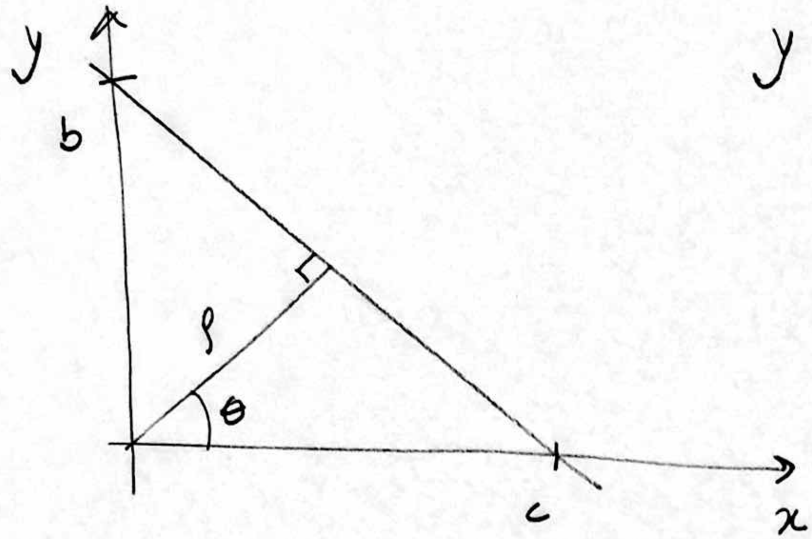

$$y = mx + b$$

$$m = \frac{\Delta y}{\Delta x} = \frac{-b}{c} = -\frac{\cos\theta}{\sin\theta}$$

$$b = \frac{\rho}{\sin\theta} \qquad c = \frac{\rho}{\cos\theta}$$

$$y = -\frac{\cos\theta}{\sin\theta}x + \frac{\rho}{\sin\theta}$$

$$\sin\theta\, y = -\cos\theta\, x + \rho$$

$$\boxed{\rho = x\cos\theta + y\sin\theta}$$

18.5.2.3 Hough Transform

The straight line $y = mx + b$ can be expressed in polar coordinates as [23]

$$\rho = x\cos(\theta) + y\sin(\theta) \tag{16}$$

where (ρ, θ) defines a vector from the origin to the nearest point on the line (Figure 18–18a). This vector will be perpendicular to the line.

We can consider a two-dimensional space defined by the two parameters ρ and θ. Any line in the x, y-plane plots to a point in that space. Thus, the Hough transform of a straight line in x, y-space is a point in ρ, θ space.

Now consider a particular point (x_1, y_1) in the x, y-plane. There are many straight lines that pass through this point, and each of these lines plots to a point in ρ,θ-space. These points, however, must satisfy Eq. (16) with x_1 and y_1 as constants. Thus, the locus of all such lines in x, y-space is a sinusoid in parameter space, and any point in the x, y-plane (Figure 18–18b) corresponds to a sinusoidal curve in ρ, θ space (Figure 18–18c).

If we have a set of edge points x_i, y_i that lie on a straight line having parameters ρ_0 and θ_0, then each edge point plots to a curve in ρ, θ space. However, all these curves must intersect at the point (ρ_0, θ_0), since this is a line they all have in common (Figure 18–18c).

