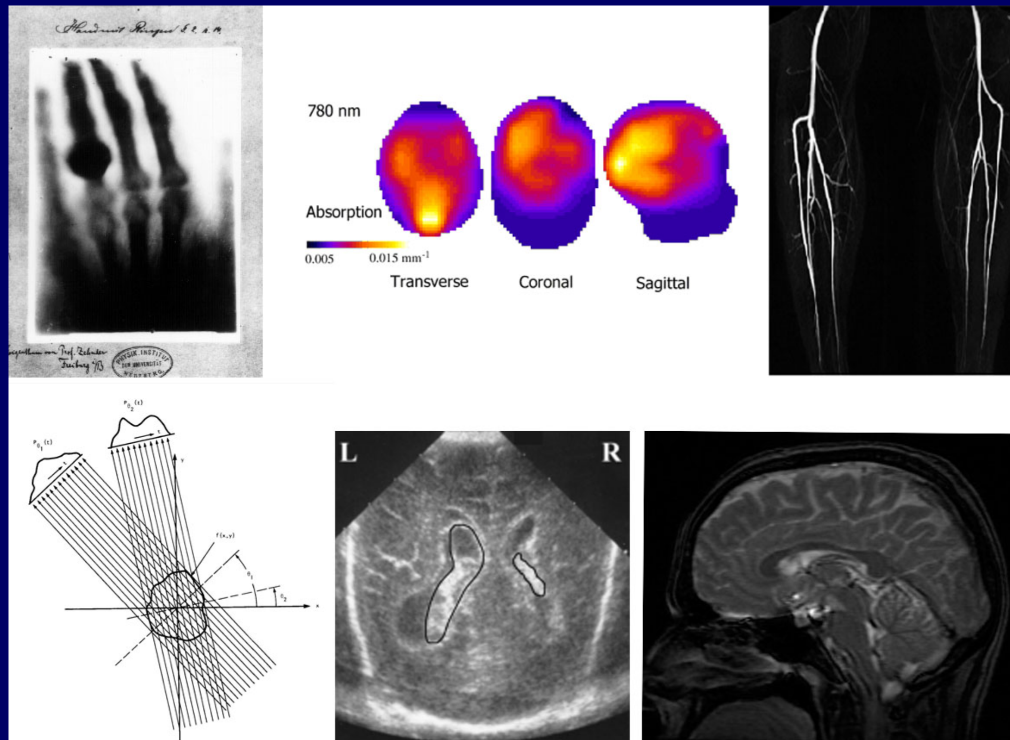
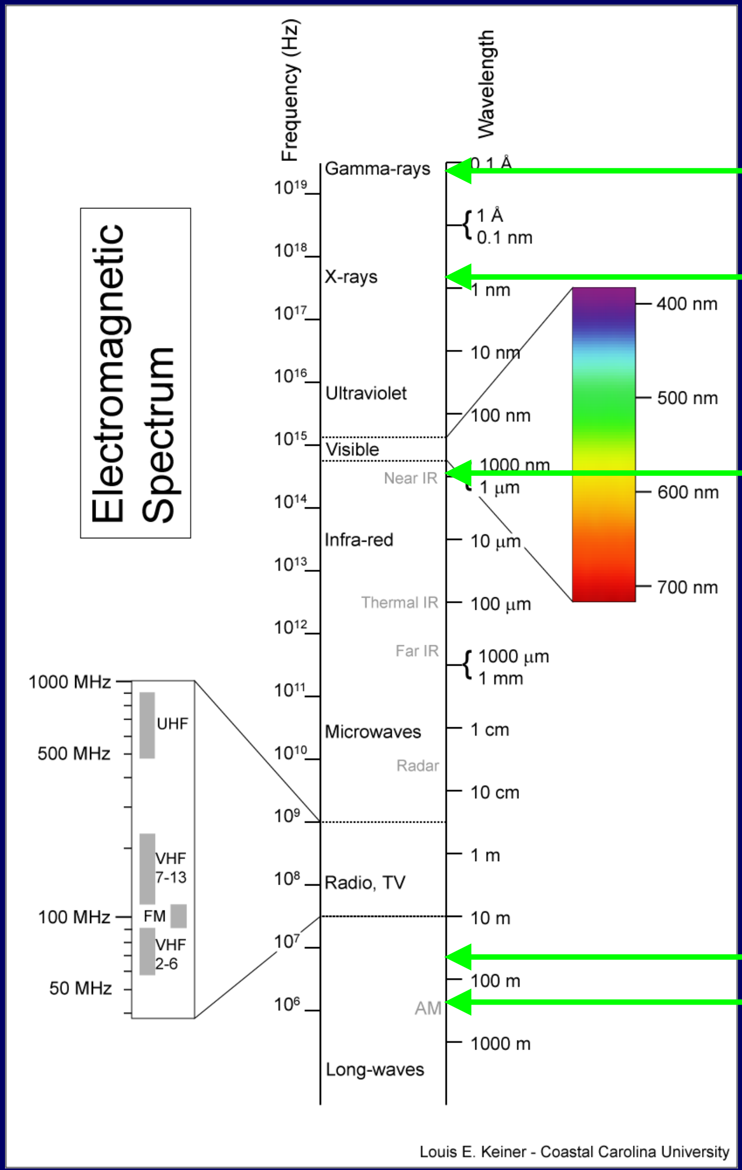
The Mathematics of Medical Imaging: What Is Essential is Invisible to the Eyes



Angel R. Pineda
Manhattan College
Riverdale, NY

Monmouth Math Colloquium
Nov. 6, 2015

Imaging Using Electromagnetic Waves



Nuclear Medicine: PET, SPECT

X-ray Imaging: Mammography, CT scans

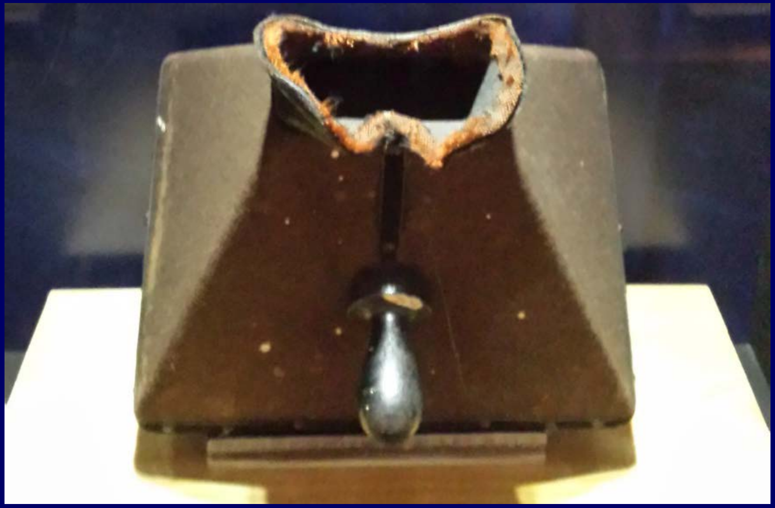
Diffuse Optical Imaging

Radio Frequency Excitation in MRI

Ultrasound

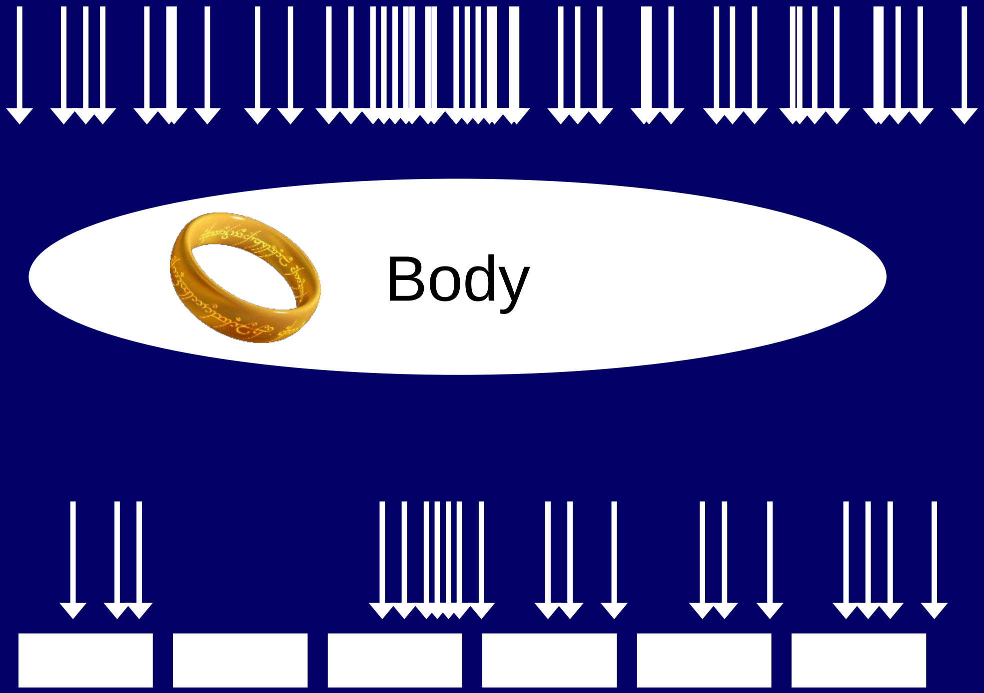
Nobel History of Medical Imaging

- In 1901 Roentgen gets first Nobel Prize in Physics for the discovery of x-rays. An immediate application is medical imaging.
- In 1979 Hounsfield and Cormack get Nobel Prize in Medicine for x-ray Computed Tomography (CT) which allows for cross-sectional imaging.
- In 2003 Lauterbur and Mansfield received the Nobel Prize in Medicine for magnetic resonance imaging (MRI).

