Centro Universitario de Ciencias Exactas e Ingenierías

Álgebra Lineal

ACTIVIDAD 3.5 Bases ortonormales
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SECCIÓN: _______D - 17__________ **CÓDIGO:** _218123444__

Instrucciones: Contesta lo que se pide, recuerda hacerlo de forma clara y con el apoyo de los recursos propuestos para esta actividad, en cada ejercicio deberás anotar el procedimiento limpio, claro y legible que justifique tu respuesta.
Especificaciones de formato: Arial 11, Interlineado sencillo, un espacio entre párrafos, margen moderado, texto justificado.

1. Escribe la definición de base ortonormal.

Un conjunto S de vectores en un espacio V con producto interno se llama ortogonal si todo par de vectores en S es ortogonal. Si, además, cada vector en este conjunto es unitario, entonces S se denomina ortonormal.

1.$\langle v_i, v_j \rangle = 0$, $i \neq j$

2.$||v_i|| = 1$, $i = 1, 2, \ldots, n$

2. Escribe a que se le llama proceso de ortonormalización de Gram-Schmidth.

1. Empezar con una base del espacio con producto interior. No se requiere que sea una base ortogonal ni que conste de vectores unitarios.
2. Convertir la base dada a una base ortogonal.
3. Normalizar cada uno de los vectores de la base ortogonal a fin de obtener una base ortonormal

3. Determine si los siguientes vectores son ortonormales

a) $\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}, \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$

$$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}} + 0 \neq 0$$

$\therefore$ *No es Ortonormal ya que el producto punto del 1° y 2° vector no es igual a 0*

b) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \\ 0 \end{pmatrix}, \begin{pmatrix} -\frac{6}{\sqrt{70}} \\ \frac{3}{\sqrt{70}} \\ \frac{5}{\sqrt{70}} \end{pmatrix}$

$$\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right) . \left(-\frac{6}{\sqrt{70}}, \frac{3}{\sqrt{70}}, \frac{5}{\sqrt{70}}\right) = -\frac{6}{\sqrt{350}} + \frac{6}{\sqrt{350}} + 0 = 0$$

$$|u_1| = \sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{2}{\sqrt{5}}\right)^2 + 0^2} = \sqrt{\frac{1}{5} + \frac{4}{4} + 0} = \sqrt{1} = 1$$

$$|u_2| = \sqrt{\left(-\frac{6}{\sqrt{70}}\right)^2 + \left(\frac{3}{\sqrt{70}}\right)^2 + \left(\frac{5}{\sqrt{70}}\right)^2} = \sqrt{\frac{36}{70} + \frac{9}{70} + \frac{25}{70}} = \sqrt{1} = 1$$

$\therefore$ *Es una base ortonormal*

4. Encuentre una base ortonormal, compruebe su respuesta, de lo contrario no contará como válida.

a) $B = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$

$$w_1 = v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$w_2 = v_2 - \frac{< v_2, w_1 >}{< w_1, w_1 >} w_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} - \frac{< \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} >}{< \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} >} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} - \frac{0}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$|w_1| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

$$|w_2| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

$$U_1 = \frac{\sqrt{2}}{2} w_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$U_2 = \frac{\sqrt{2}}{2} w_2 = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$Comprobacion: \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) . \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = -\frac{2}{4} + \frac{2}{4} = 0$$

$$|u_1| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1} = 1$$

$$|u_2| = \sqrt{\left(-\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \sqrt{1} = 1$$

b) $2x - y - z = 0 \quad y = -2x + z$

$$\begin{pmatrix} x \\ -2x+z \\ z \end{pmatrix} = x\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + z\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad B = \left\{\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right\}$$

$$w_1 = v_1 = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$$w_2 = v_2 - \frac{< v_2, w_1 >}{< w_1, w_1 >} w_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \frac{< \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} >}{< \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} >} \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \frac{(-2)}{5}\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -2/5 \\ 4/5 \\ 0 \end{pmatrix} = \begin{pmatrix} 2/5 \\ 1/5 \\ 1 \end{pmatrix}$$

$$|w_1| = \sqrt{(1)^2 + (-2)^2 + (0)^2} = \sqrt{5}$$

$$|w_2| = \sqrt{(2/5)^2 + (1/5)^2 + (1)^2} = \sqrt{\frac{4}{25} + \frac{1}{25} + \frac{25}{25}} = \sqrt{6/5}$$