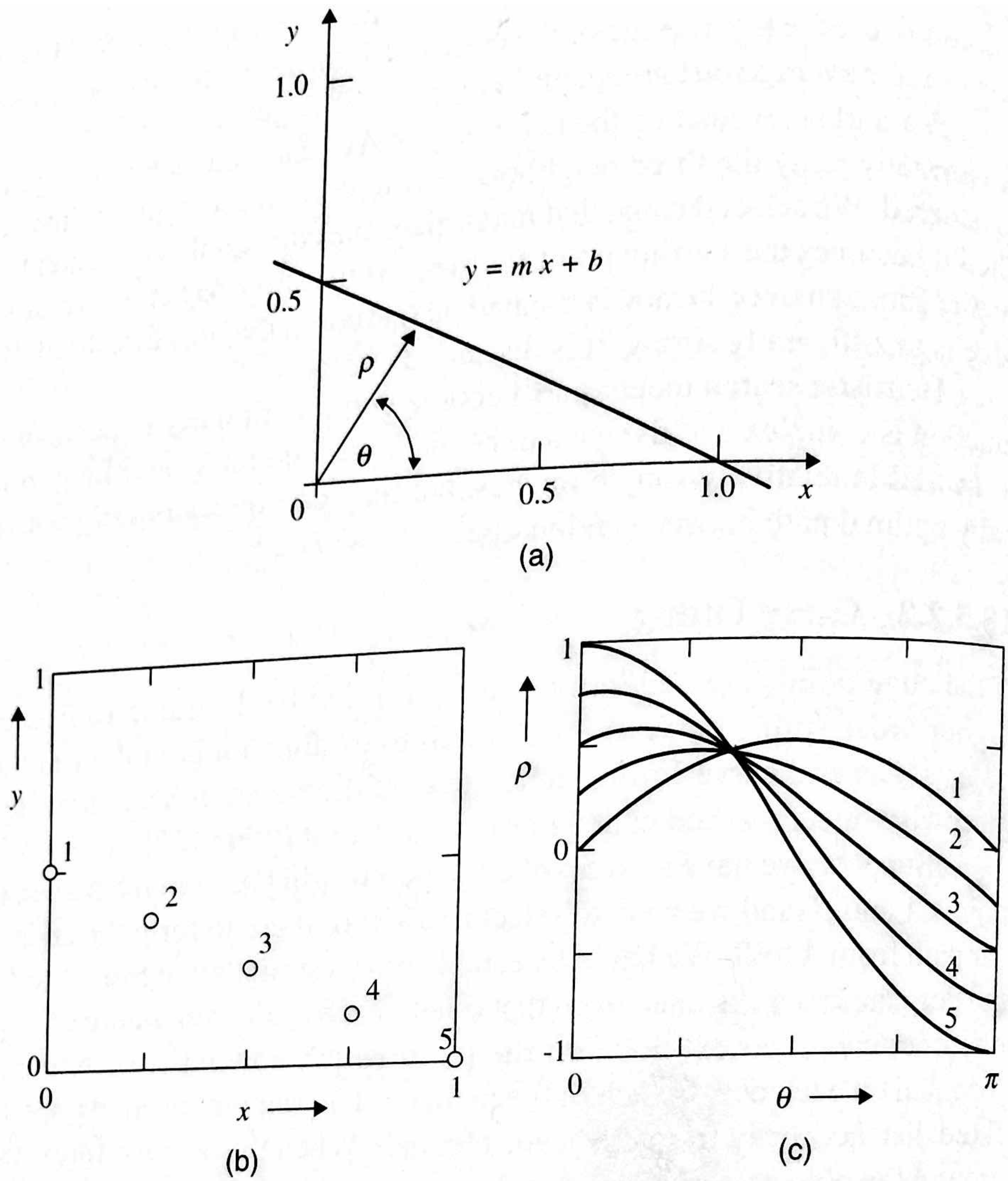

Figure 18–18 The Hough transform: (a) polar coordinate expression of a straight line; (b) x, y plane; (c) ρ, θ plane

Thus, to find the straight-line segment that the points fall upon, we can set up a two-dimensional histogram in ρ, θ space. For each edge point, (x_i, y_i), we increment all the histogram bins in ρ, θ space that correspond to the Hough transform (sinusoidal curve) for that point. When we have done this for all the edge points, the bin containing (ρ_0, θ_0) will be a local maximum. Thus, we search the ρ, θ space histogram for local maxima and obtain the parameters of linear boundary segments.