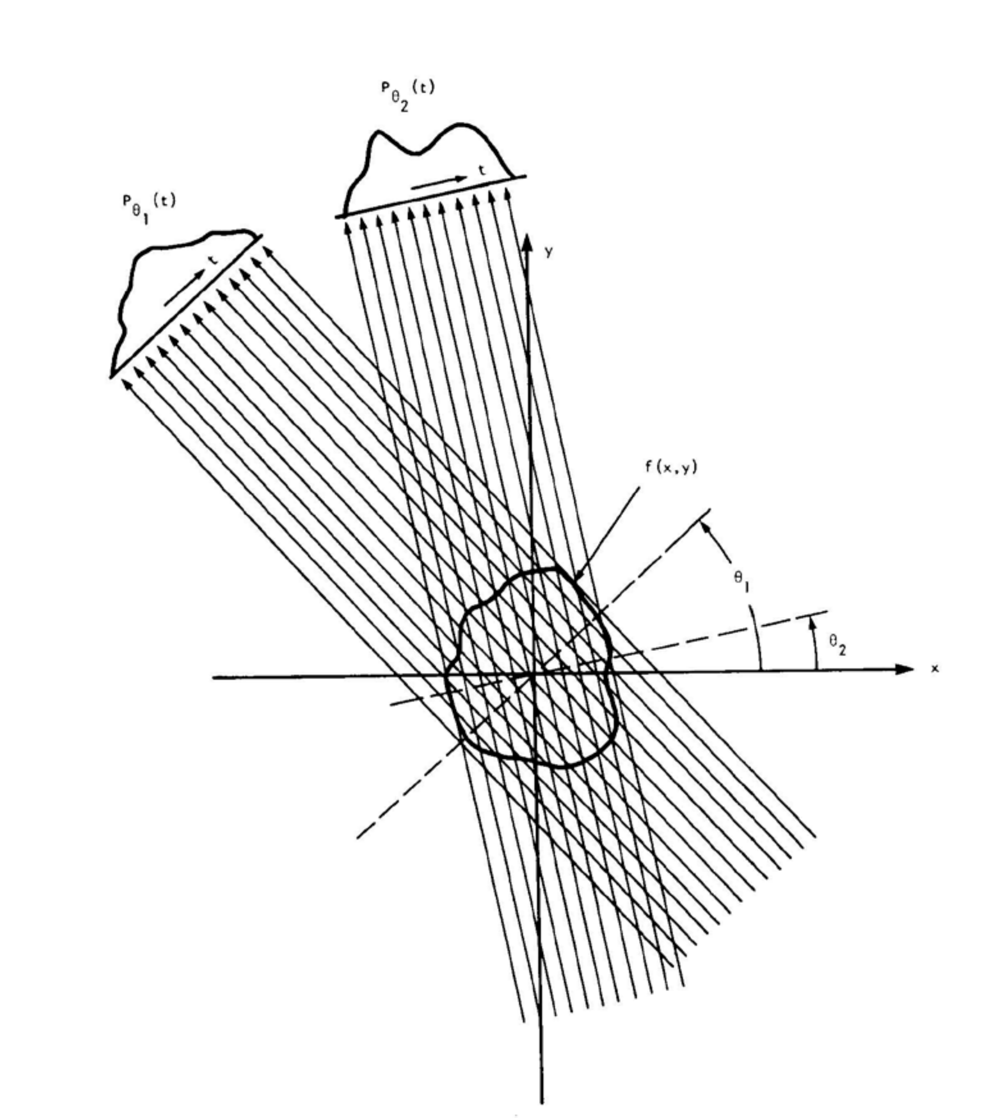


X-ray Imaging (Single Projection)

Image of Anna Bertha Ludwig's hand.
First x-ray image ever produced.



X-ray Projections (2-D Radon Transform)



$$P_\theta(t) = \int_{(\theta,t)\text{ line}} f(x, y)\, ds.$$

From Kak and Slaney, Principles of Computerized Tomographic Imaging

Inverting the Radon Transform

- Take the Fourier Transform of Projections
- Multiply by Judicious Weighting Function (Filter)
- Smear Back Filtered Projections (Backproject)
- Modern reconstruction in CT is based on Filtered Backprojection (Note: Hounsfield iteratively solved the linear system.)

Show MATLAB Simulation of CT

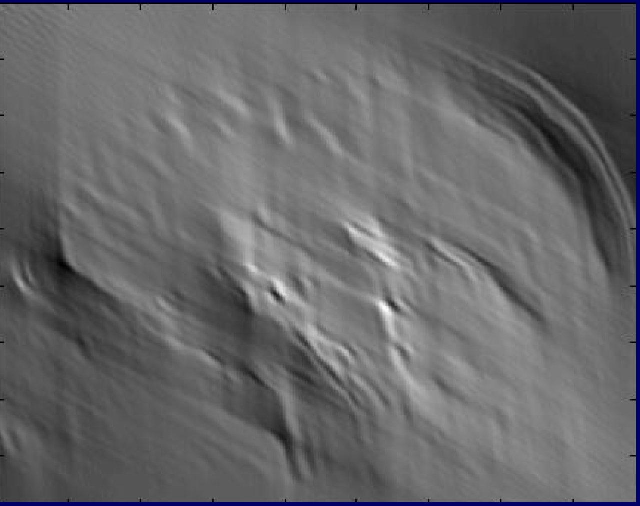


Image with 60 projections

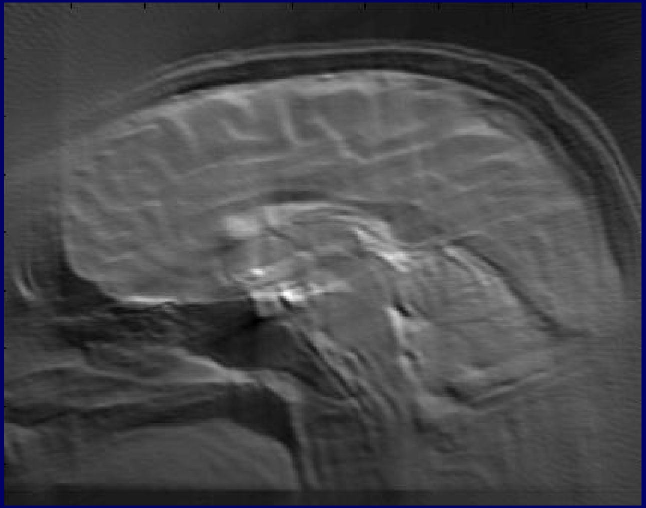


Image with 120 projections

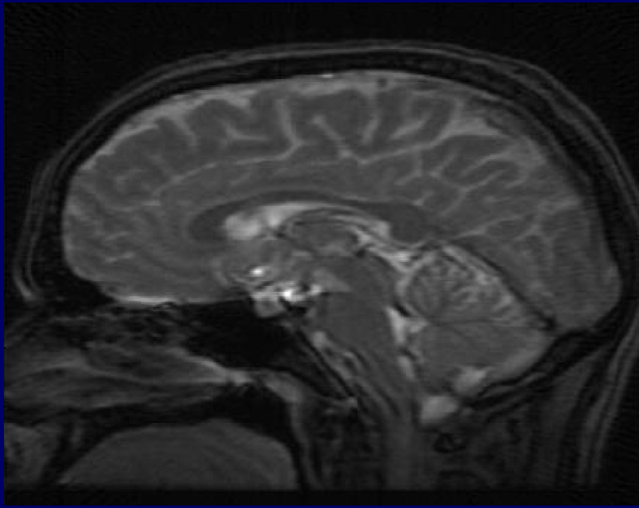
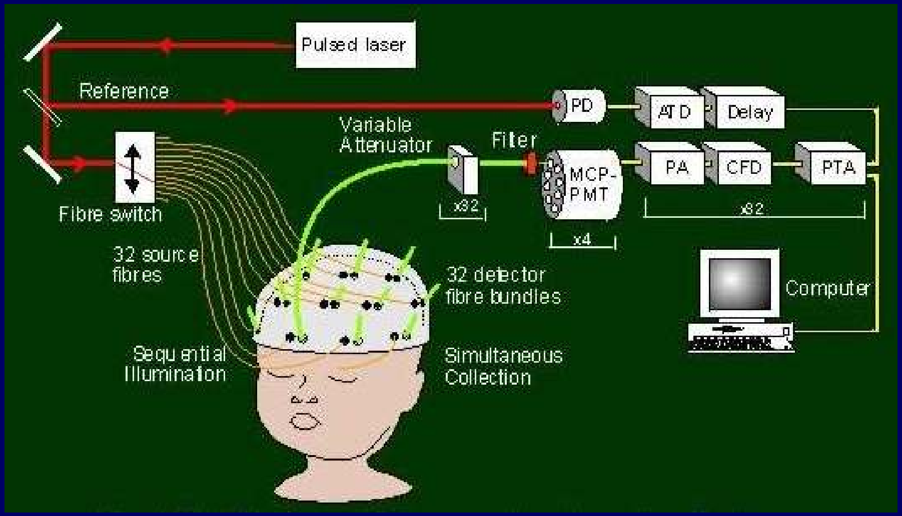


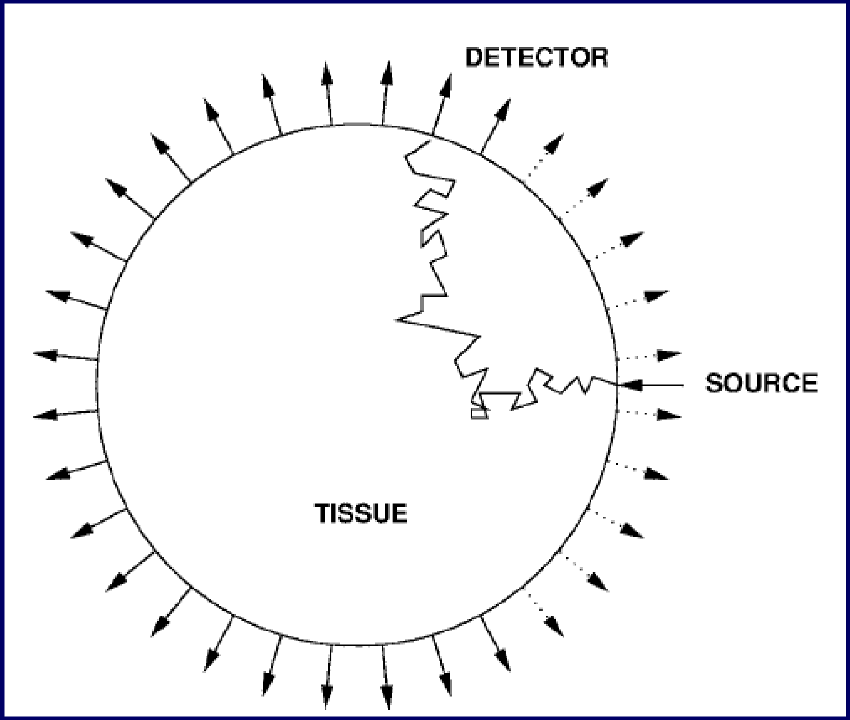
Image with 180 projections

Optical Tomography: The Instrument



MONSTIR
(**M**ulti-channel **O**pto-electronic **N**ear-infrared **S**ystem for **T**ime-resolved **I**mage **R**econstruction) at University College London.