$$U_1 = \frac{\sqrt{5}}{5} w_1 = \left(\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}, 0\right)$$

$$U_2 = \frac{\sqrt{6/5}}{6/5} w_2 = \left(\frac{2\sqrt{6/5}}{6}, \frac{\sqrt{6/5}}{6}, \frac{\sqrt{6/5}}{6/5}\right) o \left(\frac{2}{5\sqrt{(6/5)}}, \frac{1}{5\sqrt{(6/5)}}, \frac{1}{\sqrt{(6/5)}}\right)$$

$$Comprobacion: \left(\frac{\sqrt{5}}{5}, -\frac{2\sqrt{5}}{5}, 0\right) . \left(\frac{2\sqrt{\frac{6}{5}}}{6}, \frac{\sqrt{\frac{6}{5}}}{6}, \frac{\sqrt{\frac{6}{5}}}{\frac{6}{5}}\right)$$

$$= -\frac{2\sqrt{5}}{25\sqrt{(6/5)}} + \frac{2\sqrt{5}}{25\sqrt{(6/5)}} + 0 = 0$$

$$|u_1| = \sqrt{\left(\frac{\sqrt{5}}{5}\right)^2 + \left(-\frac{2\sqrt{5}}{5}\right)^2 (0)^2} = \sqrt{\frac{5}{25} + \frac{20}{25} + 0} = \sqrt{1} = 1$$

$$|u_2| = \sqrt{\left(\frac{2}{5\sqrt{(6/5)}}\right)^2 + \left(\frac{1}{5\sqrt{(6/5)}}\right)^2 + \left(\frac{1}{\sqrt{(6/5)}}\right)^2} = \sqrt{\frac{4}{30} + \frac{1}{30} + \frac{25}{30}} = \sqrt{1} = 1$$

c) $$\begin{matrix} 2x - y + 3z = 0 \\ -x + 6y - 2z = 0 \end{matrix}$$

$$\begin{vmatrix} 2 & -1 & 3 \\ -1 & 6 & -2 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} \begin{matrix} F1\left(\frac{1}{2}\right) = F1 \\ F1 + F2 = F2 \end{matrix}$$

$$\begin{vmatrix} 1 & -1/2 & 3/2 \\ 0 & 11/2 & -1/2 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix} \begin{matrix} F2(2/11) = F2 \\ F2(1/2) + F1 = F1 \end{matrix}$$

$$\begin{vmatrix} 1 & 0 & 16/11 \\ 0 & 1 & -1/11 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\begin{pmatrix} -16/11z \\ 1/11z \\ z \end{pmatrix} = z \begin{pmatrix} -16/11 \\ 1/11 \\ 1 \end{pmatrix} \; o \begin{pmatrix} -16 \\ 1 \\ 11 \end{pmatrix}$$

$$w_1 = v_1 = \begin{pmatrix} -16 \\ 1 \\ 11 \end{pmatrix}$$

$$|w_1| = \sqrt{(-16)^2 + 1^2 + 11^2} = \sqrt{378}$$

$$U_1 = \frac{1}{\sqrt{378}} w_1 = \left(-\frac{16}{\sqrt{378}}, \frac{1}{\sqrt{378}}, \frac{11}{\sqrt{378}}\right) =$$

$$\left(-\frac{16\sqrt{378}}{378}, \frac{1\sqrt{378}}{378}, \frac{11\sqrt{378}}{378}\right)$$

$Comprobacion$:

$$|u_1| = \sqrt{\left(-\frac{16\sqrt{378}}{378}\right)^2 + \left(\frac{1\sqrt{378}}{378}\right)^2 + \left(\frac{11\sqrt{378}}{378}\right)^2}$$

$$= \sqrt{\frac{96768}{142884} + \frac{378}{142884} + \frac{45738}{142884}} = \sqrt{1} = 1$$

d) $$\begin{matrix} a + b + c + d = 0 \\ -a + 3b + c - d = 0 \\ -8a + 4b - 5c + d = 0 \end{matrix}$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ -1 & 3 & 1 & -1 \\ -8 & 4 & -5 & 1 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \begin{matrix} F1 + F2 = F2 \\ F1(8) + F3 = F3 \end{matrix}$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 4 & 2 & 0 \\ 0 & 12 & 3 & 9 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \begin{matrix} F2\left(\frac{1}{4}\right) = F2 \\ F2(-1) + F1 = F1 \\ F2(-12) + F3 = F3 \end{matrix}$$

