pi^2}{\omega^2}-\frac{12}{\omega^4}\right]\text{sen}(\omega\pi)\right\}\cos(\omega t)\,d\omega$$

4: Obtener la integral de Fourier trigonométrica de la función

$$f(t)=\begin{cases} 0 & \text{si } t\leq -a \\ kt & \text{si } -a\leq t\leq a \\ 0 & \text{si } t>a \end{cases}$$

donde $k\neq 0$ y $a>0$. Determinar la función a la que converge.

$$F_c(\omega)=\int_{-a}^{a} kt\cos(\omega t)\,dt=\left.\frac{kt}{\omega}\operatorname{sen}(\omega t)+\frac{k}{\omega^2}\cos(\omega t)\right|_{t=-a}^{t=a}$$

$kt \searrow \cos(\omega t)$

$k \searrow \frac{1}{\omega}\operatorname{sen}(\omega t)$

$0 \quad -\frac{1}{\omega^2}\cos(\omega t)$

$$=\frac{ka}{\omega}\operatorname{sen}(\omega a)+\frac{k}{\omega^2}\cos(\omega a)-\left[\frac{-ka}{\omega}\operatorname{sen}(-\omega a)+\frac{k}{\omega^2}\cos(-\omega a)\right]$$

$$F_s(\omega)=\int_{-a}^{a} kt\operatorname{sen}(\omega t)\,dt=\left.-\frac{kt}{\omega}\cos(\omega t)+\frac{k}{\omega^2}\operatorname{sen}(\omega t)\right|_{t=-a}^{t=a}$$

$kt \searrow \operatorname{sen}(\omega t)$

$\to k \searrow -\frac{1}{\omega}\cos(\omega t)$

$0 \quad -\frac{1}{\omega^2}\operatorname{sen}(\omega t)$

$$=\frac{-ka}{\omega}\cos(\omega a)+\frac{k}{\omega^2}\operatorname{sen}(\omega a)-\left[\frac{-(-ka)}{\omega}\cos(\omega t)+\frac{k}{\omega^2}\operatorname{sen}(-\omega a)\right]$$

$$=-\frac{ka}{\omega}\cos(\omega a)+\frac{k}{\omega^2}\operatorname{sen}(\omega a)-\left[\frac{ka}{\omega}\cos(\omega t)+\frac{k}{\omega^2}\operatorname{sen}(-\omega a)\right]$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left(\frac{ka}{\omega}\operatorname{sen}(\omega a)+\frac{k}{\omega^2}\cos(\omega a)-\left[\frac{-ka}{\omega}\operatorname{sen}(-\omega a)+\frac{k}{\omega^2}\cos(-\omega a)\right]\right)\cos(\omega t)\right.$$
$$\left.+\left(-\frac{ka}{\omega}\cos(\omega a)+\frac{k}{\omega^2}\operatorname{sen}(\omega a)-\left[\frac{ka}{\omega}\cos(\omega t)+\frac{k}{\omega^2}\operatorname{sen}(-\omega a)\right]\right)\operatorname{sen}(\omega t)\right\}d\omega$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left(\frac{ka}{\omega}\operatorname{sen}(\omega a)+\frac{k}{\omega^2}\cos(\omega a)-\left[\frac{-ka}{\omega}\operatorname{sen}(-\omega a)+\frac{k}{\omega^2}\cos(-\omega a)\right]\right)\operatorname{sen}(\omega t)\right.$$
$$\left.-\left(\frac{-ka}{\omega}\cos(\omega a)+\frac{k}{\omega^2}\operatorname{sen}(\omega a)-\left[\frac{ka}{\omega}\cos(\omega t)+\frac{k}{\omega^2}\operatorname{sen}(-\omega a)\right]\right)\cos(\omega t)\right\}d\omega$$

5: Obtener la integral de Fourier en cosenos de la función

$f(t) = \begin{cases} 1 & si \quad 0 \leq t < a \\ 0 & si \quad t \geq a \end{cases}$ donde $a > 0$. Escribir la integral de Fourier en cosenos de la función cuando a) $a=1$ b) $a=\pi$

$$F_c(w) = \int_0^a 1 \cdot \cos(wt)\,dt$$

$$= \frac{1}{w}\,sen(wt)\Big|_0^a$$

$$= \frac{1}{w}\,sen(wa) //$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{w}\,sen(wa)\cdot\cos(wt)\,dw - \frac{i}{2\pi}\int_{-\infty}^{\infty} -\frac{1}{w}\,sen(wa)\cdot sen(wt)\,dw //$$