Optical Tomography: The Model



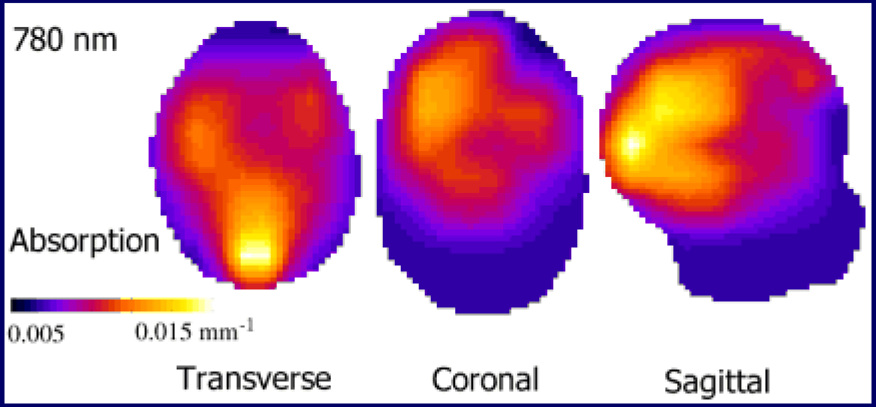
$$\frac{1}{c}\frac{\partial \Phi(\mathbf{r},t)}{\partial t} - \nabla \cdot k(\mathbf{r}) \nabla \Phi(\mathbf{r},t) + \mu_a(\mathbf{r})\Phi(\mathbf{r},t) = q_o(\mathbf{r},t)$$

Φ is the photon density,
μ_a is the attenuation coefficient,
q_0 is the source,
k is the diffusion coefficient and
c is the speed of light in the medium.

The optical parameters of tissue (k and μ_a) vary with blood volume and oxygenation.

Our goal is to estimate them from the boundary measurements.

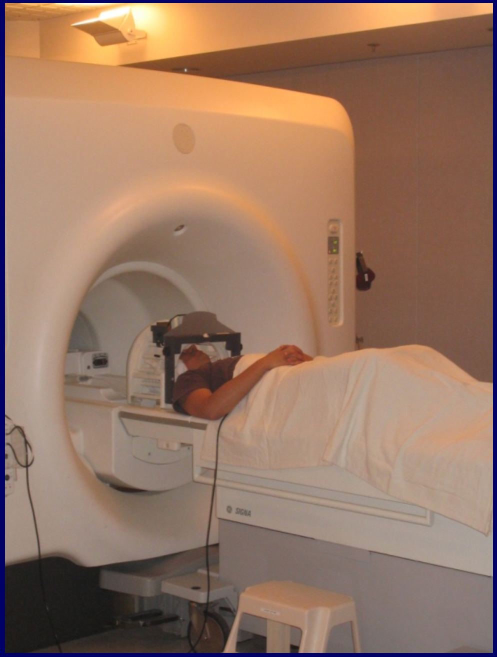
Optical Tomography: The Images

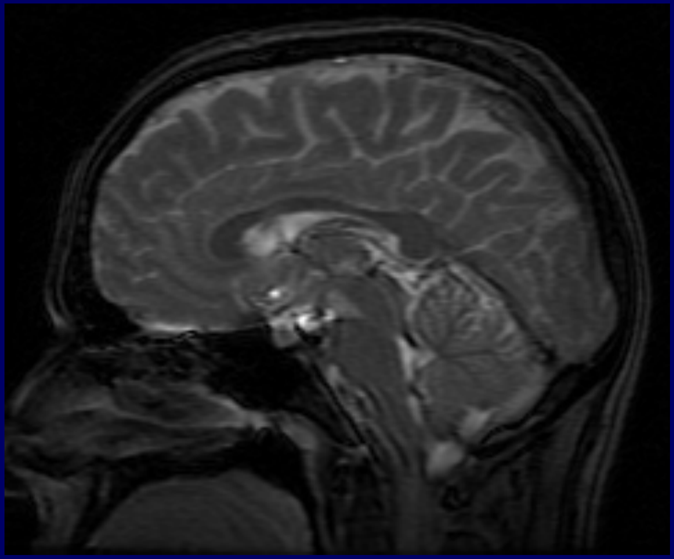


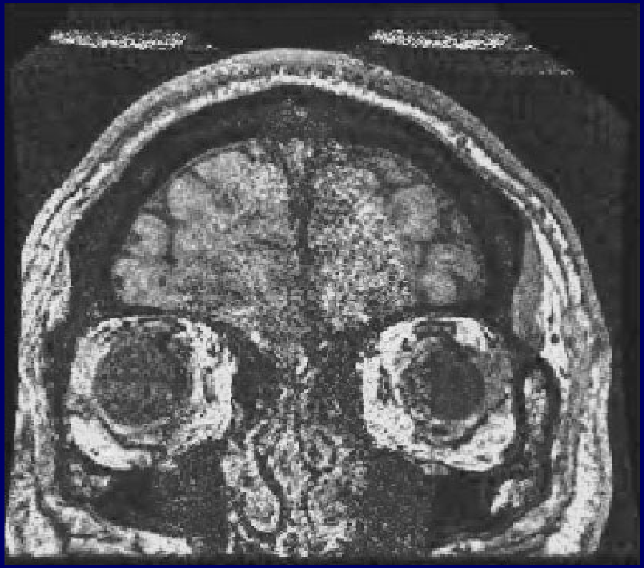
Optical Tomography

Ultrasound Image to Validate Result

Magnetic Resonance Imaging (MRI)

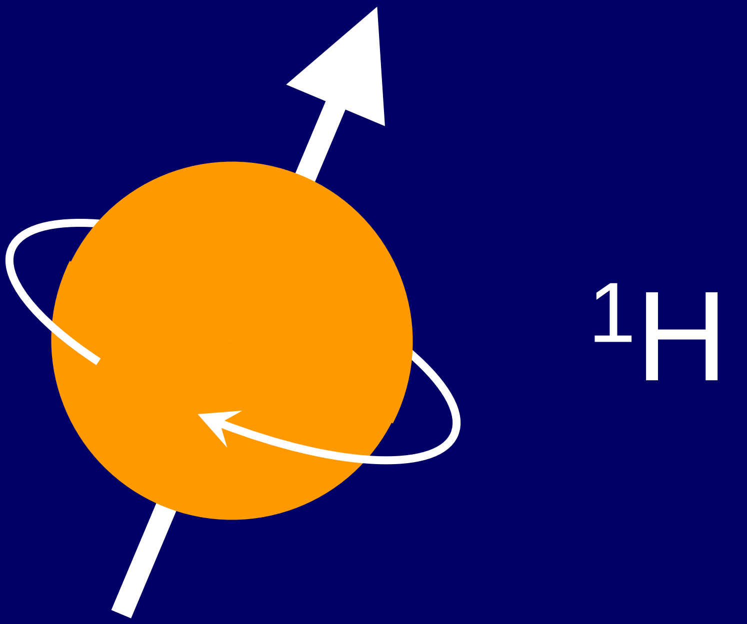






Magnetic Resonance

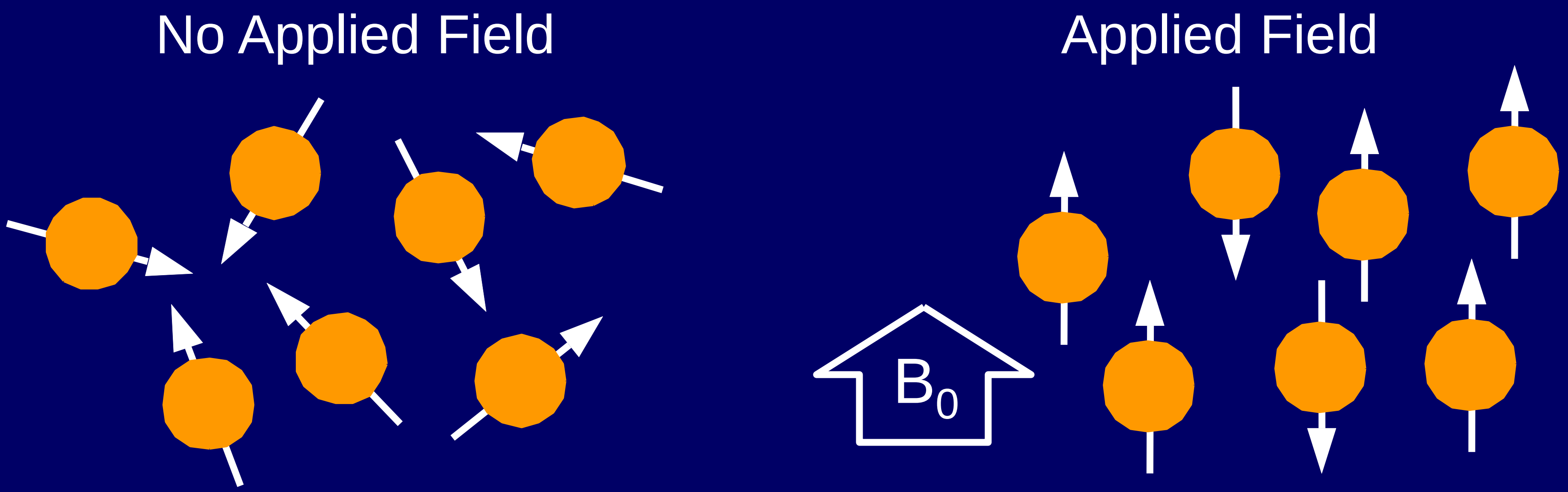
- Certain atomic nuclei including ^1H exhibit nuclear magnetic resonance.
- Nuclear "spins" are like magnetic dipoles.