$$\begin{vmatrix} 1 & 0 & 1/2 & 1 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & -3 & 9 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \begin{matrix} F3\left(-\frac{1}{3}\right) = F3 \\ F3(-1/2) + F1 = F1 \\ F3(-1/2) + F2 = F2 \end{matrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 5/2 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & -3 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{pmatrix} -\frac{5}{2d} \\ -\frac{3}{2d} \\ 3d \\ d \end{pmatrix} = d\begin{pmatrix} -\frac{5}{2} \\ -\frac{3}{2} \\ 3 \\ 1 \end{pmatrix} \ o \begin{pmatrix} -5 \\ -3 \\ 6 \\ 2 \end{pmatrix}$$

$$w_1 = v_1 = \begin{pmatrix} -5 \\ -3 \\ 6 \\ 2 \end{pmatrix}$$

$$|w_1| = \sqrt{(-5)^2 + (-3)^2 + 6^2 + 2^2} = \sqrt{74}$$

$$U_1 = \frac{1}{\sqrt{74}} w_1 = \left(-\frac{5}{\sqrt{74}}, -\frac{3}{\sqrt{74}}, \frac{6}{\sqrt{74}}, \frac{2}{\sqrt{74}}\right) =$$

$$\left(-\frac{5\sqrt{74}}{74}, -\frac{3\sqrt{74}}{74}, \frac{6\sqrt{74}}{74}, \frac{2\sqrt{74}}{74}\right)$$

$Comprobacion$:

$$|u_1| = \sqrt{\left(-\frac{5\sqrt{74}}{74}\right)^2 + \left(-\frac{3\sqrt{74}}{74}\right)^2 + \left(\frac{6\sqrt{74}}{74}\right)^2 + \left(\frac{2\sqrt{74}}{74}\right)^2}$$

$$= \sqrt{\frac{1850}{5476} + \frac{666}{5476} + \frac{2664}{5476} + \frac{296}{5476}} = \sqrt{1} = 1$$

e) $\left\{\begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} -10 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}\right\}$

$$w_1 = v_1 = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$$

$$w_2 = v_2 - \frac{< v_2, w_1 >}{< w_1, w_1 >} w_1 = \begin{pmatrix} -10 \\ 0 \\ 1 \end{pmatrix} - \frac{< \begin{pmatrix} -10 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} >}{< \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} >} \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -10 \\ 0 \\ 1 \end{pmatrix} - \frac{-1}{5}\begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1/5 \\ -1 \\ -3/5 \end{pmatrix} = \begin{pmatrix} 49/5 \\ 1 \\ 2/5 \end{pmatrix} \ o \begin{pmatrix} 49 \\ 5 \\ 2 \end{pmatrix}$$

$$w_3 = v_3 - \frac{< v_3, w_1 >}{< w_1, w_1 >} w_1 - \frac{< v_3, w_2 >}{< w_2, w_2 >} w_2$$

$$= \begin{pmatrix}0\\0\\3\end{pmatrix} - \frac{< \begin{pmatrix}0\\0\\3\end{pmatrix}, \begin{pmatrix}1\\5\\3\end{pmatrix} >}{< \begin{pmatrix}1\\5\\3\end{pmatrix}, \begin{pmatrix}1\\5\\3\end{pmatrix} >} \begin{pmatrix}1\\5\\3\end{pmatrix} - \frac{< \begin{pmatrix}0\\0\\3\end{pmatrix}, \begin{pmatrix}-10\\0\\1\end{pmatrix} >}{< \begin{pmatrix}-10\\0\\1\end{pmatrix}, \begin{pmatrix}-10\\0\\1\end{pmatrix} >} \begin{pmatrix}-10\\0\\1\end{pmatrix}$$

$$= \begin{pmatrix}0\\0\\3\end{pmatrix} - \frac{9}{35}\begin{pmatrix}1\\5\\3\end{pmatrix} - \frac{3}{101}\begin{pmatrix}-10\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\3\end{pmatrix} - \begin{pmatrix}9/35\\9/7\\27/35\end{pmatrix} - \begin{pmatrix}-30/101\\0\\3/101\end{pmatrix} = \begin{pmatrix}141/3535\\-9/7\\7773/3535\end{pmatrix}$$