a) $\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{w}\,sen(w(1))\cdot\cos(wt)\,dw - \frac{i}{2\pi}\int_{-\infty}^{\infty} -\frac{1}{w}\,sen(w(1))\cdot sen(wt)\,dw$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{w}\,sen(w)\cos(wt)\,dw - \frac{i}{2\pi}\int_{-\infty}^{\infty} -\frac{1}{w}\,sen(w)\,sen(wt)\,dw //$$

b) $\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{w}\,sen(w(\pi))\cdot\cos(wt)\,dw - \frac{i}{2\pi}\int_{-\infty}^{\infty} -\frac{1}{w}\,sen(w(\pi))\cdot sen(wt)\,dw //$

6. Obtener la integral seno de Fourier de la función

$$f(t) = \begin{cases} t^2 & \text{si } 0 \leq t < a \\ 0 & \text{si } t \geq a \end{cases}$$

donde $a > 0$. Escribir la integral seno de Fourier de la función cuando $a = \pi$

$$F_s(w) = \int_0^a t^2 \operatorname{sen}(wt)\,dt = -\frac{t^2}{w}\cos(wt) + \frac{2t}{w^2}\operatorname{sen}(wt) + \frac{2}{w^3}\cos(wt)\Big|_{t=0}^{t=a}$$

| | |
|---|---|
| $t^2$ | $\operatorname{sen}(wt)$ |
| $(-)\ 2t$ | $-\frac{1}{w}\cos(wt)$ |
| $2$ | $-\frac{1}{w^2}\operatorname{sen}(wt)$ |
| $0$ | $\frac{1}{w^3}\cos(wt)$ |

$$= -\frac{a^2}{w}\cos(wa) + \frac{2a}{w^2}\operatorname{sen}(wa) + \frac{2}{w^3}\cos(wa)$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\left[-\frac{a^2}{w}\cos(wa) + \frac{2a}{w^2}\operatorname{sen}(wa) + \frac{2}{w^3}\cos(wa)\right]\operatorname{sen}(wt)\right)dw$$

$$+ \frac{i}{2\pi}\int_{-\infty}^{\infty}\left(-\left[-\frac{a^2}{w}\cos(wa) + \frac{2a}{w^2}\operatorname{sen}(wa) + \frac{2}{w^3}\cos(wa)\right]\cos(wt)\right)dw$$

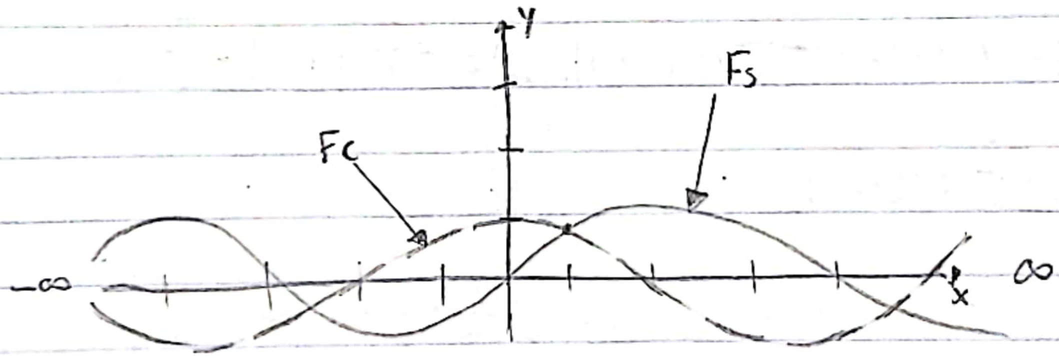
7: Dibujar, en el intervalo $(-\infty, \infty)$, las gráficas de las funciones a las que convergen las integrales de Fourier en senos y cosenos de la función

$$f(t) = \begin{cases} 1 & \text{si } 0 \le t < 1 \\ 0 & \text{si } t \ge 1 \end{cases}$$

(No es necesario obtener las correspondientes Integrales de Fourier)

$$F_c(\omega) = \int_0^1 1 \cos(\omega t)\, dt = \frac{\operatorname{sen}(\omega t)}{\omega}\Big|_0^1 = \frac{\operatorname{sen}(\omega)}{\omega}$$

$$F_s(\omega) = \int_0^1 1 \operatorname{sen}(\omega t)\, dt = \frac{-\cos(\omega t)}{\omega}\Big|_0^1 = \frac{1-\cos(\omega)}{\omega}$$



