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Research on Optimization of Vehicle Routing Problem for Ride-sharing Taxi

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Abstract

In order to improve transportation efficiency of taxi and mitigate urban traffic congestion, it is necessary to carry out ride-sharing strategy which contributes to reducing operating cost and saving road resources. This study focuses on the routing optimization of ride-sharing taxis, in which interests of both taxi drivers and passengers are taken into account. Minimization of operating cost and maximization of customer satisfaction are considered as the objective, and travel mileage, waiting time and extra riding time due to ride-sharing are used to quantify them respectively. Routing optimization model for ride-sharing taxis is established and then appropriate simulated annealing algorithm satisfying constraints of the model proposed is designed. At last, a computational experiment is conducted to verify the model and the result shows that this method is able to save 19% mileage as well as 66% taxis available.

Keywords: ride-sharing taxi; dial-a-ride problem; routing optimization; simulated annealing

1. Introduction

Taxis are able to satisfy passengers' personalized requirements and provide passengers with convenient, comfortable and prompt trips. However, taxis often employ a high unloaded ratio and make considerable demands on limited road resources. In order to improve operational efficiency of taxis, it is necessary to encourage the wide spread of ride-sharing services. Effective shared ride service needs careful organization, optimal scheduling and reliable management, and vehicle routing is one of crucial

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aspects for ride-sharing taxi in practice. Taxi is a typical representative of the DRT (Demand Responsive Transit) operating mode, and the scheduling of ride-sharing taxi can be seen as the specific practical application of dial-a-ride problem (DARP). Ride-sharing taxi's scheduling problem discussed in this article is a class of static multiple-vehicle DARP.

Much work has been reported recently in related fields. Psaraftis et al. (1980) studied the "immediate-request" case, and established a model aimed at minimizing route completion time and customer dissatisfaction. However, time windows were not taken into consideration. On the basis of study of Psaraftis et al. (1980), Desrosiers et al. (1986) proposed an improved labeling scheme, and then instances with up to 40 users could be solved. Jaw et al. (1986) studied multiple-vehicle DARP with pickup and drop-off time windows, and vehicles are not allowed to be idle when carrying passengers in the case. Oli B.G. Madsen et al. (1995) described a system to solve a static dial-a-ride routing and scheduling problem with time windows. The problem is characterized by multiple capacities and multiple objectives. Fu Liping (2002) discussed a study on the dial-a-ride paratransit scheduling problems subject to tight service time constraints and time-varying, stochastic traffic congestion, and explicitly incorporated a time-dependent, stochastic travel time model in the problem formulation. An insertion algorithm capable of handling large-scale instances was proposed. Cordeau (2003) adopted some known inequalities for the TSP or VRP, and designed a branch-and-cut algorithm for the solution. Cordeau and Laporte (2003) described and classified main features of DARP, and summarized some most critical algorithms as well. Marco Diana et al. (2004) implemented a new route initialization procedure, in which both the spatial and the temporal aspects of the problem were taken into account. Emanuel Melachrinoudis et al. (2005) proposed a double request dial-a-ride model with soft time windows and applied the model to address the CAB (Center for Addictive Behavior Health and Recovery Services, Inc.) clients' transportation needs. A regret insertion is then performed to serve the remaining requests. Qin Yun-mei et al. (2006) investigated the shared ride modes, including the static ride-sharing and the dynamic one. They proposed the vehicle routing model under static ride-sharing and analyzed the computational method of ride-sharing cost. Qin Yun-mei et al. (2008) studied the model to obtain the fare of each passenger under dynamic ride-sharing. Liu Yao-xia et al. (2008) and Zheng Li-juan (2010) explored the way of carrying out ride-sharing taxi, and elaborated significance of implementing shared ride and some safeguards needed. Wu Fang et al. (2009) developed the optimal vehicle routing and scheduling model for ride-sharing taxis, and designed improved Particle Swarm Optimization (PSO) algorithm. Zhou He-ping et al. (2011) explored the vehicle routing and fare rate optimization for ride-sharing, and then developed corresponding genetic algorithm.

By reviewing researches correlated, it is concluded that a considerable amount of research on DARP models and corresponding algorithms has been done. However, studies specified on optimization of vehicle routing problem for ride-sharing taxi are scarce. In this paper, it is attempted to take the minimization of operating cost and the maximization of customer satisfaction as the goal in the model and then routing optimization model is established considering specific characteristics of ride-sharing taxi.