$$|w_1| = \sqrt{(1)^2 + (5)^2 + (3)^2} = \sqrt{35}$$

$$|w_2| = \sqrt{(49)^2 + (5)^2 + (2)^2} = \sqrt{2430}$$

$$|w_3| = \sqrt{(141/3535)^2 + (-9/7)^2 + (7773/3535)^2} = \sqrt{\frac{22941}{3535}} = \frac{3\sqrt{2549}}{\sqrt{3535}}$$

$$U_1 = \frac{\sqrt{35}}{35} w_1 = \left(\frac{\sqrt{35}}{35}, \frac{5\sqrt{35}}{35}, \frac{3\sqrt{35}}{35}\right)$$

$$U_2 = \frac{\sqrt{2430}}{2430} w_2 = \left(\frac{49\sqrt{2430}}{2430}, \frac{5\sqrt{2430}}{2430}, \frac{2\sqrt{2430}}{2430}\right)$$

$$U_3 = \frac{1}{\sqrt{\frac{22941}{3535}}} w_3 == \left(\frac{141}{3535\sqrt{\frac{22941}{3535}}}, -\frac{9}{7\sqrt{\frac{22941}{3535}}}, \frac{7773}{3535\sqrt{\frac{22941}{3535}}}\right)$$

$$Comprobacion: \left(\frac{\sqrt{35}}{35}, \frac{5\sqrt{35}}{35}, \frac{3\sqrt{35}}{35}\right) . \left(\frac{49\sqrt{2430}}{2430}, \frac{5\sqrt{2430}}{2430}, \frac{2\sqrt{2430}}{2430}\right)$$

$$= \frac{49\sqrt{85050}}{85050} + \frac{25\sqrt{85050}}{85050} + \frac{6\sqrt{85050}}{85050} = 0$$

$$\left(\frac{\sqrt{35}}{35}, \frac{5\sqrt{35}}{35}, \frac{3\sqrt{35}}{35}\right) . \left(\frac{141}{3535\sqrt{\frac{22941}{3535}}}, -\frac{9}{7\sqrt{\frac{22941}{3535}}}, \frac{7773}{3535\sqrt{\frac{22941}{3535}}}\right)$$

$$= \frac{141\sqrt{35}}{123725\sqrt{\frac{22941}{3535}}} - \frac{45\sqrt{35}}{245\sqrt{\frac{22941}{3535}}} + \frac{23319\sqrt{35}}{123725\sqrt{\frac{22941}{3535}}} = 0$$

$$\left(\frac{49\sqrt{2430}}{2430}, \frac{5\sqrt{2430}}{2430}, \frac{2\sqrt{2430}}{2430}\right) . \left(\frac{141}{3535\sqrt{\frac{22941}{3535}}}, -\frac{9}{7\sqrt{\frac{22941}{3535}}}, \frac{7773}{3535\sqrt{\frac{22941}{3535}}}\right)$$

$$= \frac{6909\sqrt{2430}}{8590050\sqrt{\frac{22941}{3535}}} - \frac{45\sqrt{2430}}{170010\sqrt{\frac{22941}{3535}}} + \frac{15546\sqrt{2430}}{8590050\sqrt{\frac{22941}{3535}}} = 0$$

$$|u_1| = \sqrt{\left(\frac{\sqrt{35}}{35}\right)^2 + \left(\frac{5\sqrt{35}}{35}\right)^2 + \left(\frac{3\sqrt{35}}{35}\right)^2} = \sqrt{\frac{35}{1225} + \frac{875}{1225} + \frac{315}{1225}} = \sqrt{1} = 1$$

$$|u_2| = \sqrt{\left(\frac{49\sqrt{2430}}{2430}\right)^2 + \left(\frac{5\sqrt{2430}}{2430}\right)^2 + \left(\frac{2\sqrt{2430}}{2430}\right)^2}$$

$$= \sqrt{\frac{5834430}{5904900} + \frac{60750}{5904900} + \frac{9720}{5904900}} = \sqrt{1} = 1$$

$$|u_3| = \sqrt{\left(\frac{141}{3535\sqrt{\frac{22941}{3535}}},\right)^2 + \left(-\frac{9}{7\sqrt{\frac{22941}{3535}}}\right)^2 + \left(\frac{7773}{3535\sqrt{\frac{22941}{3535}}}\right)^2} = \sqrt{1} = 1$$