Centro Universitario de Ciencias Exactas e Ingenierías

Álgebra Lineal

ACTIVIDAD 4.1 Transformaciones Lineales

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Instrucciones: Contesta lo que se pide, recuerda hacerlo de forma clara y con el apoyo de los recursos propuestos para esta actividad, en cada ejercicio deberás anotar el procedimiento limpio, claro y legible que justifique tu respuesta.

Especificaciones de formato: Arial 11, Interlineado sencillo, un espacio entre párrafos, margen moderado, texto justificado.

1. Demuestra si las siguientes transformaciones son lineales o no.

a) $T(a,b,c) = \left(\frac{a}{b}, c\right)$

T:R³->R²

$u = (a_1, b_1, c_1) \quad v = (a_2, b_2, c_2) \quad u + v = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

$$T(u+v) = \left(\frac{a_1 + a_2}{b_1 + b_2}, c_1 + c_2\right)$$

$$T(u) = \left(\frac{a_1}{b_1}, c_1\right)$$

$$T(v) = \left(\frac{a_2}{b_2}, c_2\right)$$

$$T(u) + T(v) = \left(\frac{a_1 + a_2}{b_1 + b_2}, c_1 + c_2\right)$$

$$T(u+v) = T(u) + T(v)$$

$\therefore$ La Transformacion Si es Lineal

b) $T(x,y) = (4x, -y)$

T:R²->R²

$u = (x_1, y_1) \quad v = (x_2, y_2) \quad u + v = (x_1 + x_2, y_1 + y_2)$

$T(u + v) = (4(x_1 + x_2), -y_1 - y_2)$

$T(u) = (4(x_1), -y_1)$

$T(v) = (4(x_2), -y_2)$

$T(u) + T(v) = (4x_1 + 4x_2, -y_1 - y_2)$

$T(u + v) = T(u) + T(v)$

$\therefore$ La Transformacion Si es Lineal

c) $T(x, y) = (2xy, -x)$

T:R²->R²

$u = (x_1, y_1) \quad v = (x_2, y_2) \quad u + v = (x_1 + x_2, y_1 + y_2)$

$T(u + v) = (2(x_1 + x_2)(y_1 + y_2), -x_1 - x_2)$

$T(u) = (2(x_1 y_1), -x_1)$

$T(v) = (2(x_2 y_2), -x_2)$

$T(u) + T(v) = (2(x_1 y_1) + 2(x_2 y_2), -x_1 - x_2)$

$T(u + v) \neq T(u) + T(v)$

$\therefore$ La Transformacion NO es Lineal

d) $T(u, v, w) = (2u, -v, w + 1, w)$

T:R²->R²

$u = (x_1, y_1) \quad v = (x_2, y_2) \quad u + v = (x_1 + x_2, y_1 + y_2)$

$T(u + v) = (4(x_1 + x_2), -y_1 - y_2)$

$T(u) = (4(x_1), -y_1)$

$T(v) = (4(x_2), -y_2)$

$T(u) + T(v) = (4x_1 + 4x_2, -y_1 - y_2)$

$T(u + v) = T(u) + T(v)$

$\therefore$ La Transformacion Si es Lineal

2. Determina la imagen y la preimagen de las siguientes transformaciones.

a) $T(x,y) = (x+y, x-y) \qquad v = (3,-4)\,, \quad w = (3,19)$

Imagen:

$T(3,-4) = (3-4, 3+4) = (-1,7) = w$

Preimagen:

$$\begin{matrix} x+y=3 \\ x-y=19 \end{matrix} \quad \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} \begin{vmatrix} 3 \\ 19 \end{vmatrix} \; det = -2$$

$$\frac{1}{-2}\begin{vmatrix} -1 & -1 \\ -1 & 1 \end{vmatrix} = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix} \begin{vmatrix} 3 \\ 19 \end{vmatrix} = \begin{pmatrix} 11 \\ -8 \end{pmatrix} = w$$

b) $T(a,b) = (2b-a, a, b) \qquad v = (0,6) \quad w = (3,1,2)$

Imagen:

