Nombre: Arellano Granados Angel Mariano_____ Fecha: 24/05/2022

Tarea: __6__

Transformada de una derivada

1) $4y'' - y = 1, \quad y(0) = 0, \; y'(0) = \frac{1}{2}$

$4L\{y''\} - L\{y\} = L\{1\}$

$4\left(s^2Y(s) - \cancel{sy(0)} - y'(0)\right) - \left(Y(s)\right) = \frac{1}{s}$

$4\left(s^2Y(s) - \frac{1}{2}\right) - Y(s) = \frac{1}{s}$

$Y(s)4\left(s^2 - \frac{1}{2}\right) - 1 = \frac{1}{s}$

$Y(s) = \frac{1}{4s\left(s^2 - \frac{1}{2}\right)} + \frac{1}{4\left(s^2 - \frac{1}{2}\right)}$

$Y(s) = \frac{1+s}{4s\left(s^2 - \frac{1}{2}\right)}$

$y = -1 + e^{\frac{1}{2}}$

2) $y'' + y = e^{-2t}sen\,(t), \quad y(0) = y'(0) = 0$

$L\{y''\} + L\{y\} = L\{sen\,(t)|_{s\to s+2}\}$

$s^2Y(s) - \cancel{sy(0)} - \cancel{y'(0)} + Y(s) = \frac{1}{(s+2)^2+1}$

$Y(s)(s^2+1) = \frac{1}{(s+2)^2+1}$

$Y(s) = \frac{1}{(s+2)^2 + (s^2+1)}$

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$y = \frac{1}{8}e^{-2t}\cos(t) + \frac{1}{8}e^{-2t}\sin(t) - \frac{1}{8}\cos(t) + \frac{1}{8}\sin(t)$

3) $y'' + 4y = f(t), \quad donde\, f(t) = \begin{cases} 0 & 0 \le t < 3 \\ t & t \ge 3 \end{cases}, \quad y(0) = 0, y'(0) = 0$

$f(t) = 0 + (t-0)u(t-3) = t\,u(t-3)$

$L\{y''\} + 4L\{y\} = L\{t\,u(t-3)\}$

$$s^2Y(s) - \cancel{sy(0)} - \cancel{y'(0)} + 4Y(s) = \frac{-3e^{-3s}s - e^{-3s}}{s^2}$$

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$$y = 3\left(\frac{1}{4} - \frac{1}{4}\cos(2(t-3))\right) + \frac{t-3}{4} - \frac{1}{8}\sin\big(2(t-3)\big)$$