p-values: a leading cause of the lack of replicability in Science?

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CRISIS? WHAT CRISIS? (Supertramp 1975)

Introductory words

The problem

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Testing and replicability

Suppose that we conclude that $\boldsymbol{x}$ provides strength evidence in favour of certain H. A replicability issue appears if H is not similarly endorsed by a repetition $\boldsymbol{x}^\star$ of the experiment.

Typical situation

Example: two treatments

If μ_i is the mean of time to recover of certain disease for treatment i then $H : \mu_1 - \mu_2 > 0$ express the hypothesis that treatment 2 is better than treatment 1 (in the sense that, in average, less time to recover is needed).

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• The above example represents a very typical situation where we have two hypotheses (one normally being the complement of the other) and we test one against the other

$$H_0 : \boldsymbol{\theta} \in \Theta_0, \;\; H_1 : \boldsymbol{\theta} \in \Theta - \Theta_0.$$

Hypothesis testing: revealing observations

1. Testing and model selection

Hypotheses define different statistical models (say f_H) with a common parametric form but differing in the location of the parameters.

For instance:

$$x \sim N(\mu, \sigma^2), \quad H_0 : \mu = 0, \; H_1 : \mu \neq 0$$

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Naive approach to testing:

To measure the degree of compatibility ("better fit") of hypotheses/models with data.

Drawback: lack of natural interpretation in terms of evidence.

2. Impossible theories

A particular (and unfortunately ubiquitous in applied studies) testing situation contains hypotheses where Θ_H consists on only one point $\boldsymbol{\theta}_0$ (e.g. $\mu_1 = \mu_2$, or $\beta = 0$). These are called *point* or *precise* hypotheses.

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• How to mix “better fit” and “complexity” we do not know but we all agree in

Ockham’s razor

For a similar fit choose the simplest explanation.

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There are limits even for statistics!

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p-value (p for short)=

the probability of obtaining the observed data, or more extreme, if the null hypothesis is true.

• Nothing wrong with p-value itself (it is just a number!) it is how we use and interpret it.

What we teach to our students

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- Researchers often wish to turn a p-value into a statement about the truth of a null hypothesis, or about the probability that random chance produced the observed data. The p-value is neither.

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- By itself, a p-value does not provide a good measure of evidence regarding a model or hypothesis.

The issues with p-values are not new

Many statisticians have alerted in the past about the dangers associated with significant testing and p-values:

Hogben 1957

We can already detect signs of such deterioration in the growing volume of published papers...recording so-called significant conclusions which an earlier vintage would have regarded merely as private clues for further exploration.

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J. O. Berger 2015

...few people actually understand what a p-value means; and the rampant misinterpretation of p-values is largely responsible for the well-documented lack of reproducibility of science.

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- A radical position is the one taken by the editors of *Basic and Applied Social Psychology* to ban p-values:

BASP editorial (2015)

...authors will have to remove all vestiges of the (null significance test procedure) NHSTP (p-values, t-values, F-values, statements about significant differences or lack thereof, and so on).

...we believe that the $p < .05$ *bar is too easy to pass and sometimes serves as an excuse for lower quality research.*

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Matthews n.d. (1998)

The plain fact is that 70 years ago R. Fisher gave scientists a mathematical machine for turning baloney into breakthroughs and flukes into fundings.

A high tendency to declare positives

Why are p-values so well-trained to declare positives? (or equivalently to reject null hypotheses). Three possible reasons:

- R1: Because of its definition,
- R2: because of the effect of n,
- R3: because of a distorted interpretation in frequentist terms.

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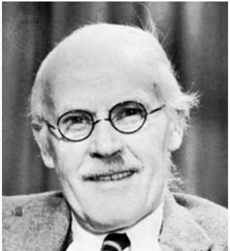

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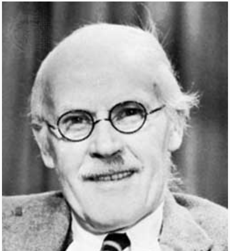

Jeffreys 1961 A hypothesis, that may be true, may be rejected because it has not predicted observable results that have not occurred.

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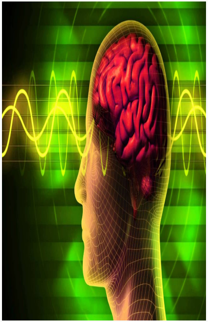

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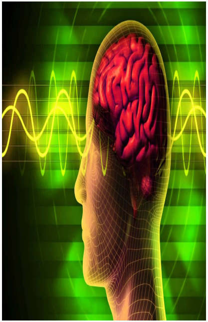

Parapsychology and extra sensory perception (ESP)

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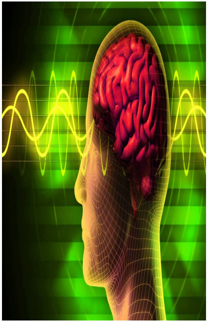

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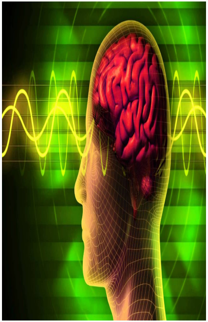
• An electronic device (REG) is arranged so that one gets a random sequence of 0s and 1s with theoretically equal probability for each outcome ($\pi = 0.5$).

