3.3.5 EJERCICIOS

2. Dados los tensores:

$$R^i_{\ j} = \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix}, \quad T^i = \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix}$$

$$g^{ij} = g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Encuentre:

(a) La parte simétrica $S^i_{\ j}$ y antisimétrica $A^i_{\ j}$ de $R^i_{\ j}$.

- SIMÉTRICA

$$S^i_{\ j} = \frac{1}{2}\left(R_{ij} + R_{ji}\right) = \frac{1}{2}\left[\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 2 & 7/2 \\ 1 & 5/2 & 4 \\ 3/2 & 3 & 9/2 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{pmatrix}$$

- ANTISIMÉTRICA

$$A^i_{\ j} = \frac{1}{2}\left(R_{ij} - R_{ij}\right) = \frac{1}{2}\left[\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} - \begin{pmatrix} 1/2 & 2 & 7/2 \\ 1 & 5/2 & 4 \\ 3/2 & 3 & 9/2 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

(b) $R_{kj} = g_{ik} R^i_{\ j}$, $R^{ki} = g^{jk} R^i_{\ j}$, $T_j = g_{ij} T^i$

¿Qué se concluye de estos cálculos?

- $R_{kj} = (g_{ik})(R_{ij}) = (g_{ki})(R_{ij})$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & 1 & 3/2 \\ -2 & -5/2 & -3 \\ 7/2 & 4 & 9/2 \end{pmatrix}$$

- $R^{ki} = (g_{jk})(R_{ij}) = (R_{ij})(g_{jk})$

$$= \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & -1 & 3/2 \\ 2 & -5/2 & 3 \\ 7/2 & -4 & 9/2 \end{pmatrix}$$

- $T_j = (g_{ij})(T^i) = (T^i)(g_{ij})$

$$= \begin{pmatrix} 1/3 & 2/3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 & -2/3 & 1 \end{pmatrix}$$

$\longrightarrow$ Componentes covariantes y contravariantes del tensor R; componente covariante del tensor T.

(c) $R^i_j T_i$, $R^i_j T^j$, $R^i_j T_i T^j$

- $R^i_j T_i = (R_{ij})(T_i) = (T_i)(R_{ij}) = T_i R^i_j$

$$= \begin{pmatrix} 1/3 & 2/3 & 1 \end{pmatrix} \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 6 & 7 \end{pmatrix}$$

- $R^i_j T^j = (R_{ij})(T_j)$

$$= \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \begin{pmatrix} 1/3 \\ -2/3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

- $R^i_j T_i T^j = (R_{ij})(T_i)(T_j) = (T_i)(R_{ij})(T_j) = T_i R^i_j T^j$

$$= \begin{pmatrix} 1/3 & 2/3 & 1 \end{pmatrix} \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \begin{pmatrix} 1/3 \\ -2/3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 6 & 7 \end{pmatrix} \begin{pmatrix} 1/3 \\ -2/3 \\ 1 \end{pmatrix}$$

$$= \frac{14}{3}$$

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(d) $R^i_j S^j_i$, $R^i_j A^j_i$, $A^j_i T^i$, $A^j_i T^i T_j$

- $R^i_j S^j_i = (R_{ij})(S_{ji})$

$$= 1/2 \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \begin{pmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{pmatrix}$$

$$= 1/2 \begin{pmatrix} 11 & 17 & 23 \\ 49/2 & 79/2 & 109/2 \\ 38 & 62 & 86 \end{pmatrix}$$

- $R^i_j A^j_i = (R_{ij})(A_{ji})$

$$= 1/2 \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix}$$

$$= 1/2 \begin{pmatrix} -4 & -1 & 2 \\ -17/2 & -1 & 13/2 \\ -13 & -1 & 11 \end{pmatrix}$$

- $A^j_i T^i = (A_{ji})(T_i)$

$$= \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix}$$

$$= 1/3 \begin{pmatrix} 8 \\ 2 \\ -4 \end{pmatrix}$$

- $A^j_i T^i T_j = (A_{ji})(T_i)(T_j) = (T_j)(A_{ji})(T_i)$

$$= \begin{pmatrix} 1/3 & -2/3 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix}$$

$$= 1/3 \begin{pmatrix} 1/3 & -2/3 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ 2 \\ -4 \end{pmatrix}$$

$$= -\frac{7}{3}$$

(e) $R^i_j - 2\delta^i_j R^l_l$, $(R^i_j - 2\delta^i_j R^l_l)T_i$, $(R^i_j - 2\delta^i_j R^l_l)T_i T^j$ $\qquad g_{ik} g^{kj} = \delta^j_i$

- $R^i_j - 2\delta^i_j R^l_l = R^i_j - 2\delta^i_j (g_{lk} R^{lk})$

$$= R^i_j - 2\delta^i_j (g_{lk} g^{jl} R^k_j)$$

$$= R^i_j - 2\delta^i_j \delta^j_k R^k_j$$

$$= R^i_j - 2\delta^i_k R^k_j$$

$$= R^i_j - 2R^i_j$$

$$= -R^i_j$$

$$= \begin{pmatrix} -1/2 & -1 & -3/2 \\ -2 & -5/2 & -3 \\ -7/2 & -4 & -9/2 \end{pmatrix}$$

- $$(R^i_j - 2\delta^i_j R^l_l)\,T_i = (R^i_j - 2 g_{jl} g^{li} R^l_l)\,T_i = T_i \left[R_{ij} - 2 g_{jl} R_{ll} g_{li} \right]$$

$$= \begin{pmatrix} 1/3 & 2/3 & 1 \end{pmatrix} \begin{pmatrix} -1/2 & -1 & -3/2 \\ -2 & -5/2 & -3 \\ -7/2 & -4 & -9/2 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -6 & -7 \end{pmatrix}$$

- $$(R^i_j - 2\delta^i_j R^l_l)\,T_i T^j = (R^i_j - 2 g_{jl} g^{li} R^l_l)\,T_i T^j = T_i \left[R_{ij} - 2 g_{jl} R_{ll} g_{li} \right] T_j$$

$$= \begin{pmatrix} 1/3 & 2/3 & 1 \end{pmatrix} \begin{pmatrix} -1/2 & -1 & -3/2 \\ -2 & -5/2 & -3 \\ -7/2 & -4 & -9/2 \end{pmatrix} \begin{pmatrix} 1/3 \\ -2/3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 & 34/3 & 1 \end{pmatrix} \begin{pmatrix} 1/3 \\ -2/3 \\ 1 \end{pmatrix}$$

$$= -\frac{14}{3}$$

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