$T(0,6) = (2(6)-0, 0, 6) = (12,0,6) = w$

Preimagen:

$$\begin{matrix} 2b-a=3 \\ a=1 \\ b=6 \end{matrix} \quad \begin{pmatrix} 1 \\ 6 \end{pmatrix} = w$$

c) $T(x,y,z) = (y-x, x+y, 2x) \quad v = (2,3,0) \quad w = (-11,-1,10)$

Imagen:

$T(2,3,0) = (3-2, 2+3, 2(2)) = (1,5,4) = w$

Preimagen:

$$\begin{matrix} y-x=-11 \\ x+y=-1 \\ 2x=10 \end{matrix} \quad x=5 \quad y=-6 \quad z = Reales \quad \begin{pmatrix} 5 \\ -6 \\ z \end{pmatrix} = w$$

d) $T(x,y) = \left(\frac{\sqrt{3}}{2}x - \frac{1}{2}y, x-y, y\right) \quad v = (2,4) \quad w = (\sqrt{3}, 2, 0)$

Imagen:

$$T(2,4) = \left(\frac{\sqrt{3}}{2}(2) - \frac{1}{2}(4), 2-4,4\right) = \left(\sqrt{3}-2,-2,4\right) = w$$

Preimagen:

$$\begin{aligned} \frac{\sqrt{3}}{2}x - \frac{1}{2}y &= \sqrt{3} \\ x - y &= 2 \\ y &= 0 \end{aligned} \qquad y = 0 \quad x = 2 \quad \begin{pmatrix} 2 \\ 0 \end{pmatrix} = w$$

3. Dada la transformación lineal $T: R^3 \to R^3$ definida por:

$$T(1,0,0) = (2,4,-1),\ \ T(0,1,0) = (1,3,-2), T(0,0,1) = (0,-2,2)$$

a) Determine $T(x,y,z)$

$$B = \left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right\} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$$x\,T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y\,T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z\,T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = x\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + y\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + z\begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} =$$

$$T\begin{pmatrix} 2x+y \\ 4x+3y-2z \\ -x-2y+2z \end{pmatrix} \textit{Solucion General}$$

b) Determine $T(0,3,-1)$

$$T\begin{pmatrix} 2x+y \\ 4x+3y-2z \\ -x-2y+2z \end{pmatrix} = \begin{pmatrix} 3 \\ 11 \\ -8 \end{pmatrix}$$

c) Determine $T(2,-4,1)$

$$T\begin{pmatrix} 2x+y \\ 4x+3y-2z \\ -x-2y+2z \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix}$$

4. Dada la transformación lineal $T: R^3 \to R^3$ definida por:

$$T(1,1,1) = (2,0,-1),\ \ T(0,-1,2) = (-3,2,-1), T(1,0,1) = (1,1,0)$$

a) Determine $T(x, y, z)$

$$B = \left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}0\\-1\\2\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix}\right\} \quad \begin{bmatrix}x\\y\\z\end{bmatrix} =$$

$$\left|\begin{matrix}1 & 0 & 1\\1 & -1 & 0\\1 & 2 & 1\end{matrix}\right|\left|\begin{matrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{matrix}\right| \begin{matrix}F1(-1)+F2=F2\\F1(-1)+F3=F3\end{matrix}$$

$$\left|\begin{matrix}1 & 0 & 1\\0 & -1 & -1\\0 & 2 & 0\end{matrix}\right|\left|\begin{matrix}1 & 0 & 0\\-1 & 1 & 0\\-1 & 0 & 1\end{matrix}\right| \begin{matrix}F2(-1)=F2\\F2(-2)+F3=F3\end{matrix}$$

$$\left|\begin{matrix}1 & 0 & 1\\0 & 1 & 1\\0 & 0 & -2\end{matrix}\right|\left|\begin{matrix}1 & 0 & 0\\1 & -1 & 0\\-3 & 2 & 1\end{matrix}\right| \begin{matrix}F3\left(-\frac{1}{2}\right)=F3\\F3(-1)+F1=F1\\F3(-1)+F2=F2\end{matrix}$$