The remainder of the article is organized as follows. The next section establishes the routing optimization model for ride-sharing taxi. Section 3 develops corresponding simulated annealing algorithm to solve the model proposed. Computational experiment is presented in Section 4 and the conclusion follows in Section 5.

2. Model development

Generally speaking, passengers hope to take vehicles conveniently and quickly with least cost, whereas taxi drivers pay more attention to achieve higher income. The two objectives are usually conflicting. Reasonable routing assignment is of great importance to reach holistic optimization and improve the transportation efficiency. Therefore, the benefits of drivers and passengers should be considered in a comprehensive way for routing optimization of ride-sharing taxi. In the routing model, it is should be ensured that taxis are attractive to passengers, while taxi driver's income are guaranteed at the same time. Accordingly, this study aims to minimize operation cost and maximize passenger satisfaction as well. In the model proposed, vehicles travel smoothly and keep a constant speed; relevant information is known in advance, such as the number of requests, origin and destination (O-D) and pickup time windows; vehicles travel along the shortest path between any two requests.

2.1. Model parameters and the notation

x_{ij}^m is a binary variable: if the m th vehicle goes straight from request i to j , x_{ij}^m =1, or else x_{ij}^m =0. B_i^m denotes the time when the m th vehicle starts visiting node i . Q_i^m means the load of the m th vehicle after visiting node i . L_i^m represents the ride time of request i on the m th vehicle. The set of vehicles is defined as M . Let n be the number of requests to be served. Let K ($K = \{0,1,\ldots k\ldots\}$) denotes the set of all passengers since request i may contain more than one passenger. This problem can be defined on a complete directed graph $G = (N, A)$ where $N = P \cup D \cup \{0,2n+1\}$, $P = \{1,\ldots,n\}$ and $D = \{n+1,\ldots,2n\}$, while nodes 0 and $2n+1$ represent the origin and destination of the route. Subsets P and D contain pick-up and drop-off nodes, respectively. Taxis are equipped with an identical capacity Q . Every node i associates a amount load on the m th vehicle q_i^m and a non-negative service duration s_i .[e_i, l_i]defines the pickup time windows associated with node i , where e_i and l_i represent separately the earliest and latest time. The maximum travel time for the m th vehicle to complete all tasks on each route is T_m . Each edge (i, j) associates with a shortest travel time t_{ij} . L implies the maximum ride time of a passenger. It is calibrated as $L = (1+r)t_{ij}$, and r is the rate of acceptable detour time proportional to t_{ij} . w_k denotes waiting time of the k th passenger $(k \in K)$. Finally, the weight coefficients of operation cost, extra riding time results from ride-sharing and waiting time are stated as α_1 , α_2 and α_3 respectively.

2.2. Mathematical formulation

The objective of the proposed model is to minimize total operation cost as well as maximize passenger satisfaction under the condition that the needs of all passengers and all relative constraints are satisfied.

(1) Operation cost

The cost of operating ride-sharing taxi comprises: transportation cost, maintenance expense, as well as the taxi driver's salary etc.. This study focuses solely on the total transportation cost which is associated with travel mileage. On the supposition that transportation cost per mile is constant and that taxis travel at a constant speed, transportation cost concerns with travel time merely. Thus, the operation cost can be expressed as:

$$Z_1 = \sum_{m \in M} \sum_{i \in N} \sum_{j \in N} t_{ij} x_{ij}^m \tag{1}$$

(2) Passenger satisfaction

Passenger satisfaction is proportional to the service level. The higher service level is, the higher passenger satisfaction will be. The quality of taxi service can be measured in the following facets: service reliability (such as service availability and time reliability), comfort, security, convenience of making reservations, etc.. Some indicators are qualitative, while others are not. The reliability of the service is the easiest to quantify, therefore, this study will make use of passenger travel time to reflect passenger satisfaction. Considering operation characteristics of ride-sharing taxi, passenger satisfaction can be expressed with extra riding time Z_2 caused by ride-sharing and waiting time Z_3 :

$$Z_2 = \sum_{m\in M}\sum_{i\in N}\sum_{j\in N} Q_i^m (B_j^m - B_i^m - t_{ij}) \quad (2)$$

$$Z_3 = \sum_{k\in K} w_k \quad (3)$$

Therefore, ride-sharing taxi routing optimization model can be expressed as follows:

$$\begin{aligned} & Min\ \ Z = Min\ \ (\alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_3) \\ = Min\ \ & \sum_{m\in M}\sum_{i\in N}\sum_{j\in N}[\alpha_1 t_{ij} x_{ij}^m + \alpha_2 Q_i^m (B_j^m - B_i^m - t_{ij})] + \alpha_3 \sum_{k\in K} w_k \end{aligned} \quad (4)$$

s.t.