8. Obtener la integral compleja de Fourier de la función

$$f(t)=\begin{cases} 0 & \text{si} \quad t\leq -a \\ 2 & \text{si} \quad -a\leq t\leq 0 \\ 0 & \text{si} \quad t>0 \end{cases}$$

donde $a>0$. Escribir la integral de Fourier de la función cuando

a) $a=1$ b) $a=\pi$

$$F_c(\omega)=\int_{-a}^{0} 2\cos(\omega t)\,dt=\frac{2}{\omega}\operatorname{sen}(\omega t)\Big|_{t=-a}^{t=0}=\frac{2}{\omega}\cancel{\operatorname{sen}(\omega 0)}-\frac{2}{\omega}\operatorname{sen}(-\omega a)$$

$$=-\frac{2}{\omega}\operatorname{sen}(-\omega a)$$

$$F_s(\omega)=\int_{-a}^{0} 2\operatorname{sen}(\omega t)\,dt=-\frac{2}{\omega}\cos(\omega t)\Big|_{t=-a}^{t=0}=-\frac{2}{\omega}\cos(\omega 0)-\left[-\frac{2}{\omega}\cos(-\omega a)\right]$$

$$=-\frac{2}{\omega}+\frac{2}{\omega}\cos(-\omega a)$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[-\frac{2}{\omega}\operatorname{sen}(-\omega a)\right]\cos(\omega t)+\left[-\frac{2}{\omega}+\frac{2}{\omega}\cos(-\omega a)\right]\operatorname{sen}(\omega t)\right\}d\omega$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[-\frac{2}{\omega}\operatorname{sen}(-\omega a)\right]\operatorname{sen}(\omega t)-\left[-\frac{2}{\omega}+\frac{2}{\omega}\cos(-\omega a)\right]\cos(\omega t)\right\}d\omega$$

a) $$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[-\frac{2}{\omega}\operatorname{sen}(-\omega)\right]\cos(\omega t)+\left[-\frac{2}{\omega}+\frac{2}{\omega}\cos(-\omega)\right]\operatorname{sen}(\omega t)\right\}d\omega$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[-\frac{2}{\omega}\operatorname{sen}(-\omega)\right]\operatorname{sen}(\omega t)-\left[-\frac{2}{\omega}+\frac{2}{\omega}\cos(-\omega)\right]\cos(\omega t)\right\}d\omega$$

b) $$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[-\frac{2}{\omega}\operatorname{sen}(-\omega\pi)\right]\cos(\omega t)+\left[-\frac{2}{\omega}+\frac{2}{\omega}\cos(-\omega\pi)\right]\operatorname{sen}(\omega t)\right\}d\omega$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[-\frac{2}{\omega}\operatorname{sen}(-\omega\pi)\right]\operatorname{sen}(\omega t)-\left[-\frac{2}{\omega}+\frac{2}{\omega}\cos(-\omega\pi)\right]\cos(\omega t)\right\}d\omega$$

9. Obtener la integral compleja de Fourier de la función

$$f(t)=\begin{cases}0 & \text{si } t<-a \\ kt & \text{si } -a\le t\le 0 \\ 0 & \text{si } t>0\end{cases}$$

donde $k\neq 0$ y $a>0$. Escribir la integral de Fourier de la función cuando

a) $k=-1$ y $a=1$

b) $k=-1$ y $a=\pi$

c) $k=a$

$$F_c(w)=\int_{-a}^{0} kt\cos(wt)\,dt=$$

$$=\frac{k(0)}{w}\operatorname{sen}(w0)+\frac{k}{w^2}\cos(w0)-\left[-\frac{ka}{w}\operatorname{sen}(-wa)+\frac{k}{w^2}\cos(-wa)\right]=\frac{k}{w^2}+\frac{ka}{w}\operatorname{sen}(-wa)-\frac{k}{w^2}\cos(-wa)$$

$$F_s(w)=\int_{-a}^{0} kt\operatorname{sen}(wt)\,dt=\frac{-k(0)}{w}\cos(w(0))+\frac{k}{w^2}\operatorname{sen}(w(0))-\left[\frac{-k(-a)}{w}\cos(-wa)+\frac{k}{w^2}\operatorname{sen}(-wa)\right]$$