MRI Introductory Slides Courtesy of Brian Hargreaves at Stanford University

Polarization

- Spins are normally oriented randomly.
- In an applied magnetic field, the spins align with the applied field in their equilibrium state.
- Excess along B_0 results in net magnetization.



How MRI works?

- By modifying the magnetic field we measure the Fourier Transform of the object being imaged.
- We reconstruct the images by applying the Inverse Fourier Transform.

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x k} dx$$

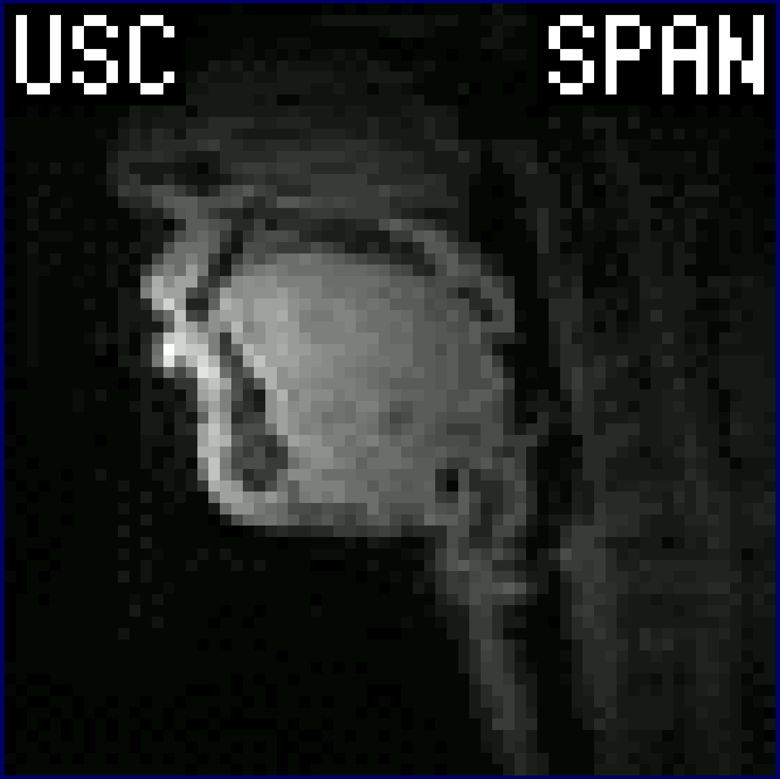
$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi i x k} dk$$

Statistical Estimation with Limited Data

- The less data (samples of the Fourier transform) one collects, the more sophisticated the mathematics required to create an image.
- An important application is time-resolved imaging in real time.
- Think in terms of constrained Least-Squares Estimation:

$$\hat{o} = \underset{o}{\operatorname{argmin}} \; \underbrace{(g - Fo)^2}_{\text{Data Agreement}} + \alpha \, \underbrace{|\nabla_t o|}_{\text{Total Variation Constraint}}$$

Accelerated MRI Movies



Hip Hop

Opera

Courtesy of Krishna Nayak at USC of
SPAN: Speech Production and Articulation kNowledge Group

Some of my Projects

- Noise Analysis of X-ray CT
- Chemical Species Separation in MRI
- Time-Resolved Angiography in MRI
- Uncertainty in Fat-Fraction Estimation using MRI

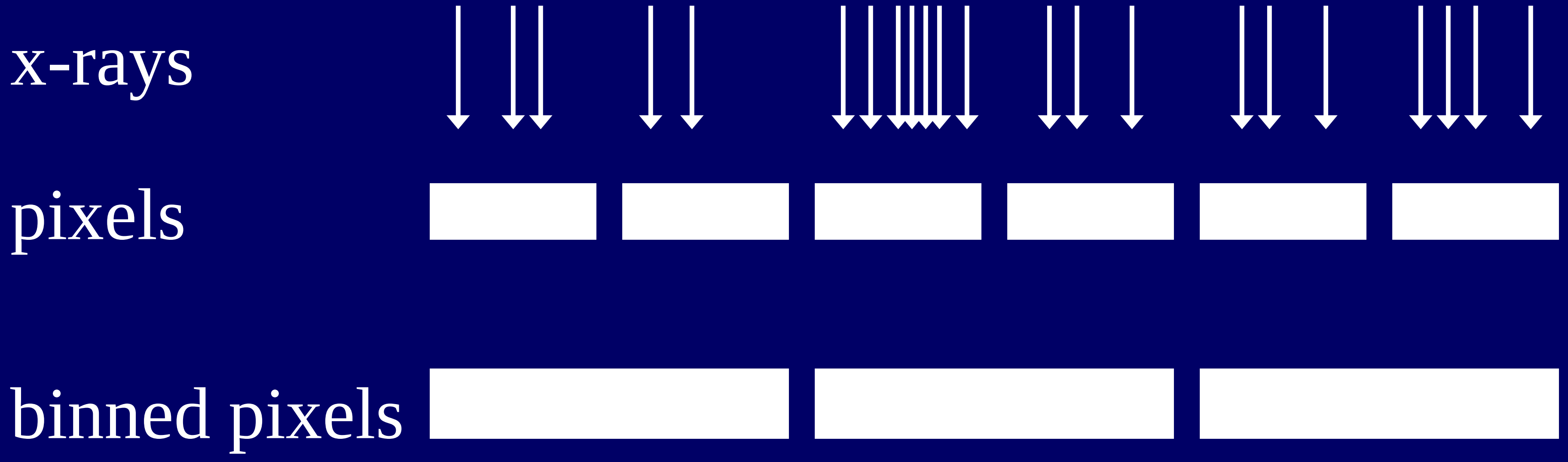
To Bin or Not to Bin in CT?

Collaboration with:

