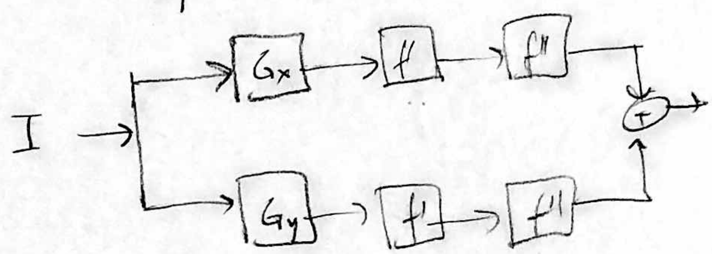
LAPLACIAN OF GAUSSIAN

$$f(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial f}{\partial x} = -\frac{1}{2\sigma^2} 2x\, e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad \frac{\partial f}{\partial y} = -\frac{1}{2\sigma^2} 2y\, e^{\frac{-x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{4\sigma^4} 4x^2 e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad \frac{\partial^2 f}{\partial y^2} = \frac{1}{4\sigma^4} 4y^2 e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} =$$



$$f(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2\pi\sigma^2} e^{-\frac{y^2}{2\sigma^2}} \frac{\partial}{\partial x}\left(e^{-x^2/2\sigma^2}\right)$$

$$= \frac{1}{2\pi\sigma^2} e^{\frac{-y^2}{2\sigma^2}} \underbrace{\frac{\partial}{\partial x}\left(\frac{-x^2}{2\sigma^2}\right)}_{\frac{-\cancel{2}x}{\cancel{2}\sigma^2}} e^{-x^2/2\sigma^2} = -\frac{x}{2\pi\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 f}{\partial x^2} = -\frac{1}{2\pi\sigma^4}\left[e^{-\frac{x^2+y^2}{2\sigma^2}} - x\frac{\cancel{2}x}{\cancel{2}\sigma^2} e^{\frac{-x^2+y^2}{2\sigma^2}}\right] = -\frac{1}{2\pi\sigma^4}\left[1 - \frac{x^2}{\sigma^2}\right] e^{\frac{-x^2+y^2}{2\sigma^2}}$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{1}{2\pi\sigma^4}\left[1 - \frac{y^2}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$\nabla^2 = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = -\frac{1}{2\pi\sigma^4}\left[2 - \frac{x^2+y^2}{\sigma^2}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$$