$$=-\frac{ka}{w}\cos(-wa)-\frac{k}{w^2}\operatorname{sen}(-wa)$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{k}{w^2}+\frac{ka}{w}\operatorname{sen}(-wa)-\frac{k}{w^2}\cos(-wa)\right]\cos(wt)+\left[-\frac{ka}{w}\cos(-wa)-\frac{k}{w^2}\operatorname{sen}(-wa)\right]\operatorname{sen}(wt)\right\}dw$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{k}{w^2}+\frac{ka}{w}\operatorname{sen}(-wa)-\frac{k}{w^2}\cos(-wa)\right]\operatorname{sen}(wt)-\left[-\frac{ka}{w}\cos(-wa)-\frac{k}{w^2}\operatorname{sen}(-wa)\right]\cos(wt)\right\}dw$$

a) $$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{-1}{w^2}+\frac{-1}{w}\operatorname{sen}(-w)+\frac{1}{w^2}\cos(-w)\right]\cos(wt)+\left[\frac{1}{w}\cos(-w)+\frac{1}{w^2}\operatorname{sen}(-w)\right]\operatorname{sen}(wt)\right\}dw$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{-1}{w^2}+\frac{-1}{w}\operatorname{sen}(-w)+\frac{1}{w^2}\cos(-w)\right]\operatorname{sen}(wt)-\left[\frac{1}{w}\cos(-w)+\frac{1}{w^2}\operatorname{sen}(-w)\right]\cos(wt)\right\}dw$$

b) $$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{-1}{w^2}+\frac{-\pi}{w}\operatorname{sen}(-w\pi)+\frac{1}{w^2}\cos(-w\pi)\right]\cos(wt)+\left[\frac{\pi}{w}\cos(-w\pi)+\frac{1}{w^2}\operatorname{sen}(-w\pi)\right]\operatorname{sen}(wt)\right\}dw$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{-1}{w^2}+\frac{-\pi}{w}\operatorname{sen}(-w\pi)+\frac{1}{w^2}\cos(-wa)\right]\operatorname{sen}(wt)-\left[\frac{\pi}{w}\cos(-w\pi)+\frac{1}{w^2}\operatorname{sen}(-w\pi)\right]\cos(wt)\right\}dw$$

c) $$\frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{a}{w^2}+\frac{a^2}{w}\operatorname{sen}(-wa)-\frac{a}{w^2}\cos(-wa)\right]\cos(wt)+\left[-\frac{a^2}{w}\cos(-wa)-\frac{a}{w^2}\operatorname{sen}(-wa)\right]\operatorname{sen}(wt)\right\}dw$$

$$+\frac{i}{2\pi}\int_{-\infty}^{\infty}\left\{\left[\frac{a}{w^2}+\frac{a^2}{w}\operatorname{sen}(-wa)-\frac{a}{w^2}\cos(-wa)\right]\operatorname{sen}(wt)-\left[-\frac{a^2}{w}\cos(-wa)-\frac{a}{w^2}\operatorname{sen}(-wa)\right]\cos(wt)\right\}dw$$

10: Obtener la integral compleja de Fourier de la función

$$f(t) = \begin{cases} 0 & \text{si} \quad t<-1 \\ -\pi t^2 & \text{si} \quad -1 \leq t \leq 0 \\ 0 & \text{si} \quad t>0 \end{cases}$$

$$F(\omega) = \int_{-1}^{0} -\pi t^2 e^{-i\omega t}\, dt$$

$$= -\pi \left[ -\frac{1}{i\omega} t^2 e^{-i\omega t} + \frac{2}{\omega^2} t e^{-i\omega t} + \frac{2}{i\omega^3} e^{-i\omega t} \right] \Bigg|_{t=-1}^{t=0}$$

$$= \frac{-\pi}{i\omega} \left[ 0 - e^{i\omega} \right] - \frac{2\pi}{\omega^3} \left[ 0 - (-1) e^{i\omega} \right] - \frac{2\pi}{i\omega^3} \left[ 1 - e^{i\omega} \right]$$

$$= \frac{i\pi}{\omega} e^{i\omega} - \frac{2\pi}{\omega^2} e^{i\omega} + \frac{2i\pi}{\omega^3} \left[ 1 - e^{i\omega} \right]$$

$$= e^{i\omega} \left[ \frac{i\pi}{\omega} \cdot \frac{2\pi}{\omega^2} - \frac{2\pi}{\omega^3} \right] + \frac{i2\pi}{\omega^3} //$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ e^{i\omega} \left[ -\frac{2\pi}{\omega^2} + i \left( \frac{\pi}{\omega} - \frac{2\pi}{\omega^3} \right) \right] + \frac{i2\pi}{\omega^3} \right\} e^{i\omega t}\, d\omega //$$