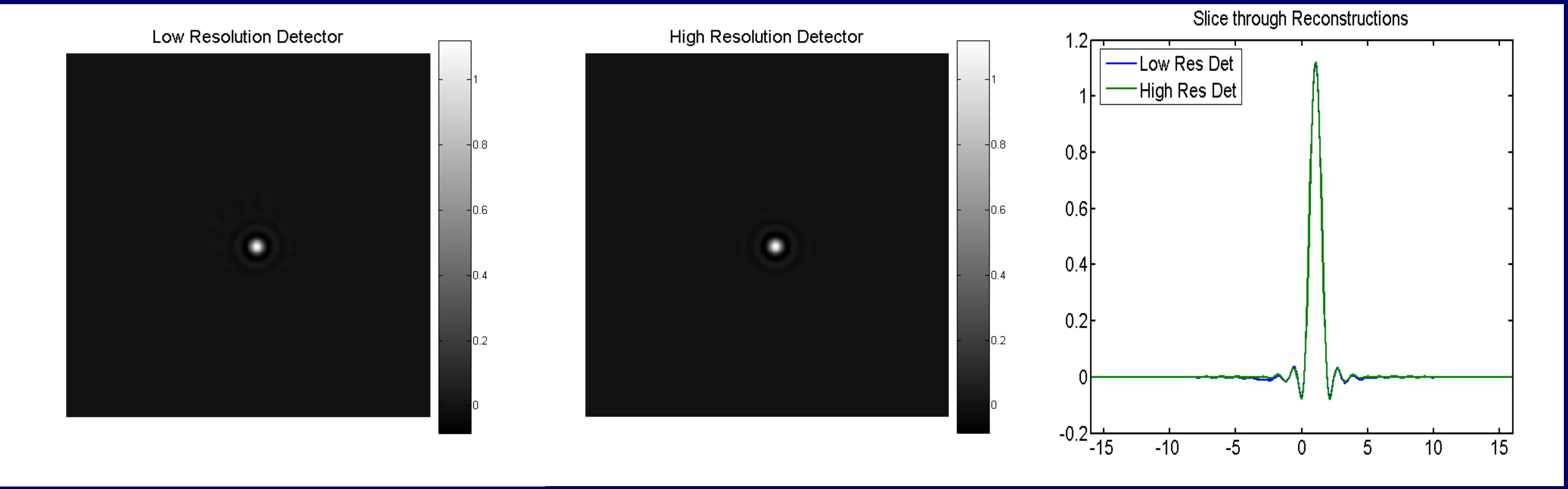
Norbert Pelc, Stanford University

Jongduk Baek, Yonsei University, Korea

Binning



Same Results for Noise-Free Data

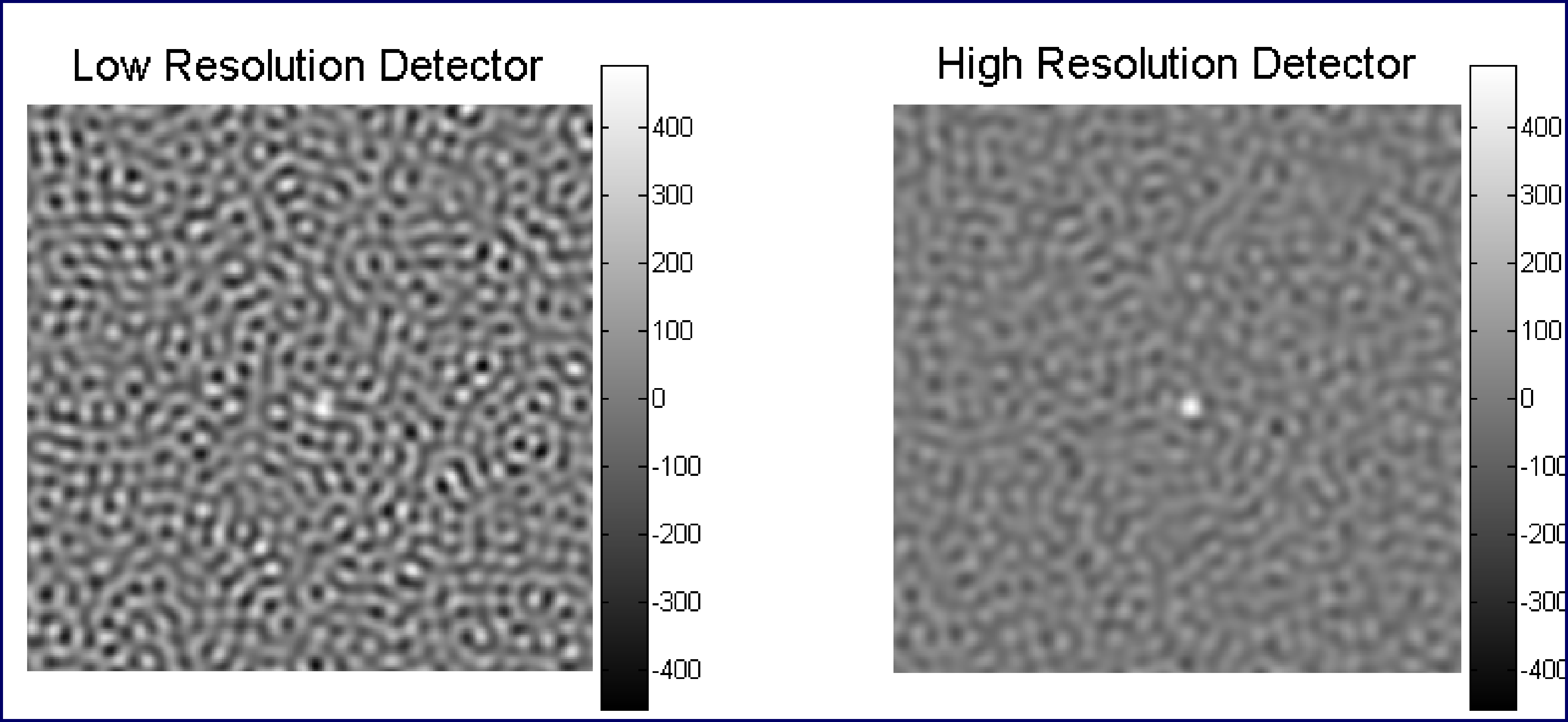


Reconstruction With Binned Data

Reconstruction With High Resolution Data

Slices of Both Reconstructions

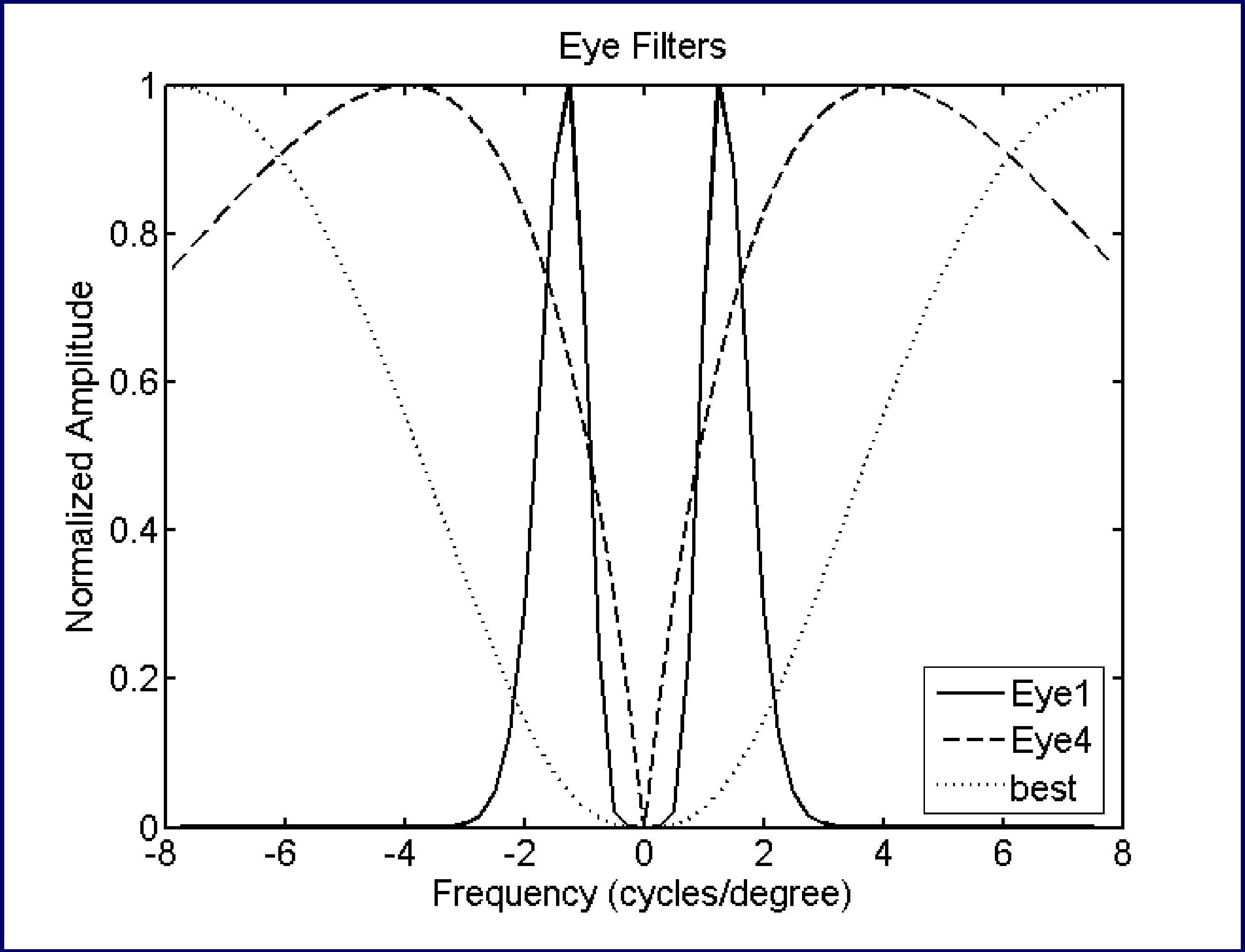
Different Results for Noisy Data!