$$e_i \le B_i^m \le l_i\ , \forall i \in N\ , \forall m \in M \quad (5)$$

$$B_{2n+1}^m - B_0^m \le T_m\ , \forall m \in M \quad (6)$$

$$B_j^m \ge (B_i^m + s_i + t_{ij}) x_{ij}^m\ , \forall i \in N\ , j \in N\ , \forall m \in M \quad (7)$$

$$L_i^m = B_{n+i}^m - (B_i^m + s_i)\ , \forall i \in P\ , \forall m \in M \quad (8)$$

$$t_{i,n+i} \le L_i^m \le L\ , \forall i \in P\ , \forall m \in M \quad (9)$$

$$Q_j^m \ge (Q_i^m + q_j^m) x_{ij}^m\ , \forall i \in N\ , j \in N\ , \forall m \in M \quad (10)$$

$$\max\{0, q_i^m\} \le Q_i^m \le \min\{Q, Q + q_i^m\}\ , \forall i \in N\ , \forall m \in M \quad (11)$$

$$\sum_j x_{ij}^m = 1\ , \forall i \in P\ , j \in N\ , \forall m \in M \quad (12)$$

$$\sum_j x_{ij}^m = \sum_i x_{ij}^m\ , \forall i \in P \cup D\ , j \in N\ , \forall m \in M \quad (13)$$

$$x_{ij}^m \in \{0,1\}\ , \forall i \in N\ , j \in N\ , \forall m \in M \quad (14)$$

The objective function (4) minimizes the weighted sum of total travel time, extra riding time and passenger waiting time. Weights α_1, α_2 and α_3 vary from case to case. Constraint (5) defines the pickup time windows. Constraint (6) bounds the duration of each route. The total travel time from origin of the first request to destination of the last one in the same route cannot exceed T_m. Vehicles at request i should not arrive j after B_j, which is assured with constraint (7). The ride time of each passenger is defined by equality (8) and constrained by inequality (9). Consistence of load variable is guaranteed by constraint (10), while capacity constraints are imposed by inequality (11). Before scheduling a vehicle, it should be examined that the residual capacity can satisfy needs of the next request. Constraints (12) and (13) ensure that each request is served exactly once and that the origin and destination nodes are visited by the same vehicle. The possible values of x_{ij}^m are given by (14).

3. Algorithm

Route optimization for ride-sharing taxi , which belongs to the NP-Hard class of problems is a particular case of CVRPTW (Capacitated Vehicle Routing Problem with Time Windows). This study attempts to solve this problem with simulated annealing (SA) algorithm.

SA is a probabilistic method proposed for finding the global minimum of a cost function that may possess several minima (Dimitris Bertsimas et al. 1993). It works by analogy with the physical annealing process. Physical annealing refers to the process of finding low energy states of a solid by initially melting the substance, and then lowering the temperature close to the freezing point (R.W.EGLESE, 1990). By emulating, the different feasible solutions to the optimization problem correspond to different states of the substance. The objective function to be minimized is in analogy with the energy of the system, while the search process of optimal solution refers to the cooling procedure.

For the routing optimization of ride-sharing taxi, at the time of implementing SA, time windows constraints are needed to consider. The following part will describe the algorithm in the following aspects: time windows processing, neighborhood solution and annealing strategy.

3.1. Time windows processing

(1) Preprocessing

To improve the computing efficiency, it is necessary to preprocess time windows before implementing SA. Remove edges that cannot be part of a feasible solution, in order to shrink search space of neighborhood solutions. The process can be carried out as follows:

If $e_i \geq l_j$ or $(l_j - e_i) < t_{ij}$, then edge (i, j) is marked infeasible.

