LAPLACIAN OF GAUSSIAN

$$f(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial f}{\partial x} = -\frac{x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad \frac{\partial f}{\partial y} = -\frac{y}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{1}{2\pi\sigma^4}\left[e^{-\frac{x^2+y^2}{2\sigma^2}} - x\frac{2x}{2\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \right]$$

$$= -\frac{1}{2\pi\sigma^4}\left[1 - \frac{x^2}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{1}{2\pi\sigma^4}\left[1 - \frac{y^2}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

h_{LoG}

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\boxed{\nabla^2 = -\frac{1}{2\pi\sigma^4}\left[2 - \frac{x^2+y^2}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}}$$