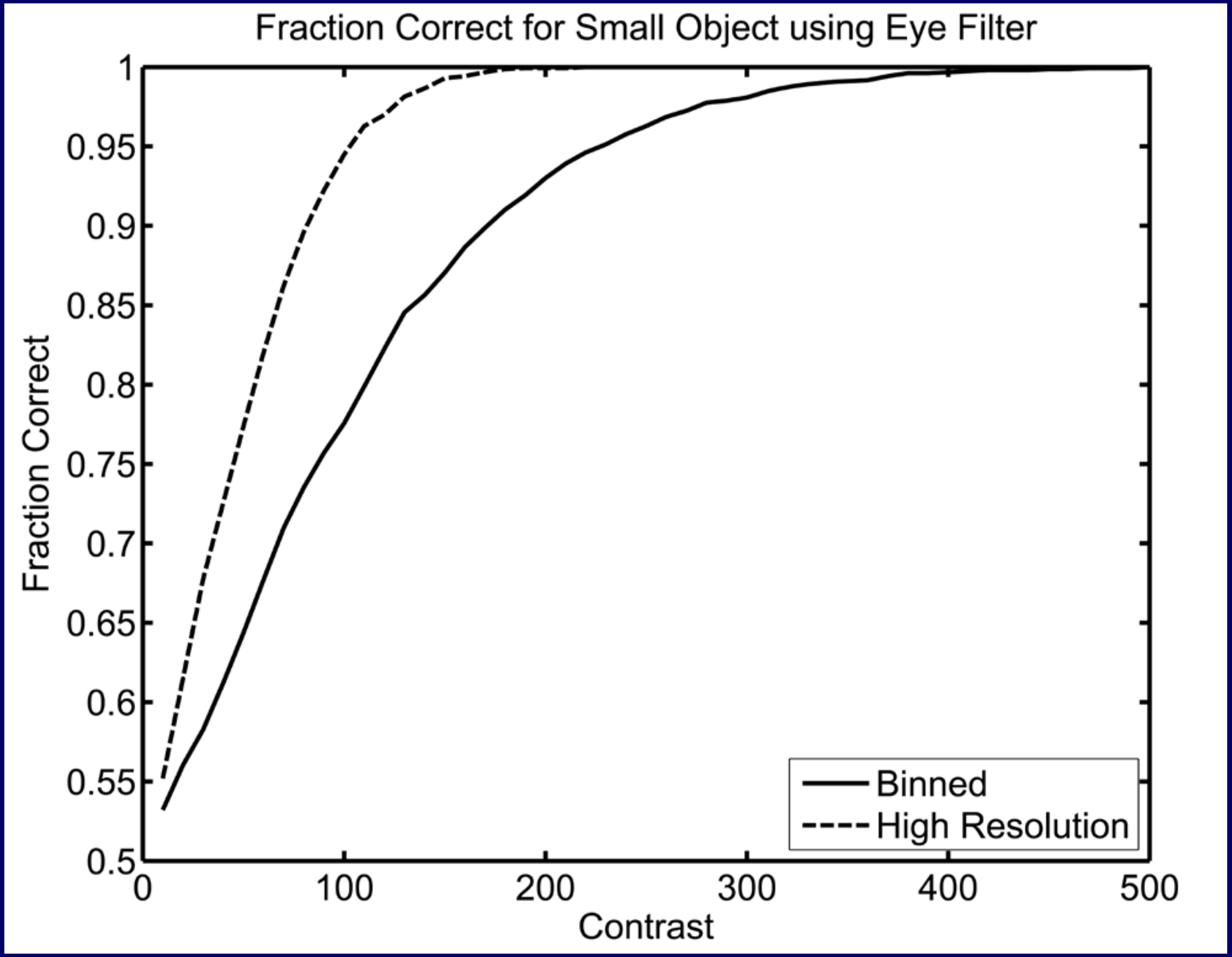
Reconstruction
With Binned Data

Reconstruction With
High Resolution Data

Eye Filters and Detection Theory



Results Small Object



Fat / Water Separation in MRI

Collaboration with:
UW Madison and GE Healthcare

CSUF Students Involved:
Emily Bice
Joaquin Alvarado

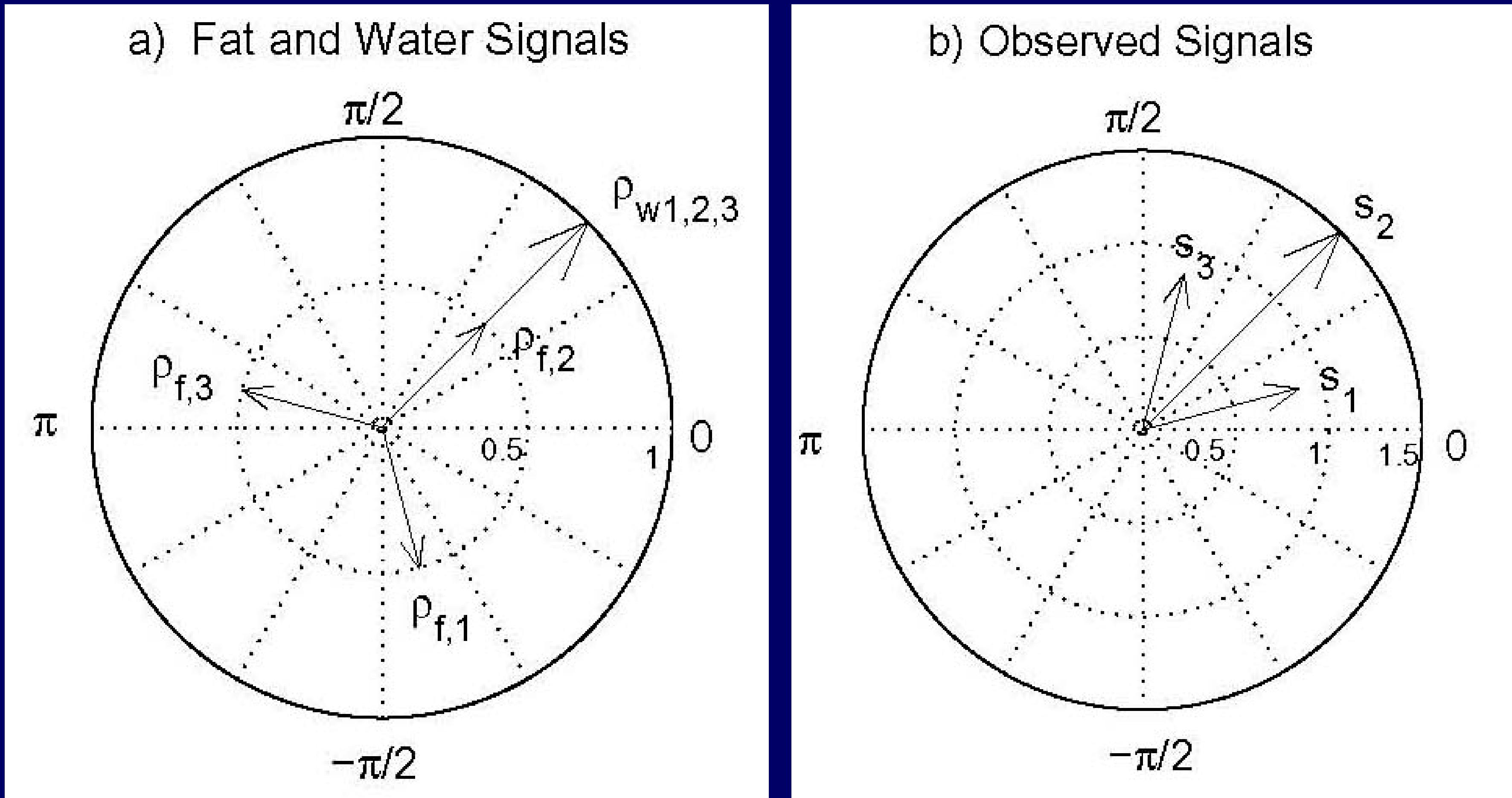
A Simple Model for the Signal

$$s(k) = \left[\rho_w \, e^{j\phi_w} + \rho_f \, e^{j\,(\phi_f + \Delta\phi_k)}\right] e^{j\psi TE_k} + \epsilon.$$

$s(k)$ is the k^{th} complex measurement,
ρ_w and ϕ_w are the magnitude and phase of water,
ρ_f and ϕ_f are the magnitude and phase of fat,
$\Delta\phi_k = \Delta w \, TE_k$ is difference in phase rotation,
TE_k is the time of the k^{th} image,
ψ is the field inhomogeneity,

and ϵ is the noise.

Geometric View of Data



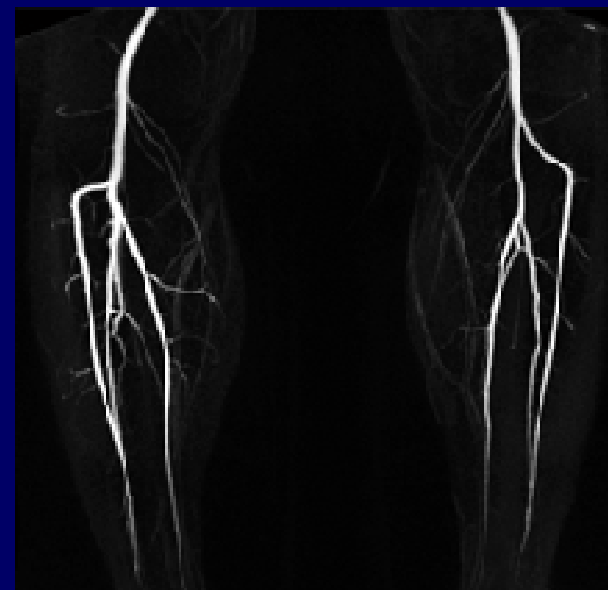
$\Delta\phi_1 = -\frac{2\pi}{3}, \Delta\phi_2 = 0, \Delta\phi_3 = \frac{2\pi}{3}$, i.e. $(-\frac{2\pi}{3}, 0, \frac{2\pi}{3})$

Theory and the Clinic Come Together!

Symmetric Acquisition Asymmetric Acquisition

HYPR Applied Math Project (for GE)