(2) Processing during route allocation

During route allocation between requests, trial insertion is required. The candidate request is inserted into each edge in the route. And then inspect whether the request to be inserted and the other requests meet time windows after insertion (Qu Xian-feng, 2008). Only when all the requests satisfy pickup time windows constraints, then the candidate one can be assigned to the appropriate route to achieve an optimal solution.

3.2. Neighbourhood solution

Neighborhood solution is achieved by solution improvement, which includes "within-route" and "between-routes" (Wu Tai-His et al. 2002). The between-route search refers to exchanging or relocating the positions of requests between any two routes in a feasible solution. The within-route search method seeks to alter the visiting sequence within a route, and 2-opt is adopted for the within-route improvement in this study. The 2-opt method tries to exchange two edges not adjacent to each other in a feasible route. This process is repeated until no feasible relocation can be found to improve the current solution.

3.3. Annealing strategy

(1) Cooling method

The temperature is gradually dropped at a constant rate, and the cooling procedure can be stated as $T_{i+1} = \beta T_i$. T_i implies the temperature after iterating i -1 times. β denotes cooling rate, at which the temperature is reduced.

(2) Acceptance rule

Metropolis Criterion (Davidson and Harel, 1996) is adopted to determine whether a new solution is accepted. At each temperature T , the new configuration j is accepted as the current one if $E_j < E_i$ (E_i denotes the objective function value at the configuration i). Or else, the probability function $p = \exp[-(E_j - E_i)/T]$ is computed. If the value p is more than the random number selected from $[0,1)$, the new configuration j is also accepted. Otherwise, the new configuration is rejected.

(3) Termination condition

The termination condition is usually based on the stop temperature and the value of the objective function (Davidson and Harel, 1996). When the objective function value remains unchanged continuously, the algorithm terminates. Otherwise, the iteration process terminates until the stop temperature is reached.

3.4. Computation procedure

Step 1: Initialize the control parameters, including setting initial temperature (T_0), the cooling rate (β), stopping temperature (T_E), the number of iterations at each temperature (l) and initial solution (E_0).

Step 2: Adopting between-route improvement and 2-opt method to construct a new configuration, and then compute corresponding value of objective function (E_1).

Step 3: Making use of Metropolis Criterion to examine whether a new configuration is accepted. If $E_1 < E_0$, the new configuration is accepted. Or else, the probability function $p = \exp[-(E_1 - E_0)/T]$ is computed. Selectin`g a random number stated as $rand$ from [0,1) , if $p > rand$, the new configuration is accepted. Otherwise, it is rejected.

Step 4: If the current solution cannot be improved, stop iterating and turn to *step 5*. Otherwise, repeat *step 2 and step 3* for $k = 1,2,...,l$ at current temperature T .

Step 5: Descend temperature gradually by $T_{i+1} = \beta T_i$. If the termination condition is met, stop iteration and terminate the procedure. Otherwise, turn to *step 2* and start a new cycle.

4. Computational Experiment

To demonstrate how the proposed model works and to verify its usefulness, the model was applied in the following computational experiment. There are total 9 nodes in the network, and the relative locations between nodes are shown in Fig. 1. 9 origin-destination (O-D) pairs are considered. Details of the requests are provided in Table 1, such as, the number of requests at every node, the number of passengers of each request and the corresponding pickup time windows. Taxis with capacity of 4 passengers travel at a constant speed of 30km/h. The ride time constraint (L) for every passenger is calibrated as $L = 1.5t_{ij}$, hence the acceptable detour distance is $0.5t_{ij} \times 30 = 15\, t_{ij}$ km. In this case, the maximum $L_{\max}$ =1.15h and detour distance is 11.5km. According to the research of Mark Wardam (2001), the weight coefficients in objective functions are set as α_1 =1, and α_2 =2, α_3 =1 in this case. Relative parameters of the SA algorithm are set as follows: initial temperature T_0 =1000, stop temperature T_E =0.001, iteration counts for each temperature l =100 and cooling rate β =0.9.