$$\left|\begin{matrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{matrix}\right|\left|\begin{matrix}-\frac{1}{2} & 1 & \frac{1}{2}\\-\frac{1}{2} & 0 & \frac{1}{2}\\\frac{3}{2} & -1 & -\frac{1}{2}\end{matrix}\right|\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\left(-\frac{x}{2}+y+\frac{z}{2}\right)\begin{pmatrix}2\\0\\-1\end{pmatrix}+\left(-\frac{x}{2}+\frac{z}{2}\right)\begin{pmatrix}-3\\2\\-1\end{pmatrix}+\left(\frac{3x}{2}-y-\frac{z}{2}\right)\begin{pmatrix}1\\1\\0\end{pmatrix} = T\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\begin{pmatrix}-x+2y+z\\0\\\frac{x}{2}-y-\frac{z}{2}\end{pmatrix}+\begin{pmatrix}\frac{3x}{2}-\frac{3z}{2}\\-x+z\\\frac{x}{2}-\frac{z}{2}\end{pmatrix}+\begin{pmatrix}\frac{3x}{2}-y-\frac{z}{2}\\\frac{3x}{2}-y-\frac{z}{2}\\0\end{pmatrix} = T\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\begin{pmatrix}\frac{4x+2y-2z}{2}\\\frac{x-2y+z}{2}\\\frac{2x-2y-2z}{2}\end{pmatrix} = \begin{pmatrix}2x+y-z\\\frac{x}{2}-y+\frac{z}{2}\\x-y-z\end{pmatrix} = T\begin{bmatrix}x\\y\\z\end{bmatrix} \ \textit{Solucion General}$$

b) Determine $T(2,1,0)$

$$\begin{pmatrix} 2x+y-z \\ \frac{x}{2}-y+\frac{z}{2} \\ x-y-z \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$$

c) Determine $T(2,-1,1)$

$$\begin{pmatrix} 2x+y-z \\ \frac{x}{2}-y+\frac{z}{2} \\ x-y-z \end{pmatrix} = \begin{pmatrix} 2 \\ 5/2 \\ 2 \end{pmatrix}$$

5. Dada la transformación lineal $T: R^2 \to R^3$ definida por:

$$T(1,1) = (2,0,-1), \;\; T(3,-1) = (2,-1,3)$$

a) Determine $T(x,y)$

$$B = \left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \end{pmatrix}\right\} \;\; \begin{bmatrix} x \\ y \end{bmatrix} = \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} det = -4$$

$$\frac{1}{-4}\begin{vmatrix} -1 & -3 \\ -1 & 1 \end{vmatrix} = \begin{vmatrix} 1/4 & 3/4 \\ 1/4 & -1/4 \end{vmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{vmatrix} \frac{x}{4}+\frac{3y}{4} \\ \frac{x}{4}-\frac{y}{4} \end{vmatrix}$$

$$\left(\frac{x}{4}+\frac{3y}{4}\right)\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \left(\frac{x}{4}-\frac{y}{4}\right)\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = T\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{pmatrix} \frac{x+3y}{2} \\ 0 \\ \frac{-x-3y}{4} \end{pmatrix} + \begin{pmatrix} \frac{x-y}{2} \\ \frac{-x+y}{4} \\ \frac{3x-3y}{4} \end{pmatrix} = T\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{pmatrix} \frac{2x-2y}{2} \\ \frac{-x+y}{4} \\ \frac{2x-6y}{4} \end{pmatrix} = \begin{pmatrix} x-y \\ -\frac{x}{4}+\frac{y}{4} \\ \frac{x}{2}-\frac{3y}{2} \end{pmatrix} = T\begin{bmatrix} x \\ y \\ z \end{bmatrix} \textit{Solucion General}$$

b) Determine $T(5,1)$

$$\begin{pmatrix} x-y \\ -\frac{x}{4}+\frac{y}{4} \\ \frac{x}{2}-\frac{3y}{2} \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

c) Determine $T(4,,1)$

$$\begin{pmatrix} x-y \\ -\frac{x}{4}+\frac{y}{4} \\ \frac{x}{2}-\frac{3y}{2} \end{pmatrix} = \begin{pmatrix} 3 \\ -3/4 \\ 1/2 \end{pmatrix}$$