Students Not Pictured:
Nasser Abbasi and Siavash Jalal



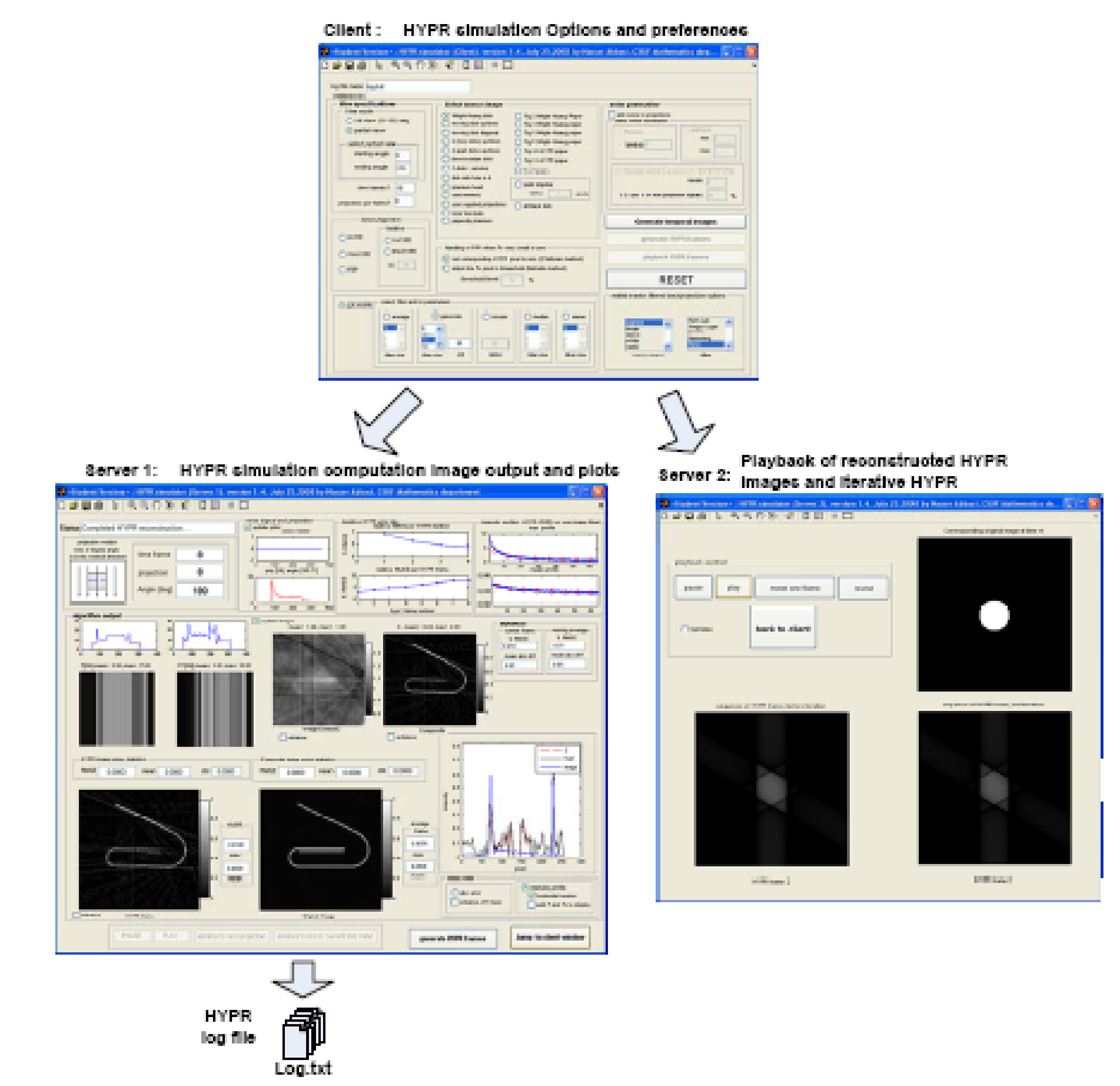
Leg Angiogram

Co-advisor: Bill Gearhart

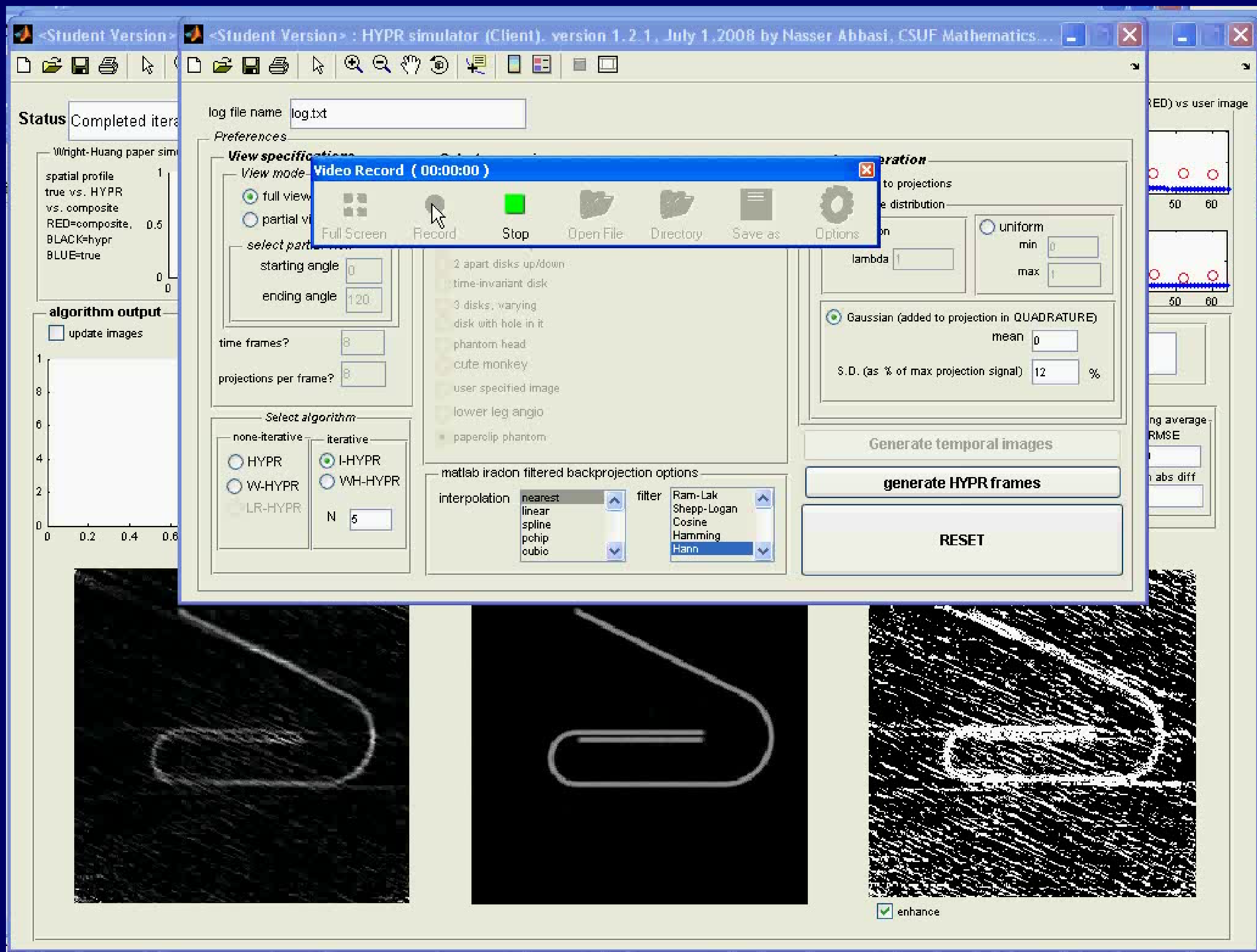
Explaining HYPR!

MLEM-1 Algorithm	HYPR
$g = Hf + \varepsilon$	$s_t = R_{\phi_n}\left[I_t\right]$
$f_n^{(1)} = f_n^{(0)} \frac{1}{z_n}\left[H^{\mathrm{T}}\left[\frac{g}{(Hf^{(0)})}\right]\right]_n$	$J_n = \frac{1}{N_p} C_n \cdot R_\phi^u \left(\frac{s}{R_\phi(C)}\right)_n$
H	R_ϕ – *Radon Transform*
H^T	R_ϕ^u – *Unfiltered backprojection*
z_n	N_p – *Projections per time frame*
g	s – *Original projection*
$f^{(0)}$	C – *Composite image*

The HYPR Simulator



The Movie



<Student Version> : HYPR simulator (Client). version 1.2.1, July 1.2008 by Nasser Abbasi, CSUF Mathematics...
Status
Completed itera
log file name
log.txt
Preferences
View mode
full view
partial vi
select part
starting angle
0
ending angle
120
time frames?
8
projections per frame?
8
Video Record (00:00:00)
Full Screen
Record
Stop
Open File
Directory
Save as
Options
2 apart disks up/down
time-invariant disk
3 disks, varying
disk with hole in it
phantom head
cute monkey
user specified image
lower leg angio
paperclip phantom
Select algorithm
none-iterative
HYPR
W-HYPR
LR-HYPR
iterative
I-HYPR
WH-HYPR
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5
matlab iradon filtered backprojection options
interpolation
nearest
linear
spline
pchip
cubic
filter
Ram-Lak
Shepp-Logan
Cosine
Hamming
Hann
to projections
distribution
uniform
min
max
lambda
Gaussian (added to projection in QUADRATURE)
mean
0
S.D. (as % of max projection signal)
12
%
Generate temporal images
generate HYPR frames
RESET
Wright-Huang paper sim
spatial profile
true vs. HYPR
vs. composite
RED=composite,
BLACK=hypr
BLUE=true
algorithm output
update images
ng average
RMSE
abs diff
enhance

CURM project

Student Collaborators:
Anne Calder
Eden Ellis
Li-Hsuan Huang
Kevin Park

Introduction

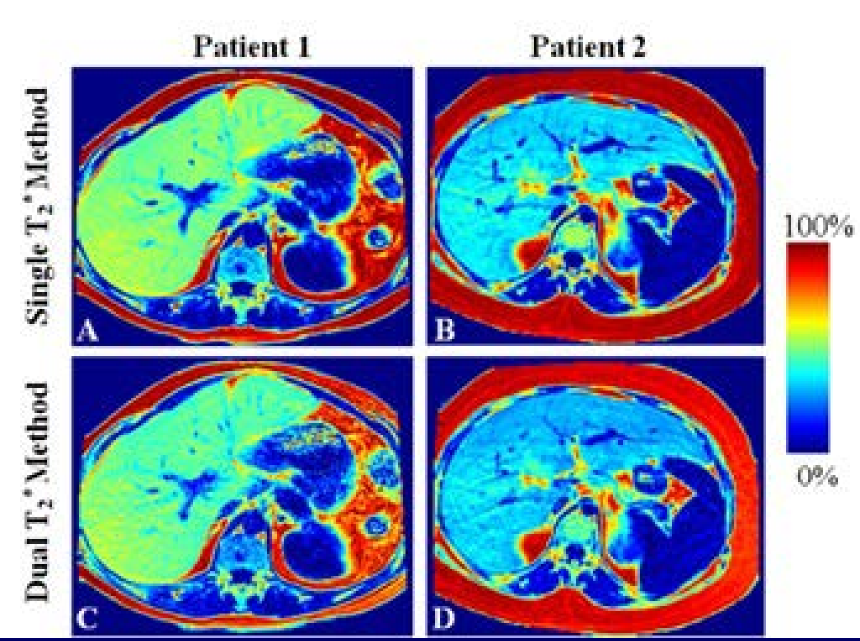
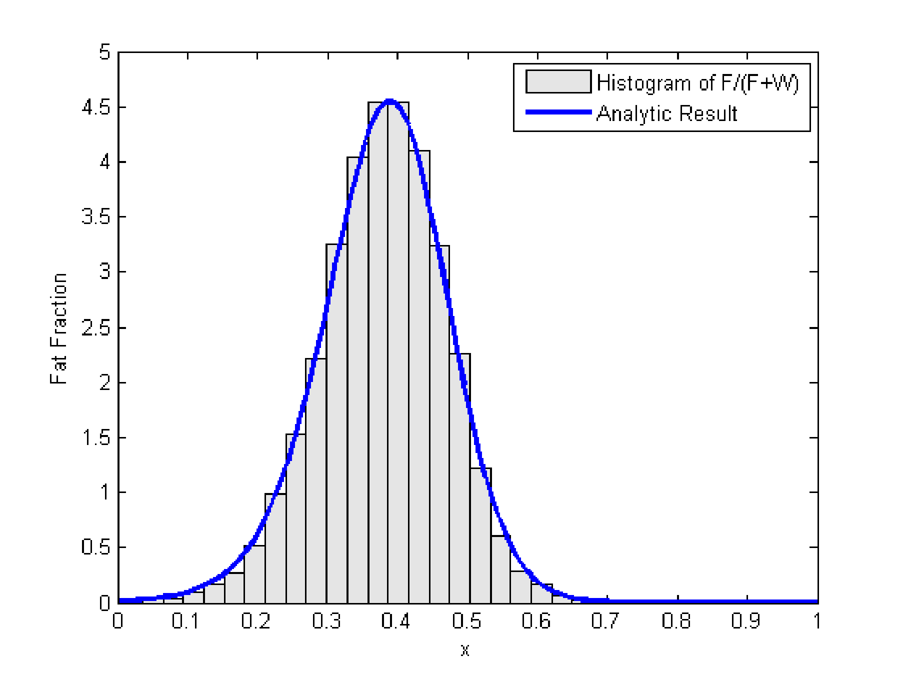


