CONVERSATORIO MÉTODOS MATEMÁTICOS 1

TENSORES

2. Coordenadas elípticas (v, w)

$x = \cosh(w)\cos(v)$, $y = \operatorname{senh}(w)\operatorname{sen}(v)$

a. Base de vectores unitarios.

$$|e_u\rangle = \frac{1}{\left\| \frac{\partial|r\rangle}{\partial u} \right\|} \frac{\partial|r\rangle}{\partial u}$$

- $|e_v\rangle$

$$\frac{\partial|r\rangle}{\partial v} = \frac{\partial x(v,w)}{v}|e_x\rangle + \frac{\partial y(v,w)}{\partial v}|e_y\rangle$$

$$= \underbrace{-\cosh(w)\operatorname{sen}(v)}_{a}\,\hat{\imath} + \underbrace{\operatorname{senh}(w)\cos(v)}_{b}\,\hat{\jmath}$$

$$|e_v\rangle = \frac{a\hat{\imath} + b\hat{\imath}}{(a^2+b^2)^{1/2}}$$

- $|e_w\rangle$

$$\frac{\partial|r\rangle}{\partial w} = \frac{\partial x(v,w)}{\partial w}|e_x\rangle + \frac{\partial y(v,w)}{\partial w}|e_y\rangle$$

$$= \underbrace{\operatorname{senh}(w)\cos(v)}_{c}\,\hat{\imath} + \underbrace{\cosh(w)\operatorname{sen}(v)}_{d}\,\hat{\jmath}$$

$$|e_w\rangle = \frac{c\hat{\imath} + d\hat{\jmath}}{(c^2+d^2)^{1/2}}$$

b. Exprese $|a\rangle = 5\hat{\imath} + 2\hat{\jmath}$ en la base $\{|e_v\rangle, |e_w\rangle\}$

$$\tilde{a}^i = \frac{\partial \tilde{x}^i}{\partial x^k} a^k \rightarrow \tilde{a}^i = \left(\frac{\partial x^k}{\partial \tilde{x}^i}\right)^{-1} a^k$$

$$\begin{pmatrix} -\cosh(w)\operatorname{sen}(v) & \operatorname{senh}(w)\cos(v) \\ \operatorname{senh}(w)\cos(v) & \cosh(w)\operatorname{sen}(v) \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \tilde{a}^i$$

$|a\rangle$ en la base

$5 = \cosh(w)\cos(v) \rightarrow v = \cos^{-1}(5/\cosh(w))$

$2 = \operatorname{senh}(w)\operatorname{sen}(v)$ $\quad 2 = \operatorname{senh}(w)\operatorname{sen}\left(\cos^{-1}\left(\frac{5}{\cosh(w)}\right)\right)$

$w \approx -2{,}37055$

$v \approx 5{,}896616$

$$\begin{pmatrix} 2.03525 & -4{,}91346 \\ -4{,}91346 & -2.03525 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$\tilde{a}^i \approx \begin{pmatrix} 0{,}01235 \\ -1{,}01249 \end{pmatrix}$$

CONVERSATORIO MÉTODOS MATEMÁTICOS 1 FINAL

VARIABLE COMPLEJA

3. a) Polos y residuos

$$f(z) = \frac{1}{z^4 + 5z^2 + 6}$$

$z^4 + 5z^2 + 6$
↳ $(z^2 + 2)(z^2 + 3)$

$$f(z) = \frac{1}{(z^2+2)(z^2+3)}$$

$z^2 + 2 = 0$
↳ $z^2 = -2$
$z = \pm \sqrt{2}\, i$

$z^2 + 3 = 0$
↳ $z^2 = -3$
$z = \pm \sqrt{3}\, i$

Polos:
- $z = \pm\sqrt{2}\, i$, $z = \pm \sqrt{3}\, i$
↳ polos simples

$$\frac{P(z_0)}{q'(z_0)} = \frac{1}{4z^3 + 10z}$$

RESIDUOS:
- $z = +\sqrt{2}\, i$

$$\text{Res } f(z) = \frac{1}{4(\sqrt{2}\, i)^3 + 10(\sqrt{2}\, i)} = \frac{1}{-8\sqrt{2}\, i + 10\sqrt{2}\, i} = \frac{1}{2\sqrt{2}\, i} = \frac{2\sqrt{2}\, i}{4 \cdot 2 i^2} = -\frac{\sqrt{2}}{4} i$$