• A subject then attempts to influence the REG and obtain a sequence of pulses with a distribution that is different from the baseline. In testing terms $H_0 : \pi = 0.5$ against the alternative $H_1 : \pi \neq 0.5$.



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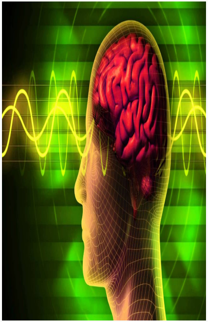

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- The experiment had $n = 104,490,000$ and they obtained $s = 52,263,471$ succeses implying an observed proportion of $\hat{\pi} = 0.5001768$.

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• Nevertheless, we can experiment these facts using a small program in R (adapted by Hector Perpian from Sellke, Bayarri, and J. O. Berger 2001)

Simulation

Many different experiments $D_1, D_2, \ldots, ..., D_L$ performed to test

$$H_0 : \theta = 0, \;\; H_1 : \theta \neq 0.$$

where θ is the mean of

$$x_1, \ldots x_n \sim N(\theta, \sigma^2).$$

Suppose H_0 is true with certain probability, and under H_1: $\theta \sim N(0, \sigma_P^2)$. Other possibilities:

- $\theta = a$ (fixed value),
- $\theta \sim Un(-a, a)$

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A main message here is that

> *Bayarri's talk; 2013 Knowing that data is "rare" under H_0 is of little use unles one determines whether or not it is also "rare" under H_1*

- In general, $p \approx 0.05$ is as rare under H_0 than under H_1.

p-values, alphas and a possible solution

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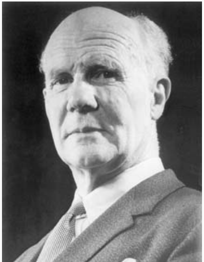

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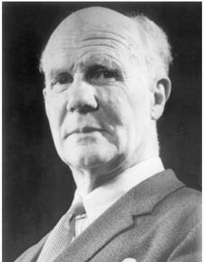

J. Neyman- E. Pearson significance testing

Construct a critical region (CR) subject to a prespecified type-I error rate α. If the sample falls in CR then reject H_0 and report α as the error measure.

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The p vaue can be used SIMPLY as a convenient tool to avoid the explicit construction of the critical region.

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- Fiher would report: $p = 0.021$,
- Neyman-Pearson would report $\alpha = 0.05$,
- Jeffreys would report $Pr(H_0 \mid data) = 0.28$.

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- So for instance, associated with $p = 0.05$, you obtain $\underline{\alpha} = 0.29$. This means that rejecting H_0 in experiments with $p \approx 0.05$ implies a type-I error probability of at least 29%.
- Alternatively, $\underline{\alpha}$ can be interpreted in a Bayesian fashion:

$$\text{Bayes Factor} = B_{A0} \leq \frac{1}{\underline{\alpha}} - 1, \text{ or equiv. if } Pr(H_0) = Pr(H_A), \text{ then } Pr(H_0 \mid data) \geq \underline{\alpha}$$

- In the example with $p = 0.05$ you can interpret $B_{A0} \leq 2.45$ or $Pr(H_0 \mid data) \geq 0.29$.

Almost finished!

Conclusion

The regular adoption of the "-eplog(p) rule" could be of much help in reducing the impact on replicability issues of p values since it dramatically disminishes their argued prediposition towards declaring positive findings.

References

- Agulló, J., V. Carratalá, and J. Gimeno (1999). *Inferencia estadística para economía y empresa*. Textos docentes. Publicaciones de la Universidad de Alicante.
- Berger, J. O. (2003). "Could Fisher, Jeffreys and Neyman have agreed on Testing?" In: *Statistical Science* 18, pp. 1–32.
- – (2015). "Invited contribution to: "BANNING NULL HYPOTHESIS SIGNIFICANCE TESTING"". In: *The ISBA Bulletin* 22.1, p. 5.
- Berger, James O. and T. Sellke (1987). "Testing a Point Null Hypothesis: The Irreconcialability of P-values and Evidence". In: *Journal of American Statistical Association* 82, pp. 112–122.
- Hogben, L. (1957). *Statistical theory*. Read Books.
- Hubbard, R. and M. J. Bayarri (2003). "Confusion over measures of evidence (p's) versus errors (alpha's) in classical statistical testing". In: *The American Statistician* 57.171-182.
- Jefferys, W.H. (1990). "Bayesian Analysis of Random Event Generator Data". In: *Journal of Scientific Exploration* 4.2, pp. 153–169.

Bibliography II

- Jeffreys, Harold (1961). *Theory of Probability*. 3rd. Oxford University Press.
- Johnson, R.A. and G.K. Bhattacharyya (1992). *Statistics Principles and Methods (2nd ed)*. John Wiley & Sons.
- Lang, J., K. Rothman, and C.I. Cann (1998). "That confounded p-value". In: *Epidemiology* 9.1, pp. 7–8.
- Matthews, R. *http://www2.isye.gatech.edu/ brani/isyebayes/bank/pvalue.pdf*.
- Sellke, T., M. J. Bayarri, and J. O. Berger (2001). "Calibration of P-values for Testing Precise Null Hypotheses". In: *The American Statistician* 55, pp. 62–71.
- Velez, R. and A. García (1993). *Principios de inferencia estadística*. UNED.