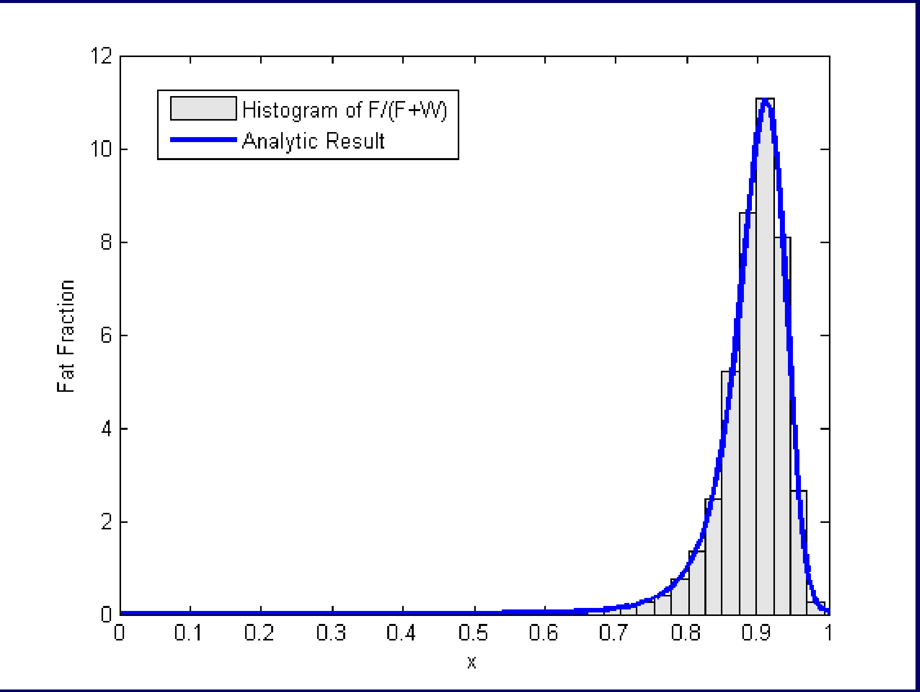
Image courtesy of Reeder, SB.

We want to characterize the uncertainty in measurements of the Fat-Fraction using MRI

$$\frac{|F|}{|W|+|F|}$$

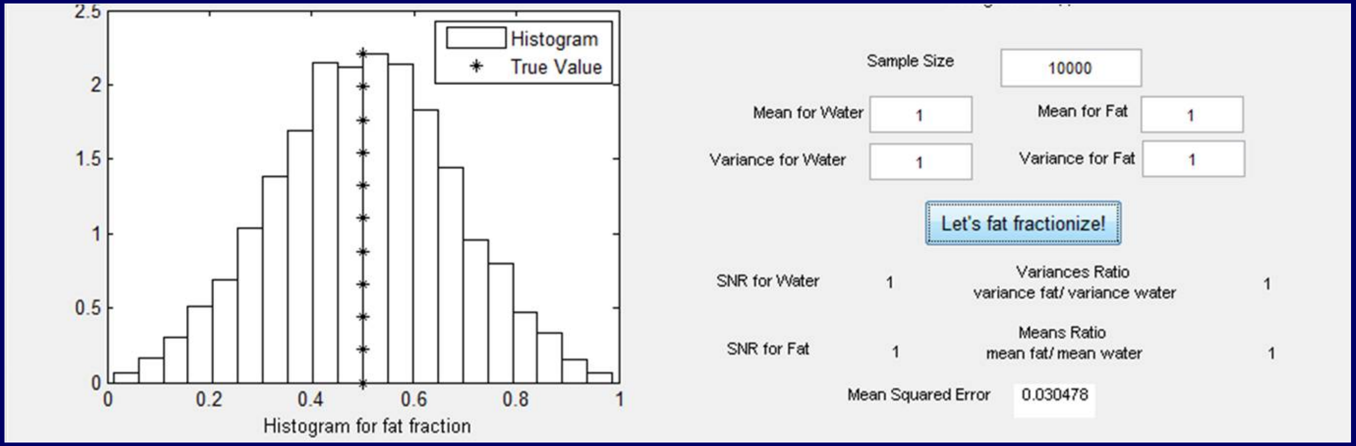
Theoretical Result



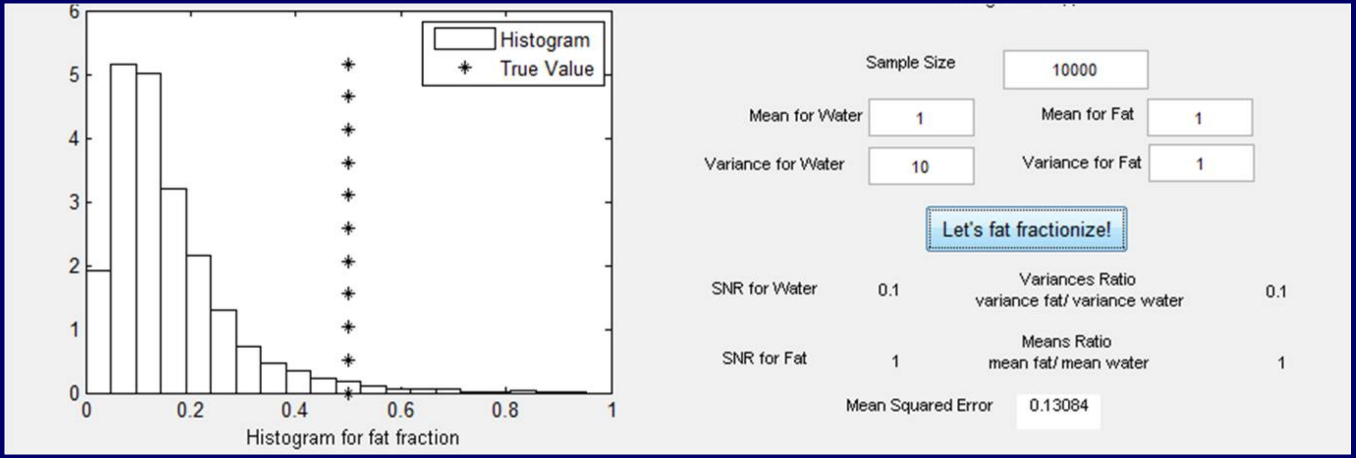


The probability density derived for the fat-fraction under the normal assumption matched simulation results: The Huang-Park Distribution!

Simulation Result



Accurate Estimation



Inaccurate Estimation

Looking Back…

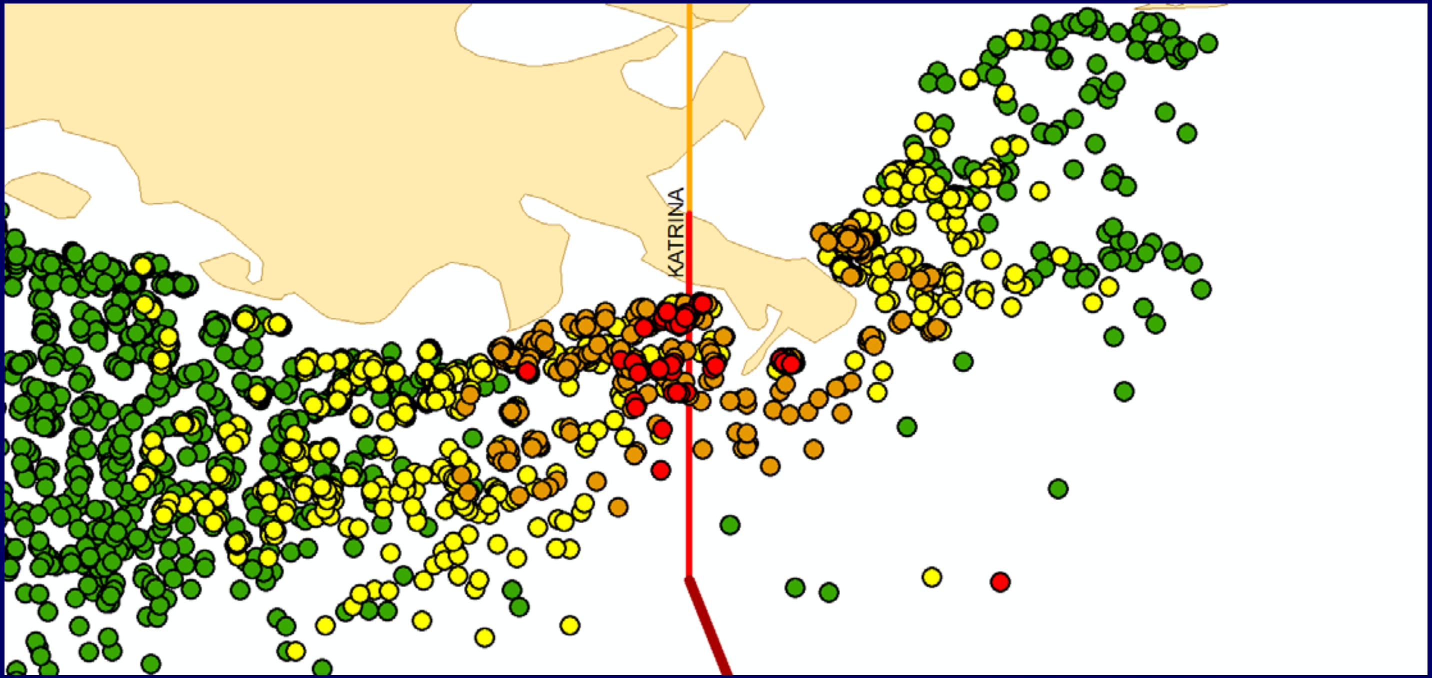
• Imaging data comes from a physical and biological process which we need to model. This is the “forward problem”.

• To create the images we see, we need to invert the process which created the image. This is the “inverse problem”.

• Mathematics, statistics and the sciences are naturally coupled in medical imaging.

Medicine and Hurricanes

Determining whether a patient has cancer from data is similar to deciding whether an oil platform will be destroyed by a hurricane (Applied Math Project with Earth Science Associates in 2015)



Acknowledgements and Support

Acknowledgements:

Norbert Pelc (at Stanford University)

Jongduk Baek (at Yonsei University)

Scott Reeder and Venkata Chebrolu (at UW Madison)

Jean Brittain and Huanzhou Yu (at GE)

John Grace and Scott Morris (at Earth Science Associates)

Financial Support:

CSUF, NSM, NIH, GE, CURM, LSAMP

Thank you… any questions?

The Usual Suspects…