6. Dada la transformación lineal $T: P_2 \rightarrow P_1$ definida por:

$$T(3+x+x^2) = x+5,\ \ T(1+x-2x^2) = -5x+1, \qquad T(-2+5x-3x^2) = -2x$$

d) Determine $T(a+bx+cx^2)$

$$B = \left\{ \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix} \right\} \ \begin{bmatrix} x \\ y \end{bmatrix} =$$

$$\begin{vmatrix} 3 & 1 & -2 \\ 1 & 1 & 5 \\ 1 & -2 & -3 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} F1\left(\frac{1}{3}\right) = F1 \\ F1(-1) + F2 = F2 \\ F1(-1) + F3 = F3 \end{matrix}$$

$$\begin{vmatrix} 1 & 1/3 & -2/3 \\ 0 & 2/3 & 17/3 \\ 0 & -7/3 & -7/3 \end{vmatrix} \begin{vmatrix} 1/3 & 0 & 0 \\ -1/3 & 1 & 0 \\ -1/3 & 0 & 1 \end{vmatrix} \begin{matrix} F2(3/2) = F2 \\ F2(-1/3) + F1 = F1 \\ F2(7/3) + F3 = F3 \end{matrix}$$

$$\begin{vmatrix} 1 & 0 & -7/2 \\ 0 & 1 & 17/2 \\ 0 & 0 & 35/2 \end{vmatrix} \begin{vmatrix} 1/2 & -1/2 & 0 \\ -1/2 & 3/2 & 0 \\ -3/2 & 7/2 & 1 \end{vmatrix} \begin{matrix} F3\left(\frac{2}{35}\right) = F3 \\ F3(7/2) + F1 = F1 \\ F3(-17/2) + F2 = F2 \end{matrix}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1/5 & 1/5 & 1/5 \\ 8/35 & -1/5 & -17/35 \\ -3/35 & 1/5 & 2/35 \end{vmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left(\frac{x}{5}+\frac{y}{5}+\frac{z}{5}\right)\binom{5}{1}+\left(\frac{8x}{35}-\frac{y}{5}-\frac{17z}{35}\right)\binom{1}{-5}+\left(-\frac{3x}{35}+\frac{y}{5}+\frac{2z}{35}\right)\binom{0}{-2}=T\begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{pmatrix}x+y+z\\ \frac{x}{5}+\frac{y}{5}+\frac{z}{5}\end{pmatrix}+\begin{pmatrix}\frac{8x}{35}-\frac{y}{5}-\frac{17z}{35}\\ -\frac{40x}{35}+y+\frac{85z}{35}\end{pmatrix}+\begin{pmatrix}0\\ \frac{6x}{35}-\frac{2y}{5}-\frac{4z}{35}\end{pmatrix}=T\begin{bmatrix}x\\y\end{bmatrix}$$

$$\begin{pmatrix}\frac{43x+28y+18z}{35}\\ \frac{-27x+28y+88z}{35}\end{pmatrix}=T\begin{bmatrix}x\\y\end{bmatrix}\ \mathit{Solucion\ General}$$

e) Determine $T(2x-1)$

$$\begin{pmatrix}\frac{43x+28y+18z}{35}\\ \frac{-27x+28y+88z}{35}\end{pmatrix}=\begin{pmatrix}\frac{13}{35}\\ \frac{83}{35}\end{pmatrix}=\left(\frac{83x}{35}+\frac{13}{35}\right)$$

f) Determine $T(9x^2)$

$$\begin{pmatrix}\frac{43x+28y+18z}{35}\\ \frac{-27x+28y+88z}{35}\end{pmatrix}=\begin{pmatrix}\frac{162}{35}\\ \frac{792}{35}\end{pmatrix}=\left(\frac{792x}{35}+\frac{162}{35}\right)$$

Puedes hacer uso de vectores para realizar las operaciones, pero escribe la respuesta final en términos de polinomios.