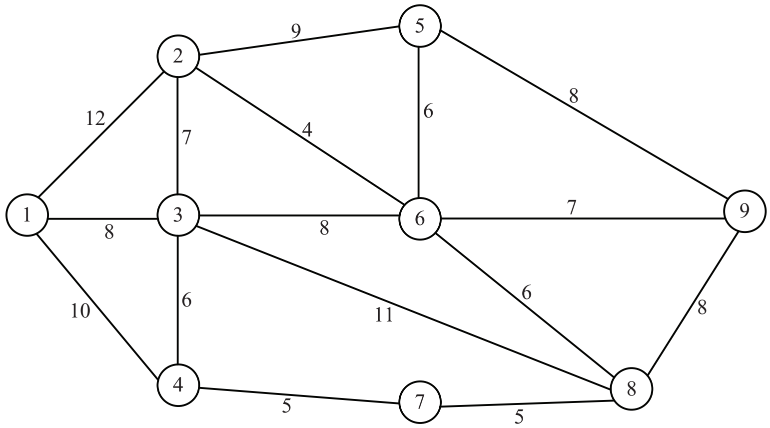

Fig. 1. An experimental road network

Table 1. Detailed requests in the road network

O-D	Number of requests	Number of passengers	Pickup time window	O-D	Number of requests	Number of passengers	Pickup time window
1-6	4	1	12:45-13:15	6-4	3	1	08:45-09:15
		1	13:00-13:30			1	08:30-09:00
		1	13:00-13:30			1	10:00-10:30
		2	10:00-10:30	7-3	3	1	09:25-09:55
2-8	3	1	10:25-10:55			2	09:30-10:00
		3	08:30-09:00			1	09:30-10:00
		1	10:20-10:50	8-7	4	1	09:30-10:00
3-9	4	1	09:30-10:00			1	11:55-12:25
		2	09:35-10:05			1	12:00-12:30
		1	09:30-10:00			2	09:30-10:00
		2	13:45-14:15	9-2	4	2	12:50-13:20
4-5	2	1	13:30-14:00			1	10:15-10:45
		1	13:35-14:05			1	10:20-10:50
5-1	2	2	10:00-10:30			1	09:30-10:00
		2	11:00-11:30				

Table 2. Computational results

No.	Path	Pickup time of first client	No.	Path	Pickup time of first client
1	1-6	13:00	6	7-3	9:30
2	1-2-6-8	10:00	7	8-7-6-4-5-1	9:30
3	3-9	9:35	8	8-7-9-2	12:00
4	4-3-5-9	13:35	9	8-7-9-2	9:30
5	2-6-8-4	8:30	10	6-4-9-5-2-1	8:30

More than one request with different pickup time windows exist between each O-D pair. Consequently, there are 29 clients in total according to O-D pairs and corresponding pickup time windows. The experiment was solved using C++ programming. The result reveals that 10 taxis are needed to satisfy the needs of all passengers. The specific routings and pickup time for every vehicle are shown in Table 2.

With the common operation mode in which each taxi serves one request at a time, 29 taxis are needed and the total travel mileage of meeting all demands is up to 375km. Nevertheless, under the premier of ride sharing, 10 taxis are sufficient to satisfy all requirements, and the travel mileage is 302km in total. The maximum detour mileage for a passenger result from ride-sharing is 6km, and the total detour distance for all requests is 13km. The maximum ride time is 0.7h for passengers, and the average waiting time is 3.34min. As a result, 19 taxis can be saved in operation and 73km mileage is reduced, which is higher than total detour distance results from ride-sharing. The average waiting time demonstrates that higher overall passenger satisfaction is achieved. Also the ride time and detour distance both are in the bounds, since the maximum ride time 0.7h<1.15h and detour distance 6km<11.5km. To sum up, by ride-sharing strategy, 19% travel mileage and 66% taxi demand can be spared.

5. Conclusions

This study proposed a routing optimization model for ride-sharing taxi and appropriate simulated annealing algorithm to address a static, multiple-vehicle dial-a-ride problem with time windows. In the model presented, benefits of both taxi drivers and passengers were considered. This feature has so far received little attention in the existing vehicle routing and scheduling literature. With the computational experiment, it can be inferred that shared ride is an effective way to save resources and mitigate traffic congestion.

Research work is in progress on the topic. Future work may be targeted at adapting the scheduling capabilities of the presented algorithm to this transit environment, and accommodate request changes dynamically through real time communication between the dispatcher and the drivers. In addition to these, the uncertain elements involved in the time windows as well as travel time between nodes could be important concerns in modifying the proposed model and solution procedure.

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