- $z = -\sqrt{2}\, i$

$$\text{Res } f(z) = i \frac{\sqrt{2}}{4}$$

- $z = +\sqrt{3}\, i$

$$\text{Res } f(z) = -i \frac{\sqrt{3}}{6}$$

- $z = -\sqrt{3}\, i$

$$\text{Res } f(z) = i \frac{\sqrt{3}}{6}$$

↳ mismo proceso del primer polo

$$f(z) = \frac{1}{(z^2 - 1)^2}$$

$z^2 - 1 = 0$
$z^2 = 1$
$z = \pm 1$

Polos:
- $z = \pm 1$
↳ polos de segundo orden

$$\frac{P(z_0)}{q'(z_0)} = \frac{1}{2(z^2-1)(2z)}$$

RESIDUOS
↳ por serie de Laurent
- $z = 1$

b. Evaluar mediante el TEOREMA DEL RESIDUO.

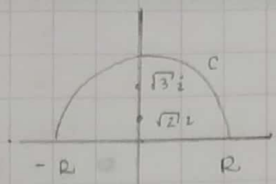
$$\int_0^\infty \frac{x^2}{x^4+5x^2+6}\,dx$$

↳ función par

$$\int_{-\infty}^{\infty} \frac{x^2\,dx}{x^4+5x^2+6}$$

EXTENSIÓN ANALÍTICA

$$\oint_{-R}^{R} \frac{z^2\,dz}{z^4+5z^2+6}$$



$$\oint = \int_C \int_{-R}^{R}$$

= TEOREMA DEL RESIDUO

POLOS:

$z = \pm\sqrt{2}\,i$, $z = \pm\sqrt{3}\,i$

RESIDUOS:

$$\frac{P(z_0)}{q'(z_0)} = \frac{z^2}{4z^3+10z}$$

① • $z = +\sqrt{2}\,i$

$$\text{Res } f(z) = -\frac{i\sqrt{2}}{4}\,(\sqrt{2}\,i)^2$$

$$= \frac{i\,2\sqrt{2}}{4}$$

$$= \frac{i\sqrt{2}}{2}.$$

• $z = -\sqrt{2}\,i$

$$\text{Res } f(z) = \frac{i\sqrt{2}}{4}\,(-\sqrt{2}\,i)^2$$

$$= -i\,\frac{2\sqrt{2}}{4}$$

$$= -i\,\frac{\sqrt{2}}{2}$$

② • $z = +\sqrt{3}\,i$

$$\text{Res } f(z) = -\frac{i\sqrt{3}}{6}\,(\sqrt{3}\,i)^2$$

$$= \frac{i\,3\sqrt{3}}{6}$$

$$= \frac{i\sqrt{3}}{2}.$$

• $z = -\sqrt{3}\,i$

$$\text{Res } f(z) = \frac{i\sqrt{3}}{6}\,(-\sqrt{3}\,i)^2$$

$$= -i\,\frac{3\sqrt{3}}{6}$$

$$= -\frac{i\sqrt{3}}{2}$$

TEOREMA DEL RESIDUO

$$\oint_C dz\,f(z) = 2i\pi\left(\frac{i\sqrt{2}}{2} + \frac{i\sqrt{3}}{2}\right)$$

$$= -\pi\,(\sqrt{2}+\sqrt{3}).$$

$$\int_0^\infty \frac{x\sin x}{x^2+a^2}\,dx$$

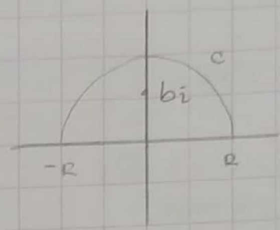
↳ función par

$$\int_{-\infty}^{\infty} \frac{x\sin x}{x^2+a^2}\,dx$$

EXTENSIÓN ANALÍTICA

$$\oint_{-R}^{R} \frac{z\sin(z)}{z^2+b^2}\,dz$$

→ $b \in \mathbb{R}$



$$\oint = \int_C \int_{-R}^{R}$$

= TEOREMA DEL RESIDUO.

$z^2 + b^2 = 0$

$z^2 = -b^2$

$z = \pm bi$

POLOS

$z = \pm bi$

↳ polos simplos.

RESIDUOS

$$\frac{P(z_0)}{q'(z_0)} = \frac{z\sin(z)}{2z}$$

• $z = bi$

$$\text{Res } f(z) = \frac{bi\sin(bi)}{2bi}$$

$$= \frac{\sin(bi)}{2}$$

TEOREMA DEL RESIDUO

$$\oint_C dz\,f(z) = 2i\pi\,\frac{\sin(bi)}{2}$$

$$= i\pi